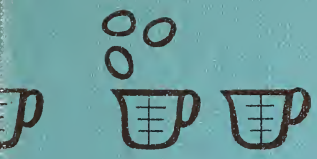

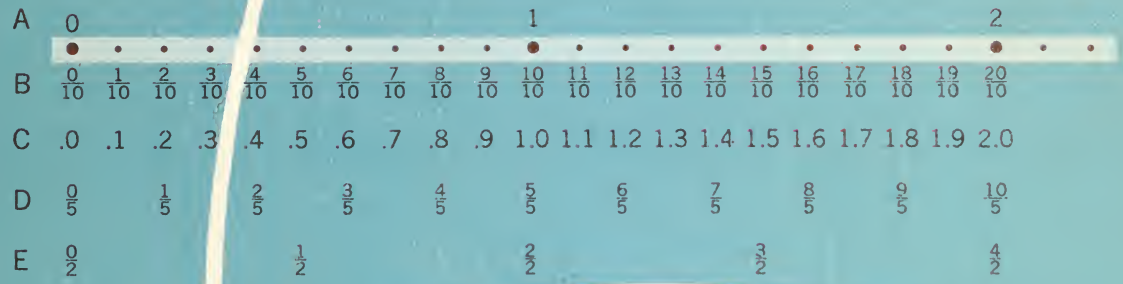

Seeing through arithmetic

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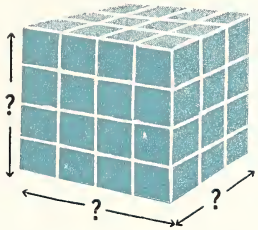

Seeing through arithmetic

6

by Maurice L. Hartung, Henry Van Engen,
Lois Knowles, and Robert A. Rennie

W. J. Gage Limited
Toronto

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Moving forward

In this lesson you will learn more about the shapes of familiar objects. You will also learn how different shapes can be shown in drawings.

A

B

B

C

D

A Are the rails of a railway track always the same distance apart, or parallel? Why did the artist draw the rails in Picture A as if they meet in the distance?

B Measure the picture of the pole marked A. Then measure the picture of the pole marked B. Would Pole B itself really be shorter than Pole A? Why?

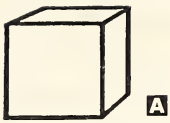
C Why did the artist draw the people who are waiting for the train at Y larger than those waiting at Z?

D When an artist draws a picture so as to suggest how things look, he is using perspective. Do you see any other examples of perspective in Picture A?

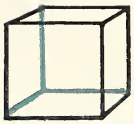
E Both Pictures B and C show a wooden block with square corners. Which picture better suggests the shape of the block? Which picture is drawn in perspective?

F Now look at the same block in Picture D. On the block itself, the edges marked o and p are parallel. Why are Lines o and p not parallel in the drawing?

G What does perspective do to the corners marked f and g?



A



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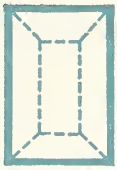


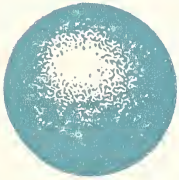
D

E

F

G

H



I

J

K

A Look at the block in Picture A. Can you see all of its sides? How many sides can you see? Is the block drawn in perspective?

B How is Picture B different from Picture A? The blue lines help you know that the block has ▒ sides in all.

C What does Picture C look like?

D Look at Picture D. What part of it is like Picture C? The dotted lines show that it is a block with 6 sides. Find the 6 sides.

E Now look at Picture E. The dotted lines show that this shape has ▒ sides.

F What does Picture F look like?

G How does the dotted line in Picture G make the shape look like a cone?

H Why is it impossible to tell whether Picture H shows a circle or a ball?

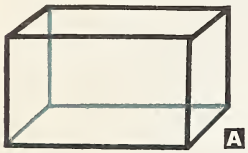
I How is Picture I different from Picture H? Does Picture I suggest the shape of a ball?

J What has the artist done to suggest that the shapes in Pictures J and K are rounded and not flat?

Now you know why pictures of objects are drawn in perspective. You also know how parts that you do not see may be suggested in drawings.

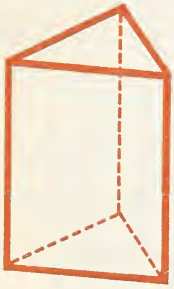
Moving forward

In this lesson you will learn the names of some important geometric shapes.



A

B



C

D

E

F

G

H

A Look at Picture A. This geometric shape has flat sides, or faces, that are rectangular. How many faces has it?

B Look at the shape in Picture B. Is it like the shape in Picture A? Why or why not?

C Look at the shape in Picture C. Three of its faces are rectangular. How many faces are triangular?

D Shape D has ▭ rectangular faces. It has how many faces that are not rectangles?

E Does Shape E appear to have any rectangular faces? How many faces are circular?

F How is Shape F like Shape E? How is it different?

G Shape G has 4 flat faces. What shape are they?

H Look at Shape H. How is it different from any other shape on this page?

I Now turn to page 3 and find as many objects as you can that are like the shapes in Pictures A and B on this page.

J What objects on page 3 are like Picture E? Picture F? Picture H?

A

straight lines
parallel lines

B

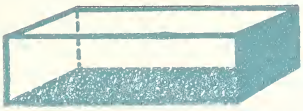
curved lines

C

right angle
perpendicular lines

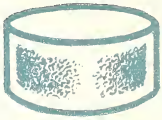
D

plane surfaces
parallel planes



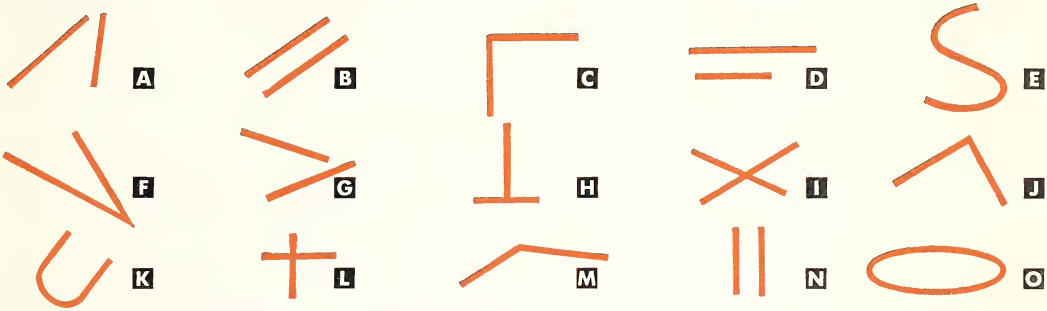
E

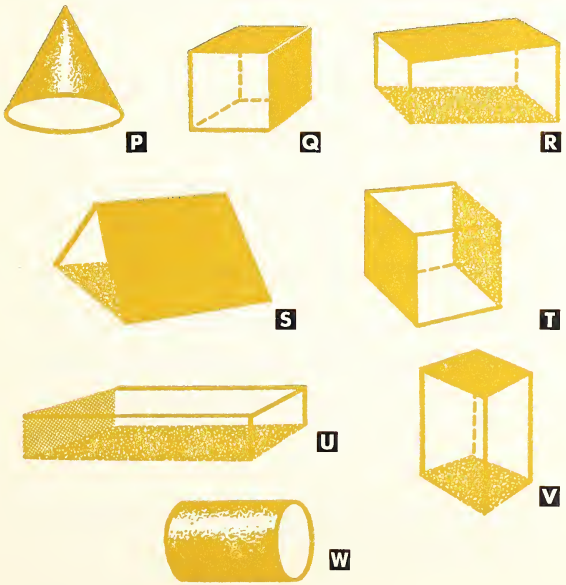
right angles
perpendicular planes



F

curved surface





A Find each set of red parallel lines in Pictures A to O above.

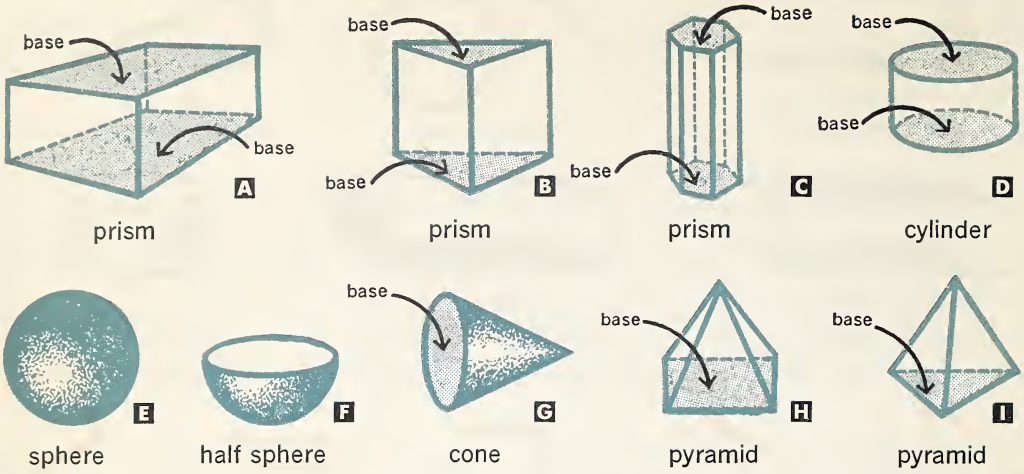
B Find each set of red perpendicular lines.

C Find each red curved line.

D Find each set of yellow parallel planes in Pictures P to W at the left.

E Find each set of yellow perpendicular planes.

F Find each yellow curved surface.



A The geometric shapes in Pictures A, B, and C are called prisms. How are these prisms alike?

B Prisms have plane sides, or faces. How many plane sides, or faces, in all has Prism A?

C The bases of Prism A are rectangular in shape. They are also parallel and of the same size. What shape are the other faces of Prism A?

D What shape are the bases of Prism B? Has Prism B any rectangular faces? How many has it?

E What shape are the bases of Prism C? How many rectangular faces has Prism C?

F The bases of Cylinder D are in the shape of circles. Have the circles equal diameters? Do the bases of Cylinder D appear to be parallel?

G Cylinder D has how many plane sides? How many curved sides?

H Has Sphere E a base? Has it any plane sides?

I How is the half sphere in Picture F different from Cylinder D?

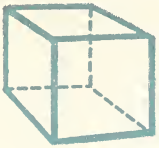
J What shape is the base of Cone G? How many plane sides has Cone G? How many curved sides has it?

K The base of Pyramid H is square. What shape is each of the other faces?

L How is Pyramid I different from Pyramid H?

M In what ways is Pyramid H different from Prism B?

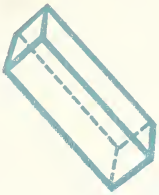
A


B

C

D

E


F

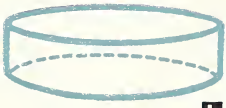
G

H

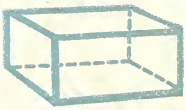
I

J

K


L

M


N

A Name the shape in each picture above. The names below will help you.

prism	sphere
cylinder	pyramid
cone	half sphere

B One of the pictures shows a prism with square faces that are the same size. This prism is a cube. It is also a rectangular prism. Which picture shows a cube?

C Which pictures show geometric shapes that have polygons as bases?

D Which pictures show shapes with circular bases?

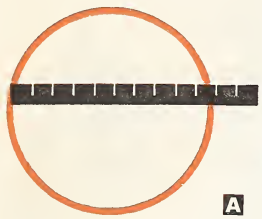
E Which shapes have only plane surfaces? Which shape has no plane surfaces?

F Which shapes have both plane and curved surfaces?

Now you should be able to recognize and name some important geometric shapes.

Moving forward

Now you will study the important dimensions of different geometric shapes.



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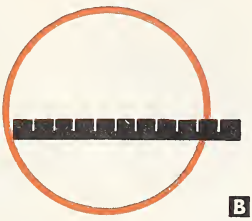

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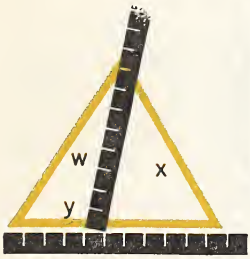

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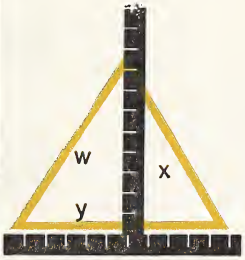


D

E F

A Suppose you want to measure the diameter of a circle. Which picture shows the correct way of measuring a diameter? Why?

B Which picture shows the correct way to measure the base and the altitude of a triangle?

C Which line is being used as the base of the triangle in Pictures C and D?

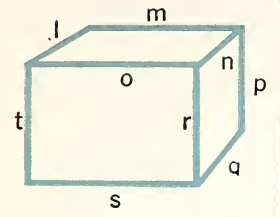
D How would you draw a line to represent the altitude of the triangle? Should the line be perpendicular to the base?

E Which picture, Picture E or Picture F, shows the correct way to measure the height of a cylinder? Why do you think so?

F Is the height of a cylinder measured at right angles to the base?

G Where would you put your ruler to measure the diameter of this cylinder? Why?

Recognizing the important dimensions of various geometric shapes; correct and incorrect ways of measuring



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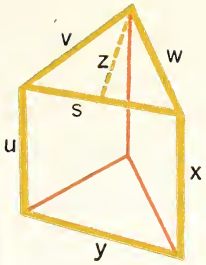

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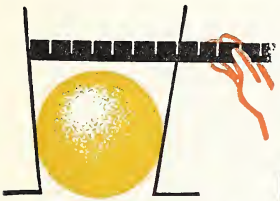
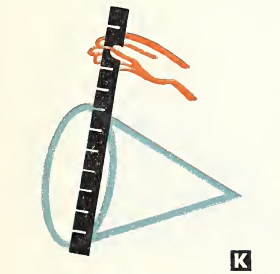


I

J



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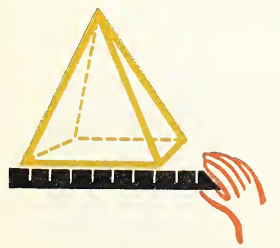
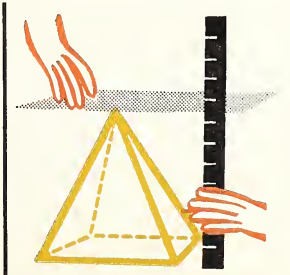
L



M



N

H Picture G shows a rectangular prism. How many dimensions do you need to measure before you can describe the size of this prism?

I Which edges of the prism would you measure? Why?

J How is Prism H different from Prism G? How is it like Prism G?

K Prism H has triangular bases. If you think of Line S as the base of a triangle, which line represents the altitude of the triangle?

L On which line would you measure the height of Prism H?

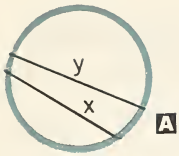
M The diameter of a sphere passes through the centre of the sphere. Which picture, I or J, shows a correct way to find the approximate diameter of a sphere? Why do you think so?

N What dimension of the cone is being measured in Picture K?

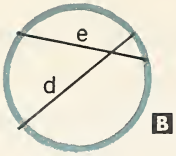
O Picture L shows a way to find the height of a cone. Why must the sheet of cardboard be held parallel to the table top?

P The base of the pyramid in Picture M is a square. How would you measure the base?

Q What dimension of the pyramid is being measured in Picture N?



A



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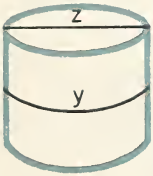

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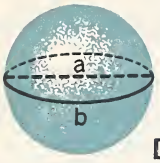

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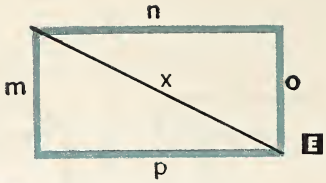

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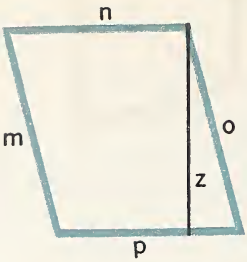

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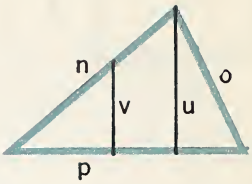
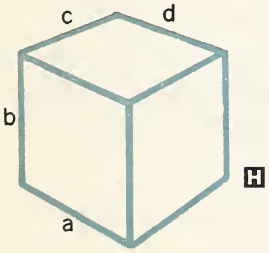


G

A For each circle, cylinder, or sphere above, which line represents a diameter?

B For each polygon above, if line p represents the base, which line represents the altitude?



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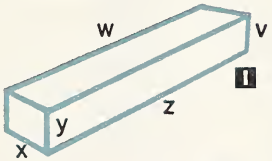
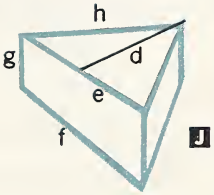


I



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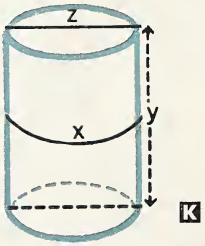
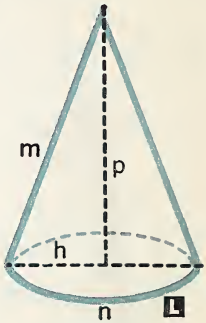


K



L

C Look at Pictures H to L. Which shapes have circular bases? For each one, which line shows the diameter?

D Which shapes have rectangular bases? For each one, which lines show the dimensions of the bases?

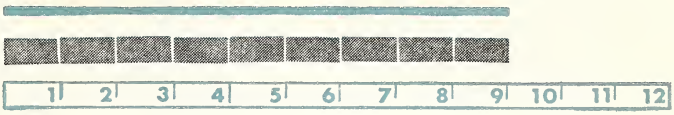
E One shape has triangular bases. Which one is it? Which lines show the base and altitude of a triangle?

F Now for each shape in Pictures H to L, name a line that represents the height.

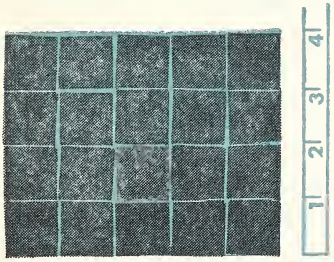
Now you should be able to recognize the important dimensions of the geometric shapes you have studied.

Moving forward

In this lesson you will learn about the measurement of area and volume.



A



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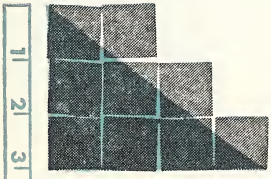

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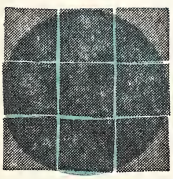


D

A How many strips does it take to measure the length of the blue stick in Picture A?

B Each strip represents 1 inch. The stick is ▒ inches long.

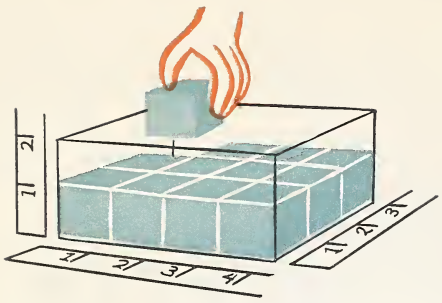
C How many squares does it take to cover the rectangle in Picture B?

D Each square represents a 1-inch square. It takes ▒ 1-inch squares to cover the rectangle.

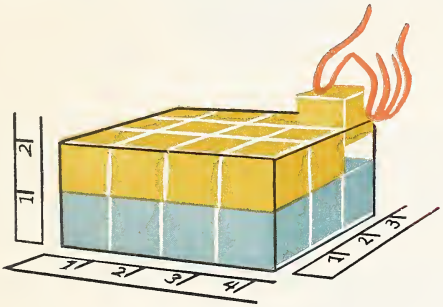
E Each 1-inch square is 1 square inch. The area of the rectangle shown in Picture B is ▒ square inches.

F Which do you think has the larger area, the triangle in Picture C or the circle in Picture D? Why do you think so?

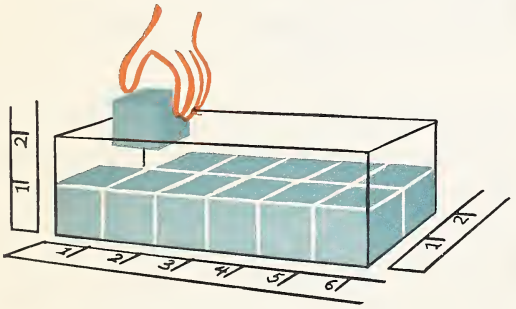
G Does an area of 1 square inch always have the shape of a 1-inch square? Explain your answer.



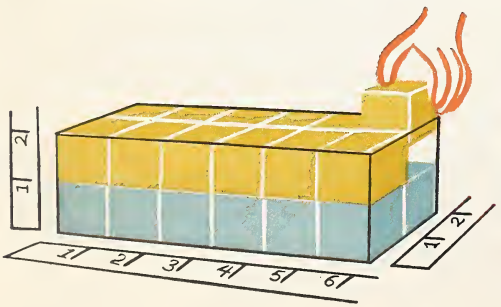
E



F



G



H

H The box in Picture E is what geometric shape?

I The box is being filled with cubes. Each cube is 1 in. long, 1 in. wide, and 1 in. high. How many of these cubes does it take to make 1 layer on the bottom of the box?

J Now look at Picture F. Will there be the same number of cubes in the second layer as in the first layer? How many cubes does the box hold?

K The volume of this box is the number of cubes the box can hold. The volume is ▒ cubes.

L Now look at Picture G. To find the volume of this box, you can fill it with 1-inch cubes. How many cubes will be in the first layer?

M Look at Picture H. How many cubes will be in the second layer? The volume of this box is ▒ cubes.

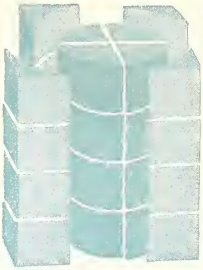
N Is the volume of this box greater than, less than, or the same as the volume of the box in Picture F?

O Have the boxes in Pictures E and G the same dimensions? Are their volumes equal?

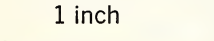
P Why is the cube a good shape to use for measuring volume?

I

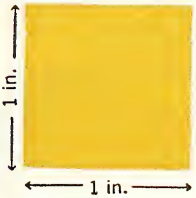
J



K

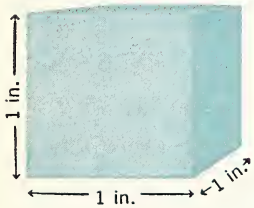


L



M

1 square inch
1 sq. in.



N

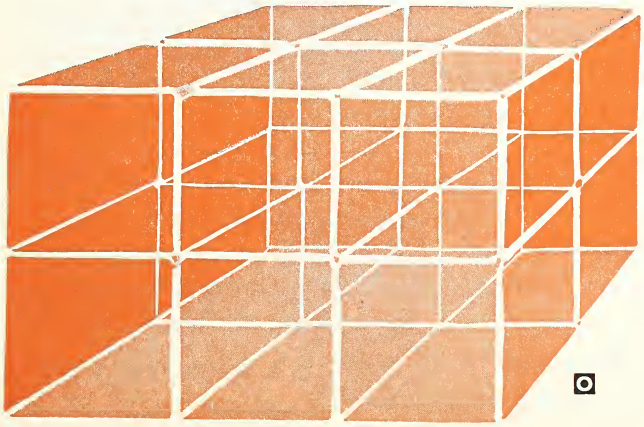
1 cubic inch
1 cu. in.

Q Look at Picture I. Why would it be hard to fill the cylinder completely with cubes?

R Picture J shows a pile of cubes as high as Cylinder I and as wide as the diameter of the cylinder. What does Picture K show? Why is it hard to know how many cubes could be made from the cylinder in Picture K?

S Do you think that you could make at least two cubes from each layer? How many layers are there?

T You can say that the volume of the cylinder is between 8 cubes and 16 cubes. Why?

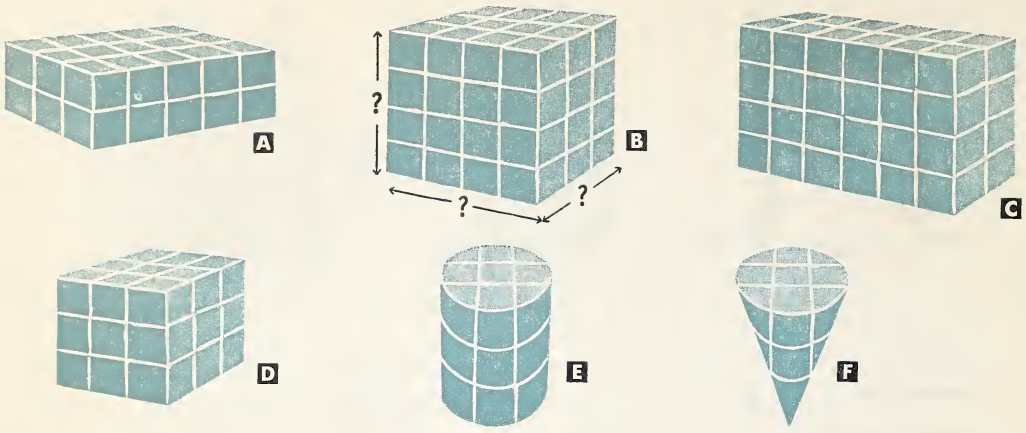


O

U What are the dimensions of the square in Picture M?

V What are the dimensions of the cube in Picture N? Is a 1-inch cube equal to 1 cubic inch?

W Prism O is made of 1-inch cubes. What are the dimensions of Prism O? What is its volume in cubic inches?



A Each shape shown above has been made of cubes, or parts of cubes. Imagine that the cubes are all 1-inch cubes. Is the volume of Prism A 15 cubic inches or 30 cubic inches? How do you know?

B How many cubes are in the top layer of Prism B? How many layers of cubes are there? Has each layer the same number of cubes? How many cubes in all are in Prism B?

C What are the dimensions of Prism B? The volume of Prism B is ▤ cu. in.

D How can you find the number of cubes in Prism C?

E What are the dimensions of Prism C? Its volume is ▤ cu. in.

F Is the volume of Prism C greater than, less than, or the same as the volume of Prism B?

G Is Prism D a cube? How do you know? Are Prisms A, B, and C also cubes?

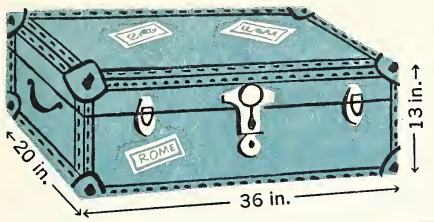
H The volume of Cube D is ▤ cu. in.

I Cylinder E has the same height as the cube in Picture D. Its diameter is 3 in. Do you think that the volume of Cylinder E is greater than, less than, or the same as the volume of Cube D? Why do you think so?

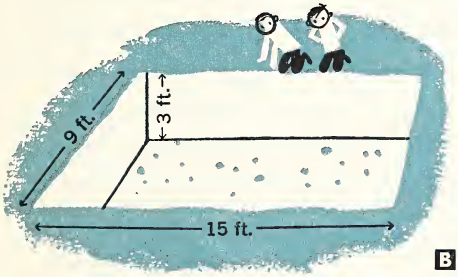
J Cone F has the same diameter and the same height as Cylinder E. Is its volume greater than, less than, or the same as the volume of Cylinder E?

Now you should know what is meant by the volume of a prism or cylinder or of other solid shapes.

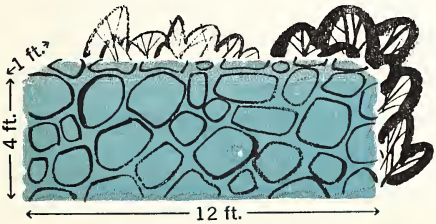
Thinking straight



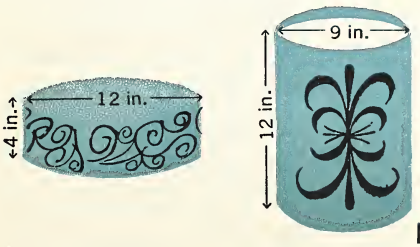
A



B



C



D

A Ruth said that the trunk in Picture A is 36 in. long, 20 in. deep, and 13 in. high. Carol said it is 36 in. wide, 20 in. long, and 13 in. deep. Which girl was right? Why do you think so?

B Would you say that the trunk has the shape of a rectangular prism? Why?

C The hole in the ground in Picture B has the shape of a prism. What kind of prism is it? Why?

D You can describe the hole as being 15 ft. by 9 ft. by 3 ft., which is often written like this: 15 ft. x 9 ft. x 3 ft. Could you also describe the hole as 15 ft. long, 9 ft. wide, and 3 ft. deep? Why?

E Would you say the hole is 3 ft. high? Why or why not?

F Describe the size of the stone wall in Picture C. For which dimension could you use the word "thick"?

G The wall is ▒ ft. x ▒ ft. x ▒ ft.

H How would you describe the size of the box in Picture D? Why? How would you describe the size of the wastebasket?

I Henry said the altitude of a certain triangle was 15 inches. John said the height of the same triangle was 15 inches. Do you think both boys were talking about the same dimension? Why?

Checking up

Test 1

Find the sum.

A 15, 38, 26, 19, 47
B 274, 501, 386, 630
C 432, 195, 548, 313
D 169, 74, 280, 455, 729
E 6318, 2467, 992
F 7526, 4008, 6276
G 5109, 1378, 4629, 3260
H 8321, 796, 3554, 5437

Test 2

A $473 - 228 = n$
B $116 - 67 = n$
C $705 - 649 = n$
D $500 - 206 = n$
E $3499 - 1765 = n$
F $6002 - 3850 = n$
G $27134 - 8693 = n$
H $94500 - 37298 = n$
I $85623 - 49795 = n$

Test 3

A $6 \times 48 = n$
B $4 \times 73 = n$
C $86 \times 5 = n$
D $3 \times 281 = n$
E $7 \times 420 = n$
F $9 \times 194 = n$
G $632 \times 8 = n$
H $2 \times 5069 = n$
I $4 \times 3780 = n$

Test 4

A $52 \times 43 = n$
B $18 \times 67 = n$
C $77 \times 77 = n$
D $48 \times 329 = n$
E $25 \times 630 = n$
F $545 \times 15 = n$
G $64 \times 78 = n$
H $102 \times 75 = n$
I $839 \times 90 = n$

Test 5

A $117 \times 240 = n$
B $336 \times 481 = n$
C $509 \times 275 = n$
D $628 \times 742 = n$
E $493 \times 207 = n$
F $806 \times 509 = n$
G $565 \times 738 = n$
H $684 \times 420 = n$
I $729 \times 587 = n$

Test 6

A $79 \div 2 = n$
B $252 \div 6 = n$
C $145 \div 5 = n$
D $83 \div 4 = n$
E $376 \div 8 = n$
F $109 \div 3 = n$
G $2481 \div 7 = n$
H $6102 \div 9 = n$
I $1537 \div 5 = n$

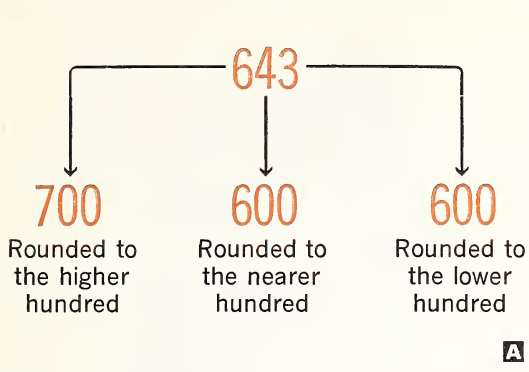
Test 7

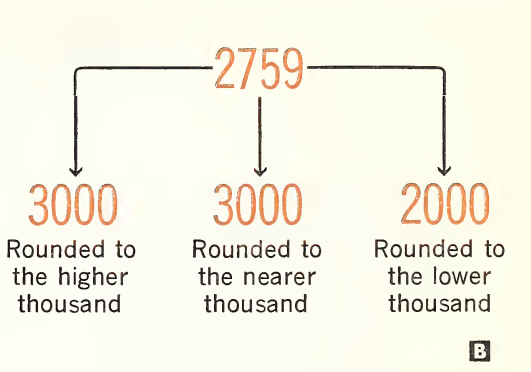
A $n + 326 = 700$
B $561 - n = 279$
C $n - 608 = 1327$
D $1463 + n = 3088$
E $n - 2729 = 4306$
F $n + 816 = 1002$
G $7205 + n = 19134$
H $9365 - n = 5220$
I $n - 6994 = 8379$

Test 8

A 4 ft. = ▒ in.
B 30 pt. = ▒ qt.
C 9 lb. = ▒ oz.
D 91 da. = ▒ wk.
E 72 ft. = ▒ yd.
F 10 bu. = ▒ pk.
G 3 c. = ▒ fl. oz.
H 8 T. = ▒ lb.
I 7 hr. = ▒ min.
J 16 gal. = ▒ pk.
K 5 yr. = ▒ mo.
L 48 qt. = ▒ gal.
M 6 doz. = ▒ things
N 18 tbsp. = ▒ fl. oz.
O 4 yd. = ▒ in.
P 40 qt. = ▒ pk.
Q 6 da. = ▒ hr.
R 3 mi. = ▒ ft.

Thinking straight





Round each of the following numbers to the higher hundred, the nearer hundred, and the lower hundred. For Example A you would have 2600, 2600, and 2500.

A	2573	E	885	I	7024
B	467	F	138	J	962
C	312	G	4550	K	533
D	1108	H	941	L	6247

Round each of the following numbers to the higher thousand, the nearer thousand, and the lower thousand.

M	3180	Q	12578	U	19738
N	5674	R	30294	V	41629
O	9302	S	71889	W	86940
P	9836	T	92115	X	20070

A When 224 is divided by 32, the answer is 7. Now divide both 224 and 32 by 4. Next divide 56 by 8. Is the answer the same as when 224 is divided by 32?

B $240 \div 10 = 24$, and $30 \div 10 = 3$. Now divide 24 by 3. Why has $24 \div 3$ the same answer as $240 \div 30$?

C The short-cut way to divide 240 by 30 is first to divide both 240 and 30 by 10. $240 \div 30 = 24\not{0} \div 3\not{0}$. The answer is ▒.

D The short-cut way to divide 35000 by 700 is first to divide each number by 100. $35000 \div 700 = 350\not{0}\not{0} \div 7\not{0}\not{0}$. The answer is ▒.

Use the short cut and then divide.

E	$560 \div 80 = n$	K	$2000 \div 400 = n$
F	$2100 \div 30 = n$	L	$4800 \div 600 = n$
G	$810 \div 90 = n$	M	$63000 \div 900 = n$
H	$3600 \div 40 = n$	N	$42000 \div 700 = n$
I	$4000 \div 50 = n$	O	$36000 \div 600 = n$
J	$720 \div 80 = n$	P	$8400 \div 200 = n$

Learning how

This lesson will help you remember how to divide by numbers that have two or three figures.

see

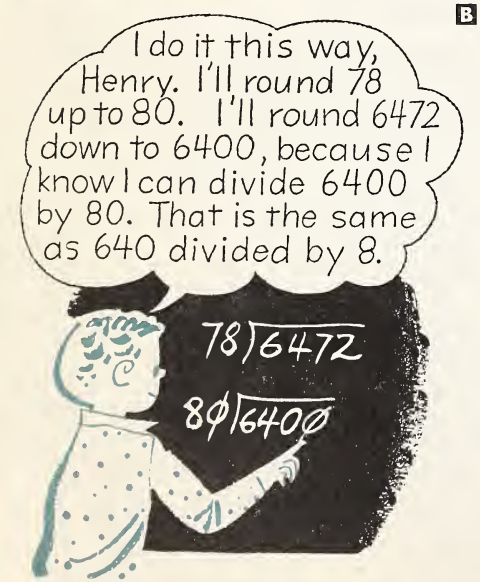
A

Henry made this equation for a problem: $6472 \div 78 = n$. To find the numeral that replaces n, Henry had to divide. When he finished dividing, his work looked like this.

```
78)6472 | 50
   3900 |
   2572 | 30
   2340 |
    232 |  2
    156 |
     76 | 82
```

3900 ← 50 × 78 = 3900. Henry wrote 3900 here so that he could subtract.

B


I do it this way, Henry. I'll round 78 up to 80. I'll round 6472 down to 6400, because I know I can divide 6400 by 80. That is the same as 640 divided by 8.
78)6472
8ø)640ø

6472 ÷ 78 = 82 and ▒ remainder

Bill's work looked like this.

8ø)640ø ← Bill chose 6400 because the 64 in 6400 can be divided by the 8 in 80.

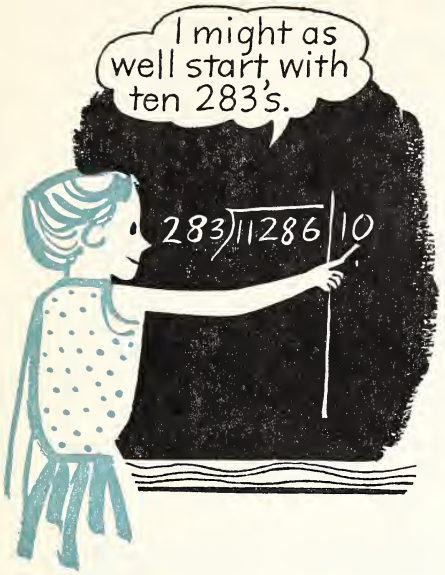
```
78)6472 | 80
   6240 |
    232 |  2
    156 |
     76 | 82
```

6240 ← 80 × 78 = 6240

2 ← Think of 16ø ÷ 8ø. 2 × 78 = 156

see

A

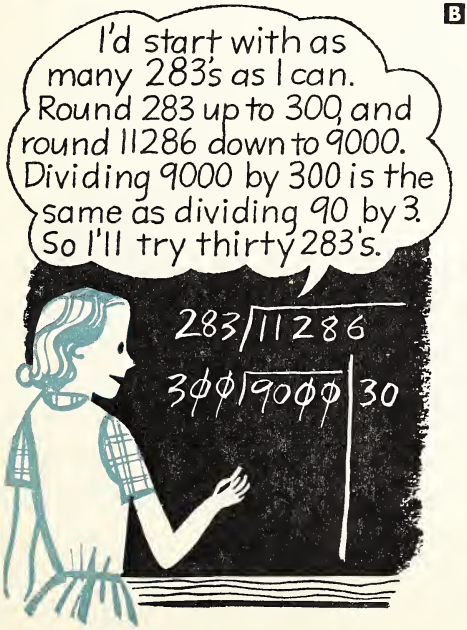


Ruth made this equation for a problem: $11286 \div 283 = n$. Her work looked like this when she finished dividing.

```
283)11286 | 10
    2830  |
    8456  | 20
    5660  |
    2796  |  5
    1415  |
    1381  |  4
    1132  |
     249  | 39
```

11286 ÷ 283 = 39 and ▒ remainder

B


I'd start with as many 283's as I can. Round 283 up to 300, and round 11286 down to 9000. Dividing 9000 by 300 is the same as dividing 90 by 3. So I'll try thirty 283's.
283)11286
3ØØ)90ØØ | 30

Sally's work looked like this.

Sally chose 9000 because the 9 in 9000 can be divided by the 3 in 300.

```
3ØØ)90ØØ
```

```
283)11286 | 30   ← 30 × 283 = 8490
    8490  |
    2796  |  9   ← Think of 27ØØ ÷ 3ØØ. Try 9.
    2547  |
     249  | 39   ← Is Sally's answer the same as Ruth's answer?
```

think $7843 \div 82 = n$. Look below at the two ways of working this example. The second way is no shorter than the first way, but the second way does help you decide what numbers to use to find the answer.

First way

```
82)7843 | 50   ← Why can you use 50 here?
 4100   |
 3743   | 40
 3280   |
  463   |  5
  410   |
   53   | 95   ← How do you get 95?
```

↑ The remainder is ▒.

Second way

↓ Round the divisor up to 90.

82)7843 ← Why do you then round 7843 down to 7200? $720\emptyset \div 9\emptyset =$ ▒

```
82)7843 | 80   ← Why can you try 80 groups of 82?
 6560   |
 1283   | 10   ← Think: 90∅ ÷ 9∅.
  820   |
  463   |  5   ← Why can you try 5 groups of 82?
  410   |
   53   | 95
```

try A $34020 \div 486 = n$

Round 486 up to 500.

Round 34020 down to 30000.

```
486)34020 | 10
 4860     |
29160     | 20
 9720     |
19440     | 20
 9720     |
 9720     | 20
 9720     |
          | 70
```

```
486)34020 | 30
14580     |
19440     | 30
14580     |
 4860     | 10
 4860     |
          | 70
```

```
486)34020 | 60
29160     |
 4860     | 10
 4860     |
          | 70
```

```
486)34020 | 70
34020     |
```

B $50000 \div 78 = n$

78	50000	200	78	50000	300	78	50000	500	78	50000	600
	15600			23400			39000			46800	
	34400	200		26600	300		11000	100		3200	40
	15600			23400			7800			3120	
	18800	200		3200	40		3200	20		80	1
	15600			3120			1560			78	
	3200	40		80	1		1640	20		2	641
	3120			78			1560				
	80	1		2	641		80	1			
	78						78				
	2	641					2	641			

C $14472 \div 536 = n$

536	14472	10	536	14472	20	536	14472	20	536	14472	20
	5360			10720			10720			10720	
	9112	10		3752	5		3752	6		3752	7
	5360			2680			3216			3752	
	3752	7		1072	2		536	1			27
	3752			1072			536				
		27			27			27			

---

do

A $8306 \div 91 = n$

B $1759 \div 28 = n$

C $3000 \div 56 = n$

D $645 \div 18 = n$

E $2979 \div 32 = n$

F $5210 \div 65 = n$

G $4631 \div 87 = n$

H $8680 \div 70 = n$

I $16300 \div 43 = n$

J $12192 \div 24 = n$

K $47008 \div 67 = n$

L $25994 \div 525 = n$

M $73125 \div 806 = n$

N $39243 \div 419 = n$

O $52041 \div 738 = n$

P $43655 \div 900 = n$

Q $26471 \div 379 = n$

R $198005 \div 600 = n$

S $26799 \div 240 = n$

T $726854 \div 858 = n$

U $315276 \div 927 = n$

Now you should be able to divide when the divisor has two or three figures.

Checking up

Test 1

A $849 \div 21 = n$
B $637 \div 14 = n$
C $1062 \div 18 = n$
D $2655 \div 40 = n$
E $906 \div 37 = n$
F $4822 \div 75 = n$
G $6000 \div 82 = n$
H $5124 \div 28 = n$
I $7909 \div 60 = n$
J $17192 \div 56 = n$
K $13609 \div 31 = n$
L $37430 \div 89 = n$
M $59867 \div 93 = n$
N $60021 \div 67 = n$

Test 2

A $9260 \div 400 = n$
B $7459 \div 135 = n$
C $15038 \div 206 = n$
D $28910 \div 578 = n$
E $47600 \div 700 = n$
F $35512 \div 386 = n$
G $203692 \div 849 = n$

Keeping skilful

A $\$8.40 \div \$.36 = n$

Divide just as you do with any numbers.

$$\begin{array}{r|l} \$.36\overline{)\$8.40} & 20 \\ 7\,20 & \\ \hline 1\,20 & 3 \\ 1\,08 & \\ \hline 12 & 23 \end{array}$$

How do you know that this 23 does not mean $.23?

This 12 means $.12.

The answer is 23 and $.12 remainder.

B $\$6.80 \div n = 15$

$$\begin{array}{r|l} 15\overline{)\$6.80} & 40 \\ 6\,00 & \\ \hline 80 & 5 \\ 75 & \\ \hline 5 & 45 \end{array}$$

This 45 means $.45. How do you know?

What does the 5 mean?

The answer is $.45 and $.05 remainder.

C $\$6.00 \div n = 8$
D $\$1.89 \div \$.25 = n$
E $\$.73 \div \$.04 = n$
F $\$8.84 \div n = 17$
G $\$2.58 \div n = 6$
H $\$5.00 \div \$.38 = n$
I $\$3.15 \div n = 35$
J $\$7.00 \div \$.60 = n$
K $\$10.00 \div n = 25$
L $\$12.50 \div n = 33$
M $\$6.27 \div \$.09 = n$
N $\$14.30 \div n = 5$
O $\$30.10 \div \$.24 = n$
P $\$23.25 \div n = 12$
Q $\$12.75 \div \$.70 = n$
R $\$50.30 \div n = 14$

A $2 \times \$22.75 = n$
B $17 \times \$.63 = n$
C $27 \times \$.08 = n$
D $5 \times \$12.67 = n$
E $56 \times \$.24 = n$
F $30 \times \$7.46 = n$
G $19 \times \$2.79 = n$
H $8 \times \$73.09 = n$

Checking up

A Kathy practises on the piano for 35 minutes a day. How many minutes does she practise in a week?

B This September there are 931 pupils at the Castle School. Last September the Castle School had 863 pupils. This September the school has how many more pupils than it had last September?

C Thirty-two sixth graders bought a plant for their room and shared the cost equally. The plant cost $3.84. How much did each sixth grader pay?

D The sixth graders were decorating the gym for a party. How many 24-ft. lengths of crepe paper ribbon could they get from a 600-ft. roll of ribbon?

E After Philip had spent $3.98 of his birthday money to buy baseball shoes, he had $2.52 left. How much money did Philip get for his birthday?

F One month the boys in Philip's ball club spent $5.07 for baseballs, $3.89 for a catcher's mask, and $3.78 for bats. What was the total cost of this equipment?

G There was $25 in the club treasury before the boys bought some equipment that cost $19.49. How much money was left in the treasury after the equipment was bought?

H The Castle School has 6 new bicycle racks. Each rack holds 54 bicycles. There is room for how many bicycles in the 6 new racks?

I John is saving money to buy a basketball. On Saturday he earned 65¢. Then he had $2.39. How much money did he have before he earned the 65¢?

J The parakeet Ann wants to buy costs $6.95. Ann has $3.47. How much more money does Ann need to buy the parakeet?

K Katherine bought 200 Mexican stamps. She traded some of them for Canadian stamps. Then she had 163 Mexican stamps. How many Mexican stamps had she traded?

L Ruth and three other girls bought a package of 1000 stamps and shared them equally. How many stamps did each girl get?

M Henry bought a package of Danish stamps for $1.30 and a package of Latin American stamps for 75¢. The Latin American stamps cost how much less than the Danish stamps?

N After Henry had traded 46 of his Danish stamps, he had 154 Danish stamps left. He had how many Danish stamps to begin with?

Exploring problems

see

A Peggy and her mother canned 26 pints of peaches. They also canned 19 quarts of peaches. They canned how many quarts of peaches in all?

Think of the pint jars and quart jars as one group of quarts.

B

Number of quarts in 26 pints

Number of quart jars

$\square + 19 = n$

Total number of quarts

First you must find how many quarts are equal to 26 pints.

compute

C

$26 \div 2$. There are 2 pt. in 1 qt.

$\square + 19 = n$

Think of the 26 pints in groups of 2. Divide 26 by 2. The 26 pints of peaches are equal to 13 quarts.

D

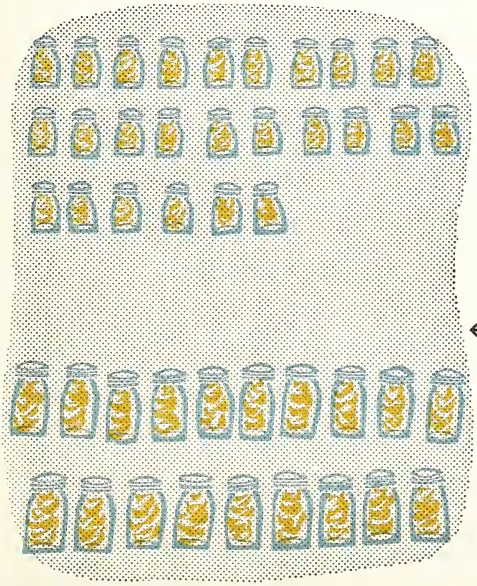
Number of quarts in 26 pints

Number of quart jars

Total number of quarts

$13 + 19 = n$

Now you can find the number that n stands for.



Think of the pints and quarts as one group of quarts.

Add 19 to 13. Peggy and her mother canned 32 quarts of peaches.

see

A

Peggy and her mother made 12 glasses of jelly on Monday, 20 glasses of jelly on Tuesday, and 10 glasses of jelly on Wednesday. They made an average of how many glasses of jelly per day?

Think of all the glasses as one group. Then imagine that the glasses are to be divided into three equal groups, one for each day.

B

Total number of glasses

Number of glasses in each group

$\square \div n = 3$

Number of groups

First you must find how many glasses of jelly in all they made.

compute

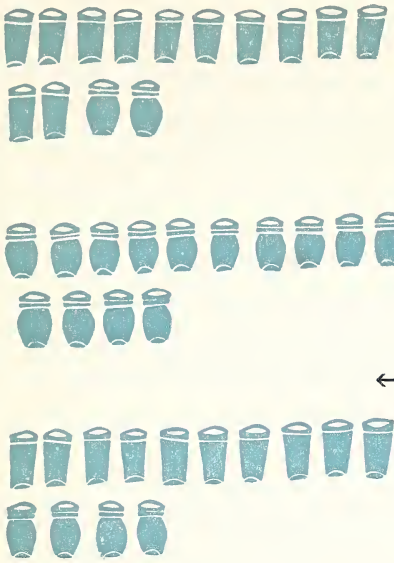
Find the total number of glasses of jelly that Peggy and her mother made.

C

$12 + 20 + 10$. Total number of glasses

$\square \div n = 3$

The sum is 42.
Put 42 in the equation.
$42 \div n = 3$



D

Total number of glasses

Number of glasses made per day

Number of days

$42 \div n = 3$ Now you can find the number that n stands for.

Think of the 42 glasses as divided into 3 equal groups. How many glasses will there be in each group?

Divide 42 by 3. The average number of glasses of jelly made per day was 14.

A Peggy's mother bought 2 bushels of peaches for $2.75 a bushel and $\frac{1}{2}$ bushel of plums for $3.39. How much did she pay for all this fruit?

Amount paid for peaches.

Amount paid for plums.

$\square + \$3.39 = n$ Total amount paid.

$(2 \times \$2.75) + \$3.39 = n$

First find how much Peggy's mother paid for the peaches. To do this, multiply $2.75 by 2. Put $5.50 in the equation.

$\$5.50 + \$3.39 = n$

Peggy's mother paid $\square$ for the fruit.

B Henry was paid $3.45 for washing 17 large screens and 9 small screens. He was paid an average of how much per screen?

Amount paid for all the screens.

Amount paid per screen.

$\$3.45 \div n = (17 + 9)$

Number of screens.

First you must find how many screens Henry washed. To do this, add 9 to 17. Put 26 in the equation.

$\$3.45 \div n = 26$

Is 13¢ or 14¢ the better answer? Why?

C One Saturday George raked leaves for 3 hr. and also walked a neighbor's dog. He was paid 35¢ for walking the dog and 50¢ an hour for raking leaves. How much did George earn that day?

THINK $\$.35 + (3 \times \$.50) = n$
Always do the work in the parentheses first.

D Alice practised on the piano for 20 min. on Monday, 35 min. on Tuesday, and 15 min. on Wednesday. Her sister Ruth practised 40 min. a day on these three days. Ruth practised how much longer than Alice?

THINK $(3 \times 40) - (20 + 35 + 15) = n$
What should you do first? What should you do next?

E Mrs. Bell spent $46.40 for groceries the first week of September, $30.72 the second week, $43.08 the third week, and $39.24 the fourth week. She spent an average of how much per week for groceries in September?

THINK $(\$46.40 + \$30.72 + \$43.08 + \$39.24) \div n = 4$

F Mrs. Bell used 3 doz. eggs in 6 days. She used an average of how many eggs per day?
$(3 \times 12) \div n = 6$

G Carol baked 2 doz. brownies one morning. After lunch only 8 brownies were left. How many brownies had been eaten?
$(2 \times 12) - n = 8$

H David read two books in 11 days. One book had 266 pages, and the other had 128 pages. David read an average of about how many pages per day?
$(266 + 128) \div n = 11$

I David's older brother built a ham radio station. He said that he spent about 60 hr. working on the station and that he finished it in 5 weeks. On the average, he spent how many hours per week working on the station?
$60 \div n = 5$

J One day David read the temperature of the air outdoors at four different times. At 8 A.M. the temperature was 58°. At noon it was 74°. At 4 P.M. it was 70°. At 8 P.M. it was 54°. What was the average of the temperatures?
$(58 + 74 + 70 + 54) \div n = 4$

K Alice bought 2 lb. of salted nuts at $1.49 per pound and paid for them with a 5-dollar bill. How much money should she have received in change?
$\$5.00 - (2 \times \$1.49) = n$

In most of these problems you had to do more than one thing to find the answer to the question in the problem.

Thinking straight

A $16+29+14$

B $(16+29)+14$ — 45

C $16+(29+14)$ — 43

D $13\times6\times32$

E $(13\times6)\times32$ — 78

F $13\times(6\times32)$ — 192

A Look at Example A. What is the sum of 16, 29, and 14?

B Look at Example B. How do you get 45? What is the sum of 45 and 14?

C In Example C, how do you get 43? What is the sum of 16 and 43?

D What do you notice about the sum in Examples A, B, and C? Does it make any difference which two numbers are added first?

In each exercise below, first use parentheses to group the numerals in different ways. Then add in different ways.

E $24+18+16+52$
F $39+32+28+47$
G $56+15+34+109$
H $37+22+40+125$

I Look at Example D. How would you find the answer? What is the answer?

J Look at Example E. How do you get 78? What is the answer when you multiply 32 by 78?

K In Example F, how do you get 192? Multiply 192 by 13.

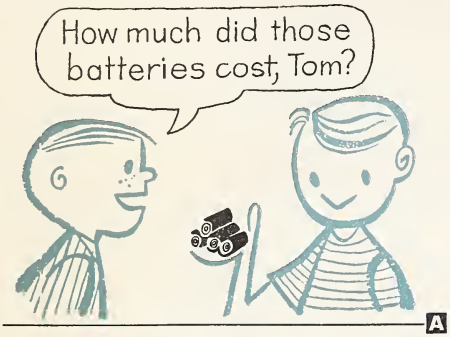
L What do you notice about the answer in Examples D, E, and F? Does it make any difference which two numbers are multiplied first?

Use parentheses to group the numerals in different ways. Then multiply.

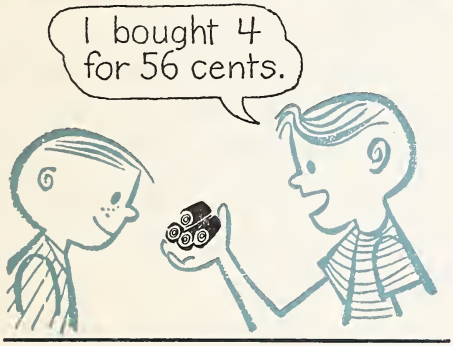
M $7\times9\times6\times4$
N $30\times23\times5$
O $2\times41\times18$
P $8\times5\times4\times7$
Q $20\times9\times32$
R $8\times50\times15$
S $14\times10\times21$
T $36\times2\times53$

Looking back

This lesson will help you remember how to use rates and how to write ratios that express these rates.



A



When Tom answered David's question, he told him the rate at which the batteries were sold. He used two numbers to do this. The rate at which the batteries were sold can be expressed in different ways.

One way to express the rate is to use the numerals 4 and 56. You can say that the batteries were sold at the rate of 4 for 56 cents.

B

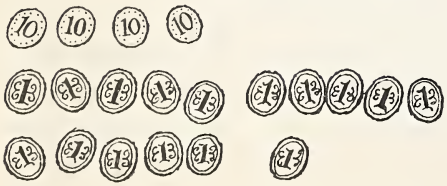
Here is the way to write 4 and 56 together to express this rate.

4 shows the number of batteries that were sold.

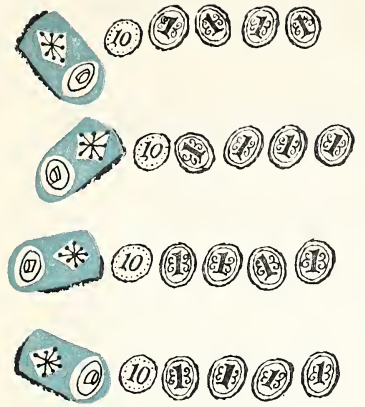
$$\frac{4}{56}$$

56 shows the number of cents needed.

4 per 56



A pair of numerals used in this way is called a ratio. This ratio expresses a rate. The rate is expressed as 4 per 56.



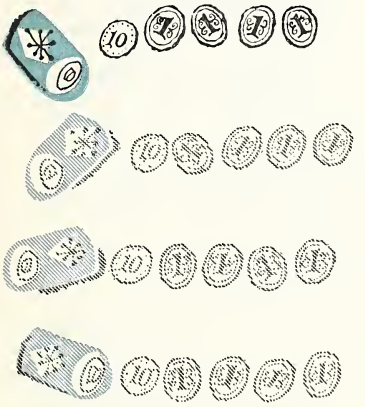
C

You can use other numerals to express the rate at which the batteries were sold.

In Picture C the 4 batteries and the 56 cents are separated into 4 equal groups. The money in each new group will buy the battery in that group.

1 battery and 14 cents are in each group.

Think of only 1 of these 4 new groups.
1 battery will sell for 14 cents.



D

Now you can express the rate as 1 battery per 14 cents. Write the ratio this way.

1 shows the number of batteries.

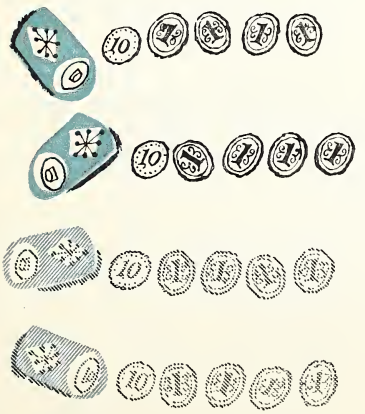
$$\frac{1}{14}$$

14 shows the number of cents needed.

1 per 14

You can now express the rate as 1 per 14.

You can use two other numerals to express the rate. Think of 2 of the 4 new groups.
2 batteries will sell for 28 cents.



E

Now you can express the rate as 2 batteries per 28 cents. Write the ratio this way.

2 shows the number of batteries.

$$\frac{2}{28}$$

28 shows the number of cents needed.

2 per 28

Now the rate is expressed as 2 per 28.

F

Picture F shows what happens when more than 4 batteries are sold at the rate Tom paid. It shows that for 6 batteries, 84 cents would be needed. This time you can express the rate as 6 per 84.

What does the 6 show? → $\frac{6}{84}$ ← What does the 84 show?

6 per 84

A Does 5 per 70 also express the rate shown in Picture F? Does 3 per 42 express the same rate?

B Now you see that 1 per 14, 2 per 28, 3 per 42, 4 per 56, 5 per 70, and 6 per 84 all express the same rate.

$$\frac{1}{14}=\frac{\blacksquare}{28}=\frac{\blacksquare}{42}=\frac{\blacksquare}{56}=\frac{\blacksquare}{70}=\frac{\blacksquare}{84}$$

When you expressed the rate as 6 batteries for 84 cents, you thought of the number of batteries first. Look at Picture F again. Now think of the amount of money first. When you think this way, you can express the rate as 84 cents for 6 batteries. Write a ratio for the rate this way.

84 shows the number of cents needed. → $\frac{84}{6}$ ← 6 shows the number of batteries.

84 per 6

C Now look at 1 group in Picture F. Think of the money first. You can express the rate as ▬ cents for ▬ battery. Write this ratio for the rate.

What does the 14 show? → $\frac{14}{1}$ ← What does the 1 show?

14 per 1

D If you express the rate as 42 per 3, are you thinking first of the money or of the batteries?

E 14 per 1, 28 per 2, 42 per 3, 56 per 4, 70 per 5, and 84 per 6 all express the same rate.

$$\frac{14}{1}=\frac{\blacksquare}{2}=\frac{\blacksquare}{3}=\frac{\blacksquare}{4}=\frac{\blacksquare}{5}=\frac{\blacksquare}{6}$$

G

Jane also expressed a rate when she told Nancy how much cocoa she was going to make. This rate, too, can be expressed in different ways.

H

You can say that Jane will make cocoa at the rate of 15 cups for 6 persons.

To show the rate, you can write the numerals 15 and 6 as a ratio.

15 shows the number of cups of cocoa. → $\frac{15}{6}$ ← 6 shows the number of persons.

$\frac{\square}{6}$ ← ■ per 6

You can express the rate as ■ per ■.

I

You can use other numerals to express the rate at which Jane will make cocoa.

The 15 cups and the 6 persons are now separated into 3 equal groups. The cups of cocoa in each new group will serve the persons in that group.

There are ■ cups and ■ persons in each new group.

Now turn the page.

J

Think of only 1 of these 3 new groups. 5 cups of cocoa will serve 2 persons.

You can say that Jane will make cocoa at a rate of ▒ cups for ▒ persons. You can write the ratio that shows the rate this way.

What does the 5 show? → $\frac{5}{2}$ ← What does the 2 show?

▒ per 2

Now the rate is expressed as 5 per ▒.

K

Picture K shows how many persons could be served if Jane made more cocoa at the same rate. It shows that if Jane made 20 cups, she could serve 8 persons.

You can express the rate now as 20 per 8.

What does the 20 show? → $\frac{20}{8}$ ← What does the 8 show?

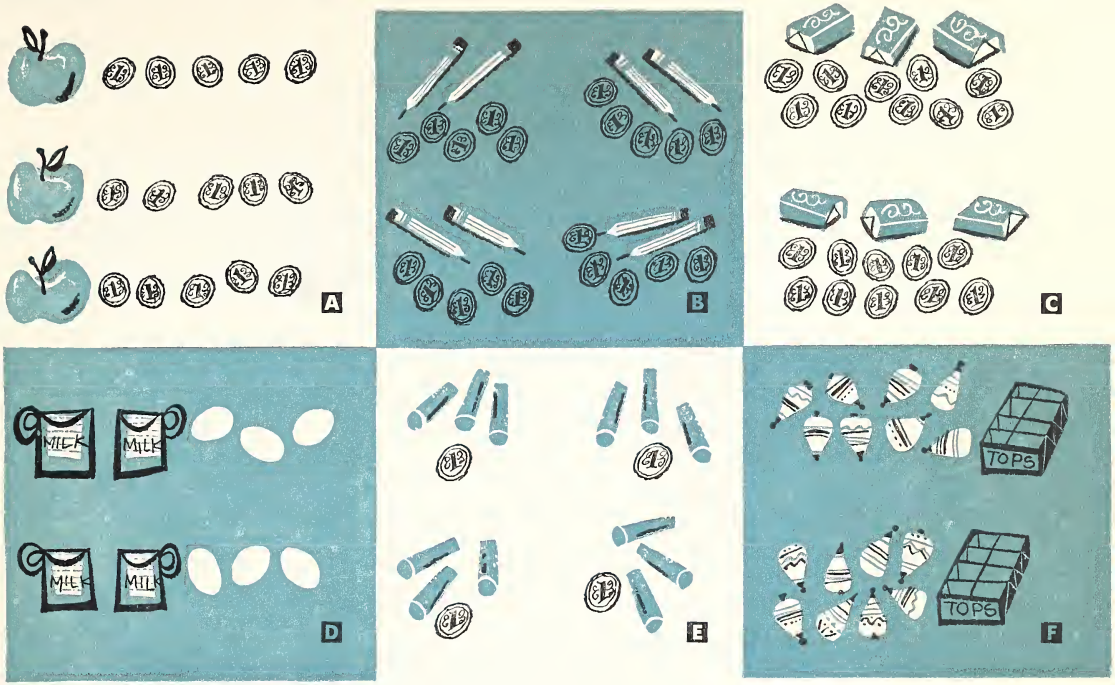
▒ per ▒

Does 10 per 4 also express the rate shown in Pictures J and K?

Do you see that 15 per 6, 5 per 2, 20 per 8, and 10 per 4 all express the same rate?

$$\frac{15}{6}=\frac{5}{2}=\frac{20}{8}=\frac{10}{4}$$

You can also express the rate at which Jane will make cocoa as 2 per 5. What do the 2 and 5 mean?



Each ratio below expresses a rate. For each ratio there is a picture. Find the picture that shows each rate.

A $\frac{2}{5}$	F $\frac{3}{2}$	K $\frac{3}{15}$	P $\frac{5}{1}$
B $\frac{10}{4}$	G $\frac{6}{20}$	L $\frac{20}{8}$	Q $\frac{10}{3}$
C $\frac{1}{8}$	H $\frac{6}{15}$	M $\frac{2}{3}$	R $\frac{16}{2}$
D $\frac{3}{10}$	I $\frac{3}{1}$	N $\frac{8}{1}$	S $\frac{6}{4}$
E $\frac{1}{3}$	J $\frac{15}{3}$	O $\frac{3}{9}$	T $\frac{2}{10}$

For each statement below, write the numerals as a ratio to show the rate.

- A 12 sandwiches for 4 boys
- B 3 bars of soap for 29¢
- C 59¢ for 2 cans of meat
- D 250 sheets of paper for 57¢
- E 4 cookies per person
- F 1 lb. of cheese for 52¢
- G 2 lb. of salted peanuts for 40 people
- H 79¢ for 16 birthday cards
- I 1 cake for 12 people
- J 5 people to a car

Now you know how numerals are used to express rates. Later on you will use rates to solve problems.

Looking back

This lesson will help you remember how ratios are used to compare one group with another.

A

Mary compared her spoons with Joan's spoons by using two numbers, 9 and 12.

B

Picture B shows both girls' spoons.

You can express the comparison that Mary made by writing the numerals 9 and 12 as a ratio. Your ratio will look like this.

Number of Mary's spoons that are compared with Joan's spoons → $\frac{9}{12}$ ← Number of Joan's spoons that Mary's spoons are compared with

9 to 12

C

You can use other numbers to compare Mary's spoons with Joan's.

Now Mary's 9 spoons and Joan's 12 spoons are separated into 3 equal groups. 3 of Mary's spoons and 4 of Joan's are in each group.

Think of one of these groups. Now you can compare Mary's spoons with Joan's by saying that there are 3 of Mary's spoons to 4 of Joan's.

D

The ratio that shows this comparison is written this way.

Number of Mary's spoons that you are thinking about now → 3

4 ← Number of Joan's spoons that Mary's spoons are compared with

3 to 4

You can now express the comparison of Mary's spoons with Joan's as 3 to 4.

E

When you think of two of the groups, you can use other numerals to express the comparison of Mary's spoons with Joan's.

Now you can express the comparison as 6 spoons to 8 spoons. Write the ratio this way.

Number of Mary's spoons that you are thinking about → 6

8 ← Number of Joan's spoons that Mary's spoons are compared with.

6 to 8

The comparison of Mary's spoons with Joan's is now expressed as 6 to 8.

F

Picture F shows that Mary and Joan collected more spoons. Now there are 15 of Mary's spoons to 20 of Joan's. Write a ratio for the comparison this way.

What does the 15 show? → $\frac{15}{20}$ ← What does the 20 show?

15 to 20

A Does 12 to 16 also express the comparison of Mary's spoons with Joan's? What part of Picture F shows this?

B 3 to 4, 6 to 8, 9 to 12, 12 to 16, and 15 to 20 all express the same comparison.

$\frac{3}{4} = \frac{\square}{8} = \frac{\square}{12} = \frac{\square}{16} = \frac{\square}{20}$

When you expressed the comparison as 15 spoons to 20 spoons, you compared Mary's 15 spoons with Joan's 20 spoons. Now look at Picture F again. This time compare Joan's 20 spoons with Mary's 15 spoons.

Number of Joan's spoons that you are thinking about → $\frac{20}{15}$ ← Number of Mary's spoons that Joan's spoons are compared with

20 to 15

Look at one group in Picture F and compare Joan's spoons with Mary's. You can express the comparison now as 4 spoons to 3 spoons. Write a ratio for the comparison.

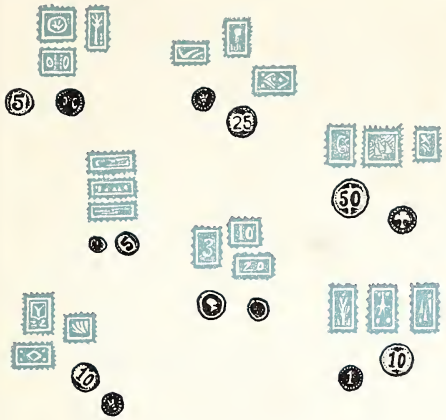
What does the 4 show? → $\frac{4}{3}$ ← What does the 3 show?

4 to 3

C If you express the comparison as 16 to 12, are you comparing Joan's spoons with Mary's or Mary's spoons with Joan's?

D 4 to 3, 8 to 6, 12 to 9, 16 to 12, and 20 to 15 all express the same comparison.

$\frac{4}{3} = \frac{\square}{6} = \frac{\square}{9} = \frac{\square}{12} = \frac{\square}{15}$



A

B

C

A Look at Picture A. There are ▒ stamps. There are ▒ coins. Finish the ratio at the right to compare all the stamps with all the coins.

21
▒

B Think of one group of stamps and coins. There are ▒ stamps and ▒ coins. Use a ratio to compare stamps with coins. Use a ratio to compare coins with stamps.

C Think of three groups of stamps and coins. There are ▒ stamps and ▒ coins. Use a ratio to compare stamps with coins. Use a ratio to compare coins with stamps.

D Use three other ratios to compare the stamps with the coins.

E Use three other ratios to compare the coins with the stamps.

F For Picture B, use a ratio to compare all the arrowheads with all the coins.

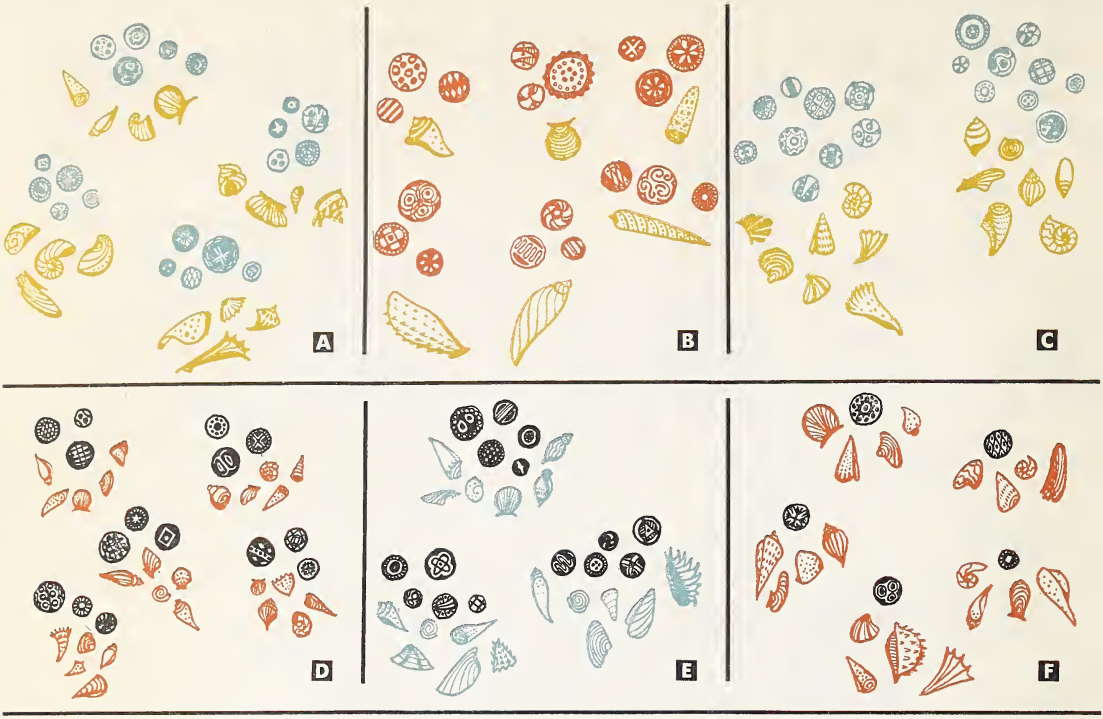
G Use two other ratios to compare the arrowheads with the coins.

H Which ratios below express the comparison of cactus plants with violet plants in Picture C?

$\frac{4}{2}$ $\frac{2}{3}$ $\frac{8}{4}$ $\frac{12}{6}$ $\frac{6}{12}$ $\frac{2}{1}$ $\frac{3}{5}$ $\frac{6}{3}$ $\frac{10}{5}$

I Which ratios below express the comparison of violet plants with cactus plants?

$\frac{2}{1}$ $\frac{5}{10}$ $\frac{3}{6}$ $\frac{4}{3}$ $\frac{6}{12}$ $\frac{2}{4}$ $\frac{1}{2}$ $\frac{6}{3}$ $\frac{4}{8}$



Each ratio below expresses a comparison. Find a picture that belongs with each ratio. For example, Ratio A can be used to express the comparison shown in Picture B.

A $\frac{1}{3}$	E $\frac{1}{4}$	I $\frac{18}{14}$	M $\frac{12}{3}$
B $\frac{10}{8}$	F $\frac{6}{10}$	J $\frac{9}{3}$	N $\frac{12}{15}$
C $\frac{12}{10}$	G $\frac{5}{6}$	K $\frac{5}{4}$	O $\frac{4}{16}$
D $\frac{7}{9}$	H $\frac{15}{18}$	L $\frac{12}{20}$	P $\frac{18}{6}$

For each comparison given below, use the numerals to write a ratio.

A 8 goldfish with 3 goldfish
B 3 goldfish with 8 goldfish
C 2 turtles with 1 turtle
D 15 shells with 8 shells
E 1 horse with 5 cows
F 16 plants with 12 plants
G 15 boys with 27 girls
H 27 girls with 15 boys
I 48 cookies with 32 cookies
J 50 cents with 45 cents
K 20 cards with 25 envelopes

Now you should be able to write a ratio to express a comparison.

Looking back

This lesson will help you remember what fractions and mixed numbers are.

A

A Look at Picture A. The paper disk is marked off in ▒ equal parts. You can use the numerals 2 and 3 to tell how much of the disk is blue.

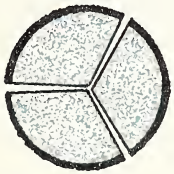
The 3 shows the number of equal parts. → $\frac{2}{3}$ ← The 2 shows you are thinking of 2 of the equal parts.

two thirds

B

$\frac{2}{3}$ is a fraction numeral. 3 is the denominator. 2 is the numerator. The numerator and the denominator are the terms of a fraction numeral. Is $\frac{2}{3}$ less than 1?

B In Picture B the same disk has been marked off in 6 equal parts. The same amount is blue, but now you can use the numerals 4 and 6 to tell how much is blue. Is $\frac{4}{6}$ equal to $\frac{2}{3}$? Is $\frac{4}{6}$ less than 1?



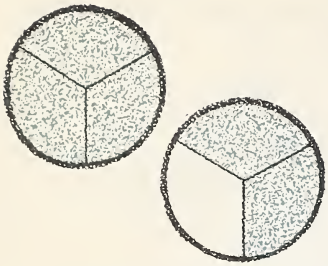
C In Picture C each disk has been cut into 3 equal parts. Each of these parts is $\frac{1}{3}$ of a whole disk. How many of these one thirds are blue? You can use the numerals 5 and 3 to tell how much of these disks is blue.

The 3 shows the number of equal parts in each disk. → $\frac{5}{3}$ ← What does the 5 show?

five thirds

C

The denominator of $\frac{5}{3}$ is ▒. The numerator is ▒. What are the terms of $\frac{5}{3}$? Is $\frac{5}{3}$ more than 1?



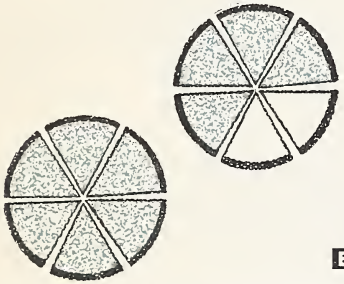
D

D You know that $\frac{5}{3}$ of the disks in Picture D are blue. You can also use $1\frac{2}{3}$ to tell how much of the disks in Picture D is blue.

What does the 1 show? → $1\frac{2}{3}$ ← What does the 2 show?

What does the 3 show? → $1\frac{2}{3}$

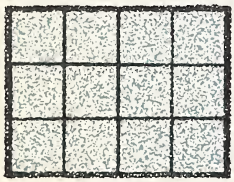
one and two thirds



E

Now look at Picture C on page 43. You can see that the blue parts of the disks in Pictures C and D are equal. $\frac{5}{3} = 1\frac{2}{3}$

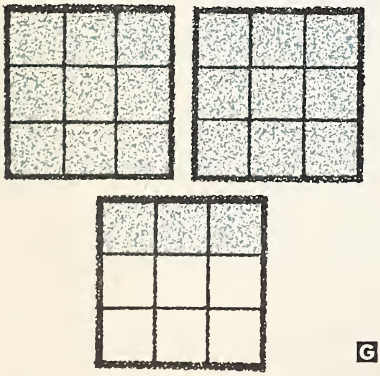
E In Picture E the same disks have been cut into 6 equal parts. Can you say that $\frac{10}{6}$ are blue? Can you say that $1\frac{4}{6}$ of the disks are blue? $\frac{10}{6} = 1\frac{4}{6}$



F

F Is $\frac{10}{6}$ equal to $\frac{5}{3}$? Is $1\frac{4}{6}$ equal to $1\frac{2}{3}$?

G The rectangular piece of paper in Picture F has been marked off in ▒ equal parts. Each part is $\frac{1}{▒}$ of the paper. How much of the paper is blue?



G

H The fraction numeral $\frac{12}{12}$ tells you how much of the paper is blue. Is $\frac{12}{12}$ equal to 1?

I The denominator of $\frac{12}{12}$ is ▒. The numerator is ▒. What are the terms of $\frac{12}{12}$?

J Which numerals can you use to tell how much of the paper in Picture G is blue?

$2\frac{1}{3}$ 3 $\frac{21}{9}$ $1\frac{3}{9}$ $\frac{7}{3}$ $2\frac{3}{9}$ $\frac{27}{9}$

Fractions which are used to tell that something is less than a whole, or less than 1, are proper fractions. Which numerals below represent proper fractions?

A	B	C	D	E	F	G	H
$\frac{10}{3}$	$\frac{5}{6}$	$\frac{8}{8}$	$\frac{7}{12}$	$\frac{1}{9}$	$\frac{13}{10}$	$\frac{2}{5}$	$\frac{5}{10}$

Fractions which are used to tell that something is equal to a whole, or more than a whole, are improper fractions. Which numerals represent improper fractions?

I	J	K	L	M	N	O	P
$\frac{5}{16}$	$\frac{9}{8}$	$\frac{3}{3}$	$\frac{1}{4}$	$\frac{6}{5}$	$\frac{7}{7}$	$\frac{4}{12}$	$\frac{20}{15}$

$1\frac{2}{3}$ expresses a mixed number. It is made up of a numeral for a whole number and a fraction numeral. Which numerals represent mixed numbers?

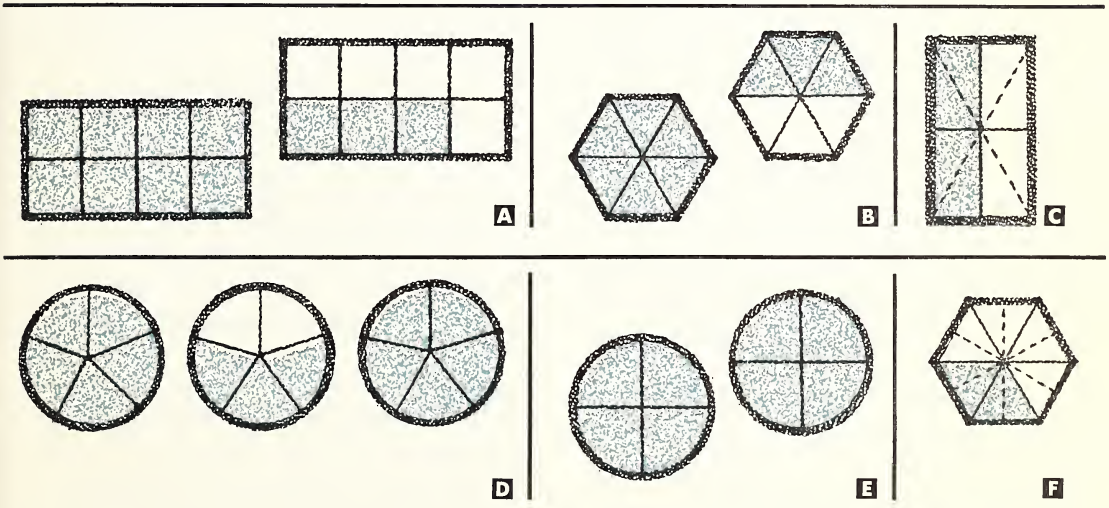
Q	R	S	T	U	V	W	X
9	$2\frac{1}{2}$	$\frac{6}{8}$	$13\frac{3}{8}$	5	$\frac{5}{12}$	$6\frac{1}{10}$	$8\frac{3}{4}$

A Write 5 numerals that represent proper fractions.

B Write 5 numerals that represent improper fractions.

C Write 5 numerals that represent mixed numbers.

For each picture below, write two numerals that tell how much of the pieces of paper is blue. You can write $1\frac{3}{8}$ and $\frac{11}{8}$ for Picture A.



Now you should be able to read and write any fraction numeral or mixed number.

Thinking straight

A Look at Picture A. Using the fraction numeral, make a statement about the picture.

B Make a statement about Picture B, using the ratio to express a rate.

C Make a statement about Picture C, using the ratio to express a comparison.

D Until you know what the pair of numerals at the right means, read it "3 over 14." If it is a ratio expressing a rate such as 3 miles in 14 minutes, read it "3 per 14." If it is a ratio expressing 3 books compared with 14 books, read it "3 to 14." How can you read the pair of numerals if it expresses a fraction?

$\frac{3}{14}$

For each exercise in the next column, write a ratio or a fraction numeral. Decide whether it expresses a rate, a comparison, or a fraction. Then write the words you would use to read the ratio or fraction numeral. For Exercise A write, "$\frac{6}{8}$, six eighths."

A Jim cut a watermelon into 8 equal pieces. He and his friends ate 6 of the pieces.

B Ann paid 55¢ for 10 yd. of ribbon.

C Mrs. Stone cut a coffee cake into 2 equal parts. She gave 1 part to Mrs. Hill.

D Last month Tony earned $7. Bill earned $5.

E Mrs. Downs made 4 jars of blueberry jam and 6 jars of strawberry jam.

F When Mrs. Hill makes cocoa, she uses 3 teaspoonfuls of cocoa for 2 cups of milk.

G The airplane travelled 800 miles in 3 hours.

H Ellen bought 3 cans of tomato juice for 25¢.

I Jane pays $2 for a piano lesson. Sally pays $3 for a piano lesson.

J Andy can walk 1 mile in 20 minutes.

K Don paid 10¢ for a birthday card.

Using arithmetic

A One day the Baker family drove to the canals at Sault Ste. Marie, which were 114 mi. away. After lunch, they still had 51 mi. to drive. How many miles had they driven in the morning?

B It took the Bakers 3 hr. to get to the canals. They travelled at an average rate of how many miles per hour?

C The Bakers took a boat trip through the canals. Tickets for grown-ups cost $2 apiece. Tickets for children cost $1. How much did the Baker family of 2 grown-ups and 3 children spend for tickets?
(?) + (?) = n

D Ships cannot go through the canals part of the year because of ice. The locks in the canals can be used about 250 days of the year. About 90 ships per day go through the locks. About how many ships go through the locks in a year?

E The oldest of the five locks was finished in 1895. The newest lock was finished in 1943. The oldest lock is how many years older than the newest lock?

THINK 1943 was how many years later than 1895?

F There are four American locks and one Canadian lock. The Canadian lock is 900 feet long. This lock is how many feet longer than or shorter than 1 mile?

G The locks are used 24 hr. a day. If 90 ships go through the locks per day, about how many ships go through the locks per hour, on an average?

THINK Which answer is better, about 3 ships per hour or about 4 ships per hour? Why?

H About 70 of the 90 ships are freighters. A freighter of today can carry about 15,000 T. About how many tons of freight could 70 fully loaded freighters carry through the canals?

I It would take 300 railway freight cars to carry 15,000 T. of freight. How many railway freight cars would be needed to carry as much freight as can be shipped in 70 freighters?

J Most of the freight that passes through the canals at Sault Ste. Marie now is ore. About 98,347,000 T. of iron ore were shipped through the canals in one year. About 30,163,000 T. of other goods were shipped through the canals that year. What was the total number of tons of freight shipped through the canals that year?

K The next year the number of tons of iron ore shipped through the canals dropped to 62,245,000 T. How many fewer tons of ore were shipped in this year than in the year before it?

L 83,616,000 tons of freight were shipped through the canals in the year described in Problem K. Was this more than or less than the total amount of freight shipped in the year described in Problem J? How much more or less was it?

M One year United States ships carried 107,301,000 tons of the total amount of 128,510,000 tons. The rest of the freight was carried by ships of other countries. How many tons of freight were carried by ships of other countries?

N Mr. Baker had $28.32 when the Bakers got back from their trip to the canals. Mr. Baker had started the trip with $75. How much money had he spent on the trip?

Keeping skilful

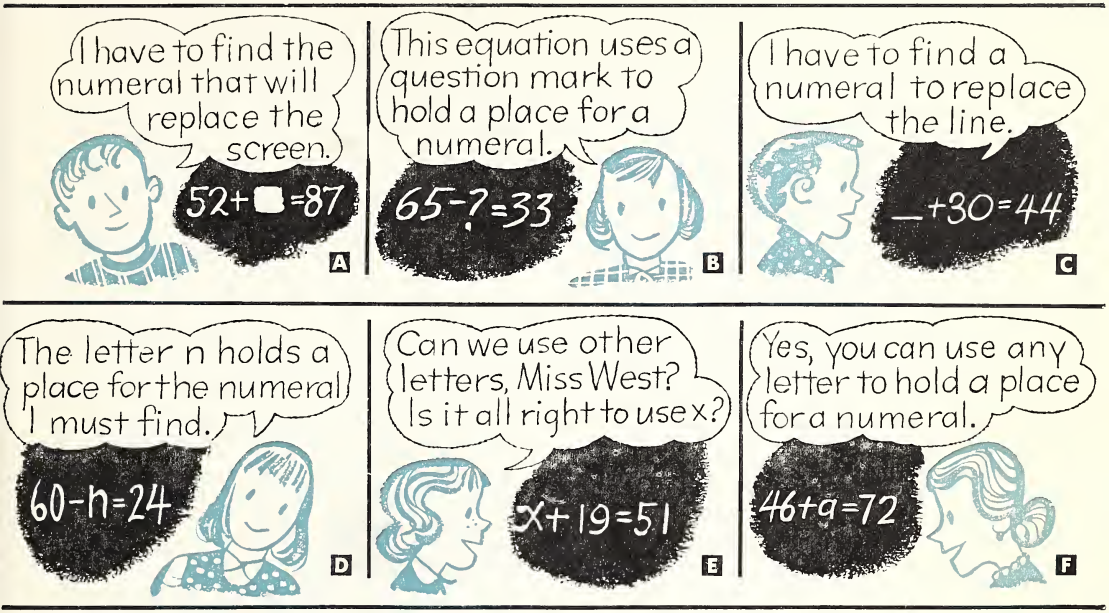
Find the sum.

A 36, 14, 72, 21
B 8, 34, 57, 29, 63
C 19, 7, 25, 9, 46
D 206, 118, 437
E 590, 386, 455
F 723, 535, 601
G 892, 78, 579
H 366, 24, 291, 86
I 68, 405, 173, 54
J 913, 874, 258, 639
K 468, 237, 150, 524
L 212, 781, 305, 677
M 475, 827, 653, 285
N 9037, 4016, 3350
O 6116, 745, 2879
P 5234, 8710, 2664
Q 3175, 2998, 1696, 4005

A $214 + n = 779$
B $n + 318 = 500$
C $683 - n = 276$
D $728 - 483 = n$
E $n - 5063 = 6119$
F $n + 2836 = 4195$
G $7603 - n = 5884$
H $3216 + n = 7520$
I $n - 5036 = 2131$
J $n + 3968 = 8002$
K $15730 - 9643 = n$
L $2848 + n = 30075$
M $53467 - n = 23397$
N $65994 - 38056 = n$
O $n - 41253 = 27864$
P $75000 - 59867 = n$
Q $n + 6308 = 24931$
R $52975 + n = 96333$

A $3 \times 206 = n$
B $47 \times 28 = n$
C $359 \div 16 = n$
D $173 \div 48 = n$
E $36 \times 57 = n$
F $204 \div n = 34$
G $25 \times 148 = n$
H $463 \times 72 = n$
I $1008 \div 45 = n$
J $390 \times 507 = n$
K $1242 \div n = 27$
L $3465 \div 63 = n$
M $228 \times 476 = n$
N $35 \times 3050 = n$
O $6792 \div 81 = n$
P $2355 \div 14 = n$
Q $83 \times 5004 = n$
R $39956 \div 830 = n$

Exploring problems


I have to find the numeral that will replace the screen.
52+□=87
A
This equation uses a question mark to hold a place for a numeral.
65-?=33
B
I have to find a numeral to replace the line.
_+30=44
C
The letter n holds a place for the numeral I must find.
60-n=24
D
Can we use other letters, Miss West? Is it all right to use x?
x+19=51
E
Yes, you can use any letter to hold a place for a numeral.
46+a=72
F

A After Henry had bought a movie ticket for 65¢, he counted the money he had left. He found that he had $1.15. How much money did he have before he bought the movie ticket?
$a - \$.65 = \$1.15$

B A book about frogs and toads that Jack wants costs $4.00. Jack has saved $1.82. How much more money does he need before he can buy the book?
$\$1.82 + s = \$4.00$

C Ellen bought a coat for $32.50 and a dress for $12.95. The coat cost how much more than the dress?
Use the letter n in your equation.

D 34 sixth-grade pupils paid $17.00 for tickets to a concert. What was the cost of each pupil's ticket?
Use the letter g in your equation.

E Kathy's piano lessons cost $3.50 per hour. Each week she takes a 1-hr. lesson. What is the cost of her piano lessons for 12 weeks?
Use the letter r in your equation.

F There are 11 classrooms in the Cook School. In September there were 396 pupils in the school. Find the average number of pupils per classroom in September.
Use the letter t in your equation.

G Joan bought 2 packages of notebook paper at 37¢ each and a package of pencils for 49¢. How much did she spend for all these things?

THINK In this equation there will be two numerals that you do not know. One stands for the amount spent for paper, and the other stands for the total amount spent. You can use letters to hold places for these numerals.

Holds a place for the amount spent for paper

$a + \$.49 = d$

Holds a place for the total amount spent

$(2 \times \$.37)$

$a + \$.49 = d$

Now you may write this to replace the letter a.

$(2 \times \$.37) + \$.49 = d$

$\$.74 + \$.49 = d$

What numeral will replace d?

H Mrs. Parks made 6 lb. of candy for a school sale. She packed it in bags, putting 12 oz. of candy into each bag. How many bags of candy did she pack?

THINK You do not know how many ounces of candy Mrs. Parks made, or how many bags she packed. Use letters in the equation to hold places for the two unknown numerals.

Holds a place for the total number of ounces

$m \div 12 = t$

Holds a place for the number of bags

You may write this to replace m.

$(6 \times 16) \div 12 = t$

$96 \div 12 = t$

What numeral will replace t?

I Jim earned 45¢ on Monday, 30¢ on Tuesday, 35¢ on Wednesday, and 50¢ on Thursday. Find the average amount he earned per day.

THINK You do not know the total amount that Jim earned. You do not know the average amount he earned per day. You do know that he worked 4 days.

Holds a place for the total amount earned

$s \div m = 4$

Holds a place for the average amount earned

Write this to replace s.

$(\$.45 + \$.30 + \$.35 + \$.50) \div m = 4$

$\$1.60 \div m = 4$

What numeral will replace m?

J Two teachers took 34 pupils on a bus trip. The fare was $.50 for each teacher and $.24 for each pupil. What was the total amount of all the fares?

THINK In Problem J there are three things you do not know. So you can use $a + b = x$ for your equation. What does x hold a place for? If a holds a place for the total amount of the teachers' fares, what does b hold a place for?

$(2 \times \$.50)$ $(34 \times \$.24)$

$a + b = x$

$(2 \times \$.50) + (34 \times \$.24) = x$

$\$1.00 + \$8.16 = x$

What numeral will replace x?

K Tony bought a model boat for $2.89, glue for 10¢, and a set of paints for $.98. He paid for all this with a five-dollar bill. How much change should he have received?

THINK How do you know that you will need to use two letters in the equation?

The letter z holds a place for what amount? $\$5.00 - z = r$

r holds a place for what amount?

This replaces z.

$\$5.00 - (\$2.89 + \$.10 + \$.98) = r$

$\$5.00 - \$3.97 = r$

L Mrs. Long bought 4 yd. of cloth at $1.39 a yard, 2 spools of thread at 10¢ a spool, and a card of buttons for 89¢. How much did all these materials cost?

THINK How many letters will you use in the equation? You can use the equation $d + t + \$.89 = y$. If d holds a place for the total cost of the cloth, what does t hold a place for? What does y hold a place for?

$(4 \times \$1.39)$ $(2 \times \$.10)$

$d + t + \$.89 = y$

$(4 \times \$1.39) + (2 \times \$.10) + \$.89 = y$

$\$5.56 + \$.20 + \$.89 = y$

M Kathy bought three 8-oz. packages of candy corn and a 14-oz. package of another kind of candy. How many ounces of candy did she buy in all?

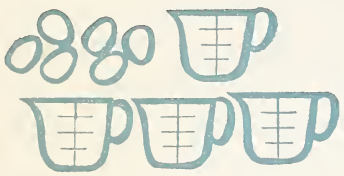
THINK There will be ▒ letters in the equation. To find the numeral that replaces one of them, why do you multiply 8 by 3? Now find the numeral that replaces the other letter.

Now you can use different letters to hold places for numerals in the equations you write for problems.

Looking back

Now you will learn how to find ratios and fraction numerals equal to other ratios and fraction numerals.

A

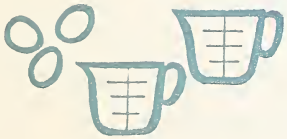


When Mrs. Stone makes custard, she uses 6 eggs for 4 cups of milk. Does the ratio at the right express the rate at which Mrs. Stone uses eggs and milk? Read the ratio ▒ per ▒.

$\frac{6}{4}$

6 and 4 are the terms of the ratio. 6 is the first term. 4 is the second term.

B



Now the 6 eggs and the 4 cups of milk have been divided into 2 equal groups. Think of 1 of the equal groups. Does the ratio at the right also express the rate at which Mrs. Stone uses eggs and milk? What is the first term of this ratio? What is the second term?

$\frac{3}{2}$

You can use arithmetic to get this other ratio that expresses the same rate. You can divide both the 6 and the 4 by 2.

$6 \div 2 = 3$

$4 \div 2 = 2$

$\frac{6}{4} = \frac{3}{2}$ 6 per 4 = 3 per 2

C

The ratio 3 over 2 expresses the rate in lower terms than the ratio 6 over 4 does.

One day Mrs. Stone made a larger amount of custard.

She used 2 × 6 eggs, or ▒ eggs.

She used 2 × 4 cups of milk, or ▒ cups of milk.

Think of all the eggs as one group. Think of all the cups of milk as one group. Does the ratio at the right also express the rate at which Mrs. Stone uses eggs and milk?

$\frac{12}{8}$

You can use arithmetic to get this new ratio that expresses the same rate. You can multiply both 6 and 4 by 2.

$2 \times 6 = \square$ $\quad\quad$ $\frac{6}{4} = \frac{\square}{\square}$

$2 \times 4 = \square$

The ratio 12 over 8 expresses the rate in higher terms than 6 over 4 does.

A You can find many ratios equal to the ratio at the right. One way is to divide both terms by the same number. Can both 16 and 20 be divided by 4?

$\frac{16}{20}$

$16 \div 4 = \square$ $\quad\quad$ $\frac{16}{20} = \frac{\square}{\square}$

$20 \div 4 = \square$

B By what other number can both 16 and 20 be divided?

$16 \div 2 = \square$ $\quad\quad$ $\frac{16}{20} = \frac{\square}{\square}$

$20 \div 2 = \square$

You can also find other ratios equal to the ratio 16 over 20 by multiplying both terms by the same numbers.

C $3 \times 16 = \square$ $\quad\quad$ $\frac{16}{20} = \frac{\square}{\square}$

$3 \times 20 = \square$

D $4 \times 16 = \square$ $\quad\quad$ $\frac{16}{20} = \frac{\square}{\square}$

$4 \times 20 = \square$

E You can use arithmetic to find a new ratio that is equal to the ratio at the right, but in lower terms. You can divide both terms by 2.

$\frac{40}{8}$

$40 \div 2 = \square$ $\quad\quad$ $\frac{40}{8} = \frac{\square}{\square}$

$8 \div 2 = \square$

F Find two more ratios equal to the ratio 40 over 8, but in lower terms.

G You can also find other ratios equal to the ratio 40 over 8, but in higher terms. You can multiply both terms by the same number.

$5 \times 40 = \square$ $\quad\quad$ $\frac{40}{8} = \frac{\square}{\square}$

$5 \times 8 = \square$

H Find two more ratios equal to the ratio 40 over 8, but in higher terms.

For each ratio below, find a ratio equal to it, but in lower terms. For Exercise A you could have 1 over 6.

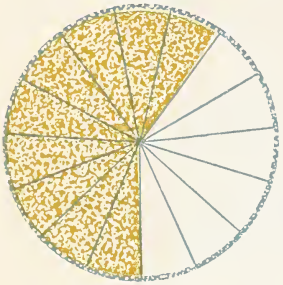
A	$\frac{6}{36}$	**C**	$\frac{8}{20}$	**E**	$\frac{35}{25}$	**G**	$\frac{27}{18}$
B	$\frac{12}{15}$	**D**	$\frac{40}{12}$	**F**	$\frac{12}{4}$	**H**	$\frac{50}{30}$

Now find a ratio equal to each ratio below, but in higher terms.

I	$\frac{2}{3}$	**K**	$\frac{3}{4}$	**M**	$\frac{11}{8}$	**O**	$\frac{7}{1}$
J	$\frac{6}{5}$	**L**	$\frac{5}{9}$	**N**	$\frac{4}{6}$	**P**	$\frac{8}{3}$

A

The paper disk in Picture A is marked off in ▒ equal parts. What fraction of the disk is yellow?

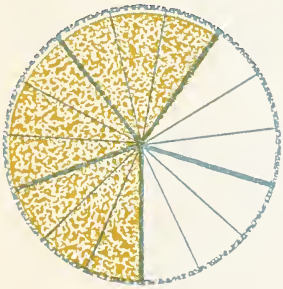


Now imagine that the disk has been marked off in only 5 equal parts. Each part is $\frac{1}{5}$ of the disk. How many one fifteenths are in each $\frac{1}{5}$?

Can you still say that $\frac{9}{15}$ of the disk is yellow?

Is $\frac{3}{5}$ of the disk yellow?

B



You can express $\frac{9}{15}$ as fifths by dividing both terms of $\frac{9}{15}$ by 3. The terms of a fraction numeral are its ~~~ and its ~~~

$9 \div 3 = 3$

$15 \div 3 = 5$

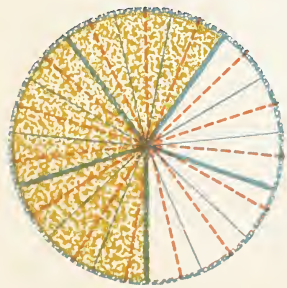
$\frac{9}{15} = \frac{3}{5}$

C

Now imagine that the same disk has been marked off in 30 equal parts. Each part is $\frac{1}{30}$ of the disk. How many one thirtieths are in each $\frac{1}{15}$?

Is $\frac{18}{30}$ of the disk yellow?

Is $\frac{9}{15}$ of the disk yellow?



You can express $\frac{9}{15}$ as thirtieths by multiplying both terms of $\frac{9}{15}$ by 2.

$2 \times 9 = ▒$

$2 \times 15 = ▒$

$\frac{9}{15} = \frac{▒}{▒}$

$\frac{9}{15}$, $\frac{3}{5}$, and $\frac{18}{30}$ all express the fraction of the disk that is yellow.

A You can find many fraction numerals that are equal to $\frac{12}{24}$. One way to find such a numeral is to divide both terms of $\frac{12}{24}$ by the same number.
12 and 24 can both be divided by 4.

$12 \div 4 = \square$
$24 \div 4 = \square$ $\qquad \frac{12}{24} = \frac{\square}{\square}$

Then $\frac{12}{24}$ is expressed in lower terms.

B Find three other fraction numerals equal to $\frac{12}{24}$ but in lower terms.

C You can also find fraction numerals equal to $\frac{12}{24}$ by multiplying both terms by the same number.

$2 \times 12 = \square$
$2 \times 24 = \square$ $\qquad \frac{12}{24} = \frac{\square}{\square}$

Then $\frac{12}{24}$ is expressed in higher terms.

D Is there any end to the equal fraction numerals you can find when you express a fraction numeral in higher terms?

For each fraction numeral below, find a numeral equal to it, but in lower terms. For Exercise A you could have $\frac{8}{12}$, $\frac{4}{\square}$, or $\frac{\square}{3}$.

A $\frac{16}{24}$	**D** $\frac{35}{25}$	**G** $\frac{40}{12}$	**J** $\frac{12}{15}$
B $\frac{12}{4}$	**E** $\frac{12}{18}$	**H** $\frac{25}{15}$	**K** $\frac{8}{20}$
C $\frac{18}{36}$	**F** $\frac{9}{12}$	**I** $\frac{10}{16}$	**L** $\frac{6}{36}$

Now find a fraction numeral equal to each one below but in higher terms.

M $\frac{2}{3}$	**P** $\frac{3}{4}$	**S** $\frac{11}{8}$	**V** $\frac{4}{5}$
N $\frac{2}{10}$	**Q** $\frac{5}{9}$	**T** $\frac{2}{6}$	**W** $\frac{8}{3}$
O $\frac{7}{10}$	**R** $\frac{10}{3}$	**U** $\frac{1}{7}$	**X** $\frac{10}{4}$

A $\frac{20}{36} = \frac{5}{9}$. By what number were both 20 and 36 divided? Can both the 5 and the 9 in $\frac{5}{9}$ be divided without a remainder by any number besides 1?

B $\frac{14}{20} = \frac{7}{10}$. By what number were both 14 and 20 divided? The 7 and the 10 cannot be divided without a remainder by any number except 1. So we say that $\frac{7}{10}$ is in lowest terms.

Which fraction numerals below are in lowest terms? How do you know?

C $\frac{10}{12}$	**F** $\frac{5}{8}$	**I** $\frac{12}{20}$	**L** $\frac{13}{24}$
D $\frac{2}{7}$	**G** $\frac{6}{14}$	**J** $\frac{3}{5}$	**M** $\frac{5}{16}$
E $\frac{11}{12}$	**H** $\frac{3}{10}$	**K** $\frac{9}{12}$	**N** $\frac{13}{15}$

For each fraction numeral below, find a fraction numeral equal to it but in lowest terms.

A $\frac{12}{16}$	**F** $\frac{20}{24}$	**K** $\frac{10}{14}$	**P** $\frac{5}{20}$
B $\frac{8}{10}$	**G** $\frac{8}{16}$	**L** $\frac{24}{30}$	**Q** $\frac{21}{24}$
C $\frac{9}{15}$	**H** $\frac{6}{9}$	**M** $\frac{18}{24}$	**R** $\frac{16}{30}$
D $\frac{8}{24}$	**I** $\frac{8}{12}$	**N** $\frac{10}{15}$	**S** $\frac{15}{20}$
E $\frac{6}{12}$	**J** $\frac{15}{30}$	**O** $\frac{10}{30}$	**T** $\frac{21}{28}$

When you multiply or divide both terms of a ratio or a fraction numeral by the same number, you get another ratio or fraction numeral equal to the first one.

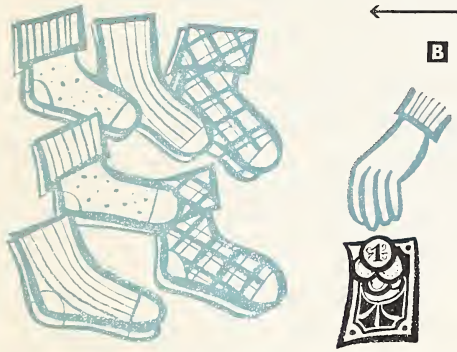
Looking back

In this lesson you will learn how to find the missing term in a pair of equal ratios or fraction numerals.

A

Look at Picture A. Now look at the ratio at the right. This ratio shows that Martha can buy socks at the rate of 3 pairs for $1.17, or 3 pairs for 117¢.

$$\frac{3}{117}$$



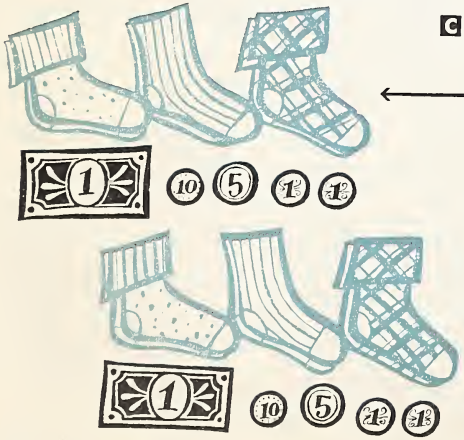
B

Now look at Picture B. Martha wants to buy 6 pairs of socks at the same rate. The ratio at the right expresses the rate. We do not know how much money Martha needs to buy 6 pairs. So we use a to hold a place for the numeral. Now we have two ratios that express the same rate. These ratios are equal.

$$\frac{6}{a}$$

$$\frac{3}{117} = \frac{6}{a}$$

You must find the numeral that replaces a.



C

It takes 2 groups of 3 pairs to make 6 pairs. It also takes 2 groups of 117¢ to buy them. Martha needs 234¢, or $2.34.

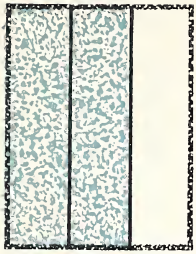
$$\frac{3}{117} = \frac{6}{a}$$

6 can be found by multiplying 3 by 2.

117 must also be multiplied by 2.

234 will replace a.

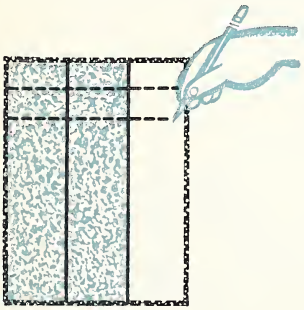
D



Here is a rectangular piece of paper marked off in thirds. The fraction numeral at the right expresses how much of the rectangle is blue.

$\frac{2}{3}$

E



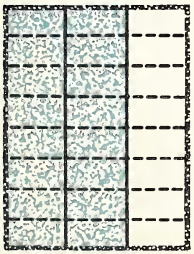
Now suppose you want to mark off the same rectangle in twenty-fourths. You will use a new fraction numeral to express how many twenty-fourths will be blue. The denominator will be 24. You do not know what numeral to use for the numerator, but you can use a screen to hold a place for the numerator.

$\frac{\square}{24}$

Since the same amount of paper is blue, $\frac{\square}{24}$ and $\frac{2}{3}$ express the same fraction.

$\frac{2}{3} = \frac{\square}{24}$ ← You must find the numerator.

F



You can see that since there are 8 twenty-fourths in each one third, there will be 8×2, or 16, twenty-fourths in 2 of these thirds.

$\frac{2}{3} = \frac{\square}{24}$

Because 24 can be found by multiplying 3 by 8, you must multiply 2 by 8 also. 16 will replace the screen.

A $\frac{3}{5}=\frac{\square}{20}$

To find the missing numerator, first find the number by which both the numerator and the denominator of $\frac{3}{5}$ must be multiplied. How do you find this number? What must you do next?

$\square \times 5 = 20$

$4 \times 5 = 20$ $\quad \frac{3}{5}=\frac{12}{20}$

$4 \times 3 = 12$

B $\frac{27}{36}=\frac{\square}{4}$

To find the missing numerator, first find the number by which both the numerator and the denominator of $\frac{27}{36}$ must be divided. How do you find this number? What must you do next?

$36 \div \square = 4$

$36 \div 9 = 4$ $\quad \frac{27}{36}=\frac{3}{4}$

$27 \div 9 = 3$

In each pair of fraction numerals below, find the missing numerator.

C $\frac{2}{3}=\frac{\square}{15}$ **I** $\frac{5}{8}=\frac{\square}{24}$ **O** $\frac{3}{5}=\frac{\square}{10}$

D $\frac{7}{8}=\frac{\square}{16}$ **J** $\frac{12}{16}=\frac{\square}{4}$ **P** $\frac{9}{18}=\frac{\square}{2}$

E $\frac{1}{6}=\frac{\square}{24}$ **K** $\frac{10}{24}=\frac{\square}{12}$ **Q** $\frac{5}{6}=\frac{\square}{12}$

F $\frac{8}{32}=\frac{\square}{4}$ **L** $\frac{4}{9}=\frac{\square}{36}$ **R** $\frac{6}{9}=\frac{\square}{3}$

G $\frac{14}{20}=\frac{\square}{10}$ **M** $\frac{12}{15}=\frac{\square}{5}$ **S** $\frac{3}{4}=\frac{\square}{20}$

H $\frac{4}{7}=\frac{\square}{21}$ **N** $\frac{25}{30}=\frac{\square}{6}$ **T** $\frac{18}{24}=\frac{\square}{4}$

A $\frac{17}{12}=\frac{51}{c}$

To find the missing term, first find the number by which both terms of the ratio 17 over 12 must be multiplied. How do you find this number? Then what must you do to get the numeral that replaces c?

$\square \times 17 = 51$

$3 \times 17 = 51$ $\quad \frac{17}{12}=\frac{51}{36}$

$3 \times 12 = 36$

B $\frac{10}{25}=\frac{a}{5}$

By what number must 25 be divided to get 5? What must you do to get the numeral that replaces a?

$25 \div \square = 5$

$25 \div 5 = 5$ $\quad \frac{10}{25}=\frac{2}{5}$

$10 \div 5 = 2$

In each pair of ratios below, find the missing term.

C $\frac{6}{19}=\frac{30}{e}$ **H** $\frac{6}{5}=\frac{d}{15}$ **M** $\frac{75}{12}=\frac{25}{n}$

D $\frac{27}{15}=\frac{m}{5}$ **I** $\frac{33}{44}=\frac{v}{4}$ **N** $\frac{4}{9}=\frac{c}{90}$

E $\frac{10}{50}=\frac{2}{s}$ **J** $\frac{21}{3}=\frac{x}{1}$ **O** $\frac{8}{18}=\frac{40}{f}$

F $\frac{4}{5}=\frac{16}{a}$ **K** $\frac{5}{13}=\frac{20}{b}$ **P** $\frac{30}{65}=\frac{6}{h}$

G $\frac{42}{9}=\frac{b}{36}$ **L** $\frac{17}{51}=\frac{1}{n}$ **Q** $\frac{49}{7}=\frac{7}{y}$

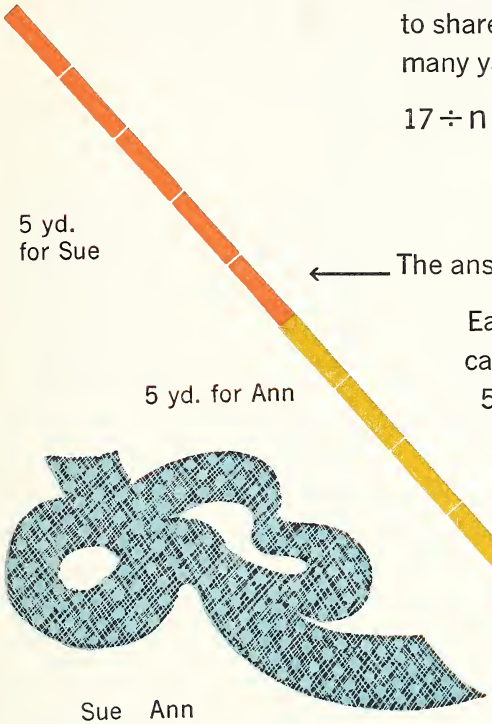
Now you should be able to find the missing term in a pair of equal ratios or fraction numerals.

Looking back

If you have a remainder when you divide, sometimes you should use it to make a fraction numeral in the answer.

A Sue, Ann, and Kathy have 17 yd. of cloth to share equally. Each girl's share will be how many yards?

$17 \div n = 3$ Divide 17 by 3.



5 yd. for Sue

5 yd. for Ann

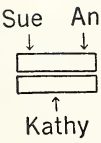
5 yd. for Kathy

2 yd. left over to be shared

The answer is 5 and 2 remainder.

Each girl can have 5 whole yards.

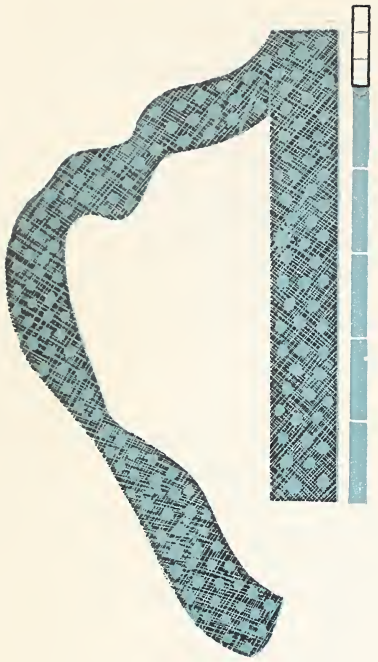
2 yards will be left over to be shared.



Think of each leftover yard as marked off into 3 equal parts.

Each girl can have 1 of the equal parts of each leftover yard. Each girl can have $\frac{1}{3}$ of each leftover yard.

Since there are 2 leftover yards, each girl can have 2 of the equal parts, or $\frac{2}{3}$ yd.



Before cutting the cloth, measure 5 whole yards and $\frac{2}{3}$ of another yard for each girl's share.

When you divide 17 by 3, you can express the answer as $5\frac{2}{3}$. Notice that the divisor, 3, tells what denominator to use in the fraction numeral. The remainder, 2, tells what numerator to use.

Number of whole yards in each girl's share → $5\frac{2}{3}$ ← Number of parts of the leftover yards each girl will get

Number of parts into which each leftover yard is marked → $5\frac{2}{3}$

Each girl's share will be $5\frac{2}{3}$ yd.

B Twelve boys picked 129 lb. of apples to share equally. What was each boy's share of the apples?

$129 \div a = 12$ Divide 129 by 12.

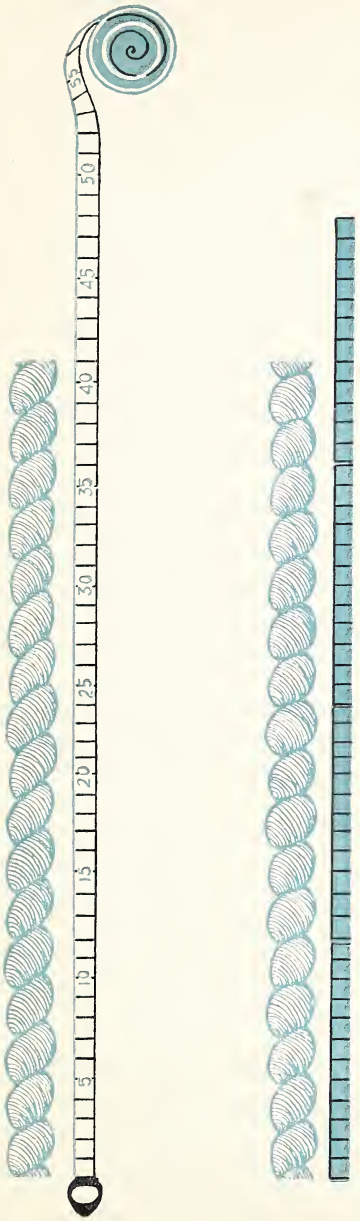
$$12\overline{)129}\quad 10\tfrac{9}{12}$$
$$\underline{120}$$
$$9$$

The divisor, 12, tells you what denominator to use. The remainder, 9, tells you what numerator to use.

Each boy's share was $10\frac{9}{12}$ lb., or $10\frac{3}{4}$ lb.

Number of whole pounds in each boy's share → $10\frac{9}{12}$ ← Number of parts of the leftover pounds in each boy's share

Number of parts into which each leftover pound is divided → $10\frac{9}{12}$



C Don has a rope 41 in. long. How many feet long is the rope?

$41 \div 12 = g$

What does the 12 represent?

$$12\overline{)41}\quad 3 \quad 36 \quad 5$$

3 ← There are 3 whole feet in the piece of rope.

5 ← There are 5 inches more than 3 ft. in the piece of rope.

Think of each inch as $\frac{1}{12}$ of a whole foot.
Think of the 5 inches as $\frac{5}{12}$ of a foot.

Number of whole feet in 41 inches

$$12\overline{)41}\quad 3\tfrac{5}{12} \quad 36 \quad 5$$

$\frac{5}{12}$ ← Number of inches more than 3 ft.

$\frac{5}{12}$ ← Number of inches in 1 ft.

When you use the fraction numeral $\frac{5}{12}$ as part of the answer, the divisor, ▒, tells what denominator to use. The remainder, ▒, tells what numerator to use.

Don's piece of rope is $3\frac{5}{12}$ ft. long.

D Mrs. Cook bought a tablecloth that was 120 in. long. Alice said, "The tablecloth is $3\frac{1}{3}$ yd. long." Was Alice right?

$120 \div 36 = b$

$36\overline{)120}\ 3\frac{12}{36}$
$\quad 108$
$\quad \overline{\ 12}$

The divisor, 36, tells you what denominator to use. The remainder, 12, tells you what numerator to use.

Number of whole yards in 120 in. → $3\frac{12}{36}$ ← Number of inches more than 3 yd.

Number of inches in 1 yd. → 36

Can the mixed number $3\frac{12}{36}$ also be expressed as $3\frac{1}{3}$? How do you know? The tablecloth was $3\frac{\square}{3}$ yd. long.

E Philip cut 25 ft. of screening into 4 equal pieces. Each piece was how many feet long?

$25 \div n = 4$ Divide 25 by 4.

What is the remainder? The answer is $6\frac{1}{4}$ ft. How do you get the $\frac{1}{4}$?

F Don earned $4.25 one week. He was paid 50¢ an hour. How many hours did he work that week?

What is the remainder? You may write $\frac{25}{50}$ or $\frac{\square}{2}$ as part of the answer. Does the $\frac{1}{2}$ mean hours or money?

G A milkman sold 46 qt. of milk one morning. This amount was equal to how many gallons?

Why do you divide by 4? What does the remainder represent? 2 qt. are $\frac{\square}{4}$ gal. or $\frac{\square}{2}$ gal. 46 qt. $= 11\frac{\square}{\square}$ gal.

H Mrs. Long bought 4 chickens that weighed a total of 11 lb. What was the average weight of the chickens?

The average weight was $2\frac{\square}{4}$ lb.

If you have a remainder in any exercise below, use a fraction numeral in lowest terms in the answer.

A	$34 \div d = 6$	J	$145 \div 20 = n$
B	$116 \div 8 = n$	K	$163 \div n = 10$
C	$76 \div 16 = c$	L	$396 \div 24 = a$
D	$85 \div f = 6$	M	$750 \div m = 50$
E	$92 \div w = 7$	N	$579 \div n = 18$
F	$118 \div 14 = n$	O	$505 \div 25 = b$
G	$145 \div a = 29$	P	$611 \div n = 47$
H	$32 \div 5 = g$	Q	$510 \div 45 = n$
I	$196 \div v = 32$	R	$252 \div c = 24$

Now you should be able to use a remainder to make a fraction numeral in an answer when you divide.

Thinking straight

A To find the average number of days per month in 1 year, use the equation $365 \div n = 12$.

1. Will the remainder represent days or months?
2. If you use the remainder to make a fraction, will the fraction numeral represent part of a day or part of a month?
3. Which do you think is the best answer, 30 days, $30\frac{5}{12}$ days, $30\frac{1}{2}$ days, or 31 days? Why?
4. Which answer above would you use if you were finding the average number of days per month in a leap year of 366 days? Why?

B To find the number of weeks in a year of 365 days, use the equation $365 \div 7 = y$.

1. Will the remainder represent weeks or days?
2. If you use the remainder to make a fraction, will the fraction numeral represent part of a day or part of a week?
3. Which do you think is the best answer, 52 weeks, $52\frac{1}{7}$ weeks, or 53 weeks? Why?
4. What answer would you use if you were finding the number of weeks in a leap year? Why?

C Mrs. Beach paid \$8.04 for 9 yd. of material. To find the cost of the material per yard, use the equation $\$8.04 \div b = 9$.

1. What will the fraction numeral in your answer represent?
2. What is the cost of the material per yard to the nearer cent?
3. If Mrs. Beach had bought only 1 yd. of the material, how much would she have had to pay for it? Why?

D Mrs. Stevens paid \$3.15 for meat that cost 84¢ per pound. Use the equation $\$3.15 \div \$.84 = s$ to find the number of pounds of meat that Mrs. Stevens bought.

1. What will the remainder represent?
2. What fraction numeral can you write in your answer? What does it represent?
3. Which do you think is the best answer, 3 lb., $3\frac{63}{84}$ lb., $3\frac{3}{4}$ lb., or 4 lb.? Why?

E Jim has 28 tadpoles that he is going to give to 5 of his friends. Can Jim give away all of the tadpoles by giving each of the 5 boys the same number? Will some of the boys get more than 5 tadpoles? Should you use a fraction numeral in the answer to a problem like this?

F Mrs. Beach can make 4 hamburgers from 1 lb. of meat. How would you find how many pounds of meat she needs to make 14 hamburgers? Can Mrs. Beach buy a part of a pound of meat? Should you use the remainder to make a fraction numeral in the answer? Or should you give the answer as 3 lb.? As 4 lb.?

G If each package of hamburger buns contains 6 buns, how would you find the number of packages Mrs. Beach needs for 14 hamburgers? Can she buy part of a package? Should you use a fraction numeral in your answer? Or should you give the answer as 2 packages? As 3 packages?

H Grain is often measured by the hundredweight. 1 hundredweight equals 100 pounds. For the word hundredweight you can use the abbreviation cwt.

1 T. = ▒ cwt. 5 cwt. = ▒ lb.

225 lb. = ▒ cwt.

I Grain also is sold by the bushel. The bushel measure is the same size everywhere. But laws have been passed that tell how much a bushel of different kinds of grain must weigh. Do you think that a bushel of one kind of grain may weigh more than or less than a bushel of another kind of grain? Why?

J The law says that a bushel of oats must weigh 34 lb. How would you find the number of bushels of oats in 1 cwt.?

K If you were telling someone how many bushels of oats there are in 1 cwt., what would you probably say?

L Will 3 bushel baskets hold 1 cwt. of oats?

M If you were buying 1 cwt. of oats, how many bushels do you think you would have to pay for?

N One bushel of shelled corn weighs 56 lb. How would you find the number of bushels of corn in 1 cwt.?

O If you were telling someone how many bushels of corn there are in 1 cwt., what would you do with the remainder?

P What would you do with the remainder if you were paying by the bushel for 1 cwt. of corn?

Q One bushel of apples weighs 48 lb. How many 5-lb. bags of apples would you say you can get from 1 bushel of apples? Should you use a fraction numeral in the answer? Or should you give the answer as 9 bags? Or should you give the answer as 10 bags?

R If you were going to sell the 5-lb. bags of apples described in Problem Q, how many would there be to sell? What should you do with the remainder?

S The 32 pupils in the sixth grade have a total of 56 brothers and sisters. When they divided to find the average number of brothers and sisters per sixth grader, their answer was 1 and 24 remainder. Should the remainder be used to make a fraction? Is it permissible to say that the average number of brothers and sisters per sixth grader is $1\frac{3}{4}$? Does this mean that it is possible to have $\frac{3}{4}$ of a brother or a sister?

T Suppose you know how many cents several items of the same kind cost when bought together. You want to buy only 1 item. When you divide to find the price of 1 item, you may have a remainder. Should you use the remainder to make a fraction numeral in the answer? Can you forget the remainder, or must you always add 1 cent to the answer?

U When you find an average, is it permissible to have a fraction numeral in the answer, even though no one can have a fraction of the thing that the fraction numeral represents?

V In division problems in which you find weight or length or some other measure, can you usually use the remainder to make a fraction numeral in your answer? Is it sensible to talk about a fractional part of a pound or of an inch or of a bushel?

W Does the way you are to use the answer help you to decide whether or not to use the remainder to make a fraction numeral in the answer? What can you do with the remainder if you do not use it to make a fraction?

Keeping skilful

Express each of these fraction numerals in lowest terms.

A	$\frac{16}{28}$	H	$\frac{18}{30}$	O	$\frac{3}{12}$
B	$\frac{4}{10}$	I	$\frac{8}{12}$	P	$\frac{16}{20}$
C	$\frac{8}{24}$	J	$\frac{4}{24}$	Q	$\frac{10}{15}$
D	$\frac{12}{15}$	K	$\frac{5}{10}$	R	$\frac{8}{16}$
E	$\frac{20}{32}$	L	$\frac{6}{16}$	S	$\frac{20}{36}$
F	$\frac{12}{16}$	M	$\frac{21}{28}$	T	$\frac{5}{25}$
G	$\frac{14}{20}$	N	$\frac{9}{30}$	U	$\frac{42}{60}$

Find the missing term in each pair of ratios.

A	$\frac{7}{8}=\frac{28}{c}$	E	$\frac{6}{7}=\frac{n}{35}$	I	$\frac{3}{2}=\frac{24}{k}$	M	$\frac{25}{40}=\frac{5}{z}$
B	$\frac{16}{32}=\frac{t}{16}$	F	$\frac{65}{15}=\frac{13}{y}$	J	$\frac{8}{14}=\frac{4}{w}$	N	$\frac{3}{11}=\frac{n}{66}$
C	$\frac{22}{10}=\frac{11}{a}$	G	$\frac{14}{42}=\frac{a}{3}$	K	$\frac{9}{5}=\frac{m}{30}$	O	$\frac{28}{14}=\frac{14}{d}$
D	$\frac{5}{9}=\frac{b}{45}$	H	$\frac{6}{18}=\frac{x}{9}$	L	$\frac{18}{36}=\frac{g}{4}$	P	$\frac{3}{8}=\frac{15}{c}$

Looking back

When you work with two or more fractions, you often need to express them as fraction numerals with denominators that are alike. Now you will learn how to find the denominator to use.

A	$\frac{3}{5}$	$\frac{6}{10}$	$\frac{9}{15}$	$\frac{12}{20}$	$\frac{15}{25}$	$\frac{18}{30}$	$\frac{21}{35}$	$\frac{24}{40}$	$\frac{27}{45}$	$\frac{30}{50}$
B	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$
C	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$
D	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$	$\frac{18}{24}$	$\frac{21}{28}$	$\frac{24}{32}$	$\frac{27}{36}$	$\frac{30}{40}$
E	$\frac{1}{6}$	$\frac{2}{12}$	$\frac{3}{18}$	$\frac{4}{24}$	$\frac{5}{30}$	$\frac{6}{36}$	$\frac{7}{42}$	$\frac{8}{48}$	$\frac{9}{54}$	$\frac{10}{60}$
F	$\frac{5}{8}$	$\frac{10}{16}$	$\frac{15}{24}$	$\frac{20}{32}$	$\frac{25}{40}$	$\frac{30}{48}$	$\frac{35}{56}$	$\frac{40}{64}$	$\frac{45}{72}$	$\frac{50}{80}$
G	$\frac{3}{10}$	$\frac{6}{20}$	$\frac{9}{30}$	$\frac{12}{40}$	$\frac{15}{50}$	$\frac{18}{60}$	$\frac{21}{70}$	$\frac{24}{80}$	$\frac{27}{90}$	$\frac{30}{100}$
H	$\frac{11}{12}$	$\frac{22}{24}$	$\frac{33}{36}$	$\frac{44}{48}$	$\frac{55}{60}$	$\frac{66}{72}$	$\frac{77}{84}$	$\frac{88}{96}$	$\frac{99}{108}$	$\frac{110}{120}$
I	$\frac{13}{16}$	$\frac{26}{32}$	$\frac{39}{48}$	$\frac{52}{64}$	$\frac{65}{80}$	$\frac{78}{96}$	$\frac{91}{112}$	$\frac{104}{128}$	$\frac{117}{144}$	$\frac{130}{160}$

A Each row of fraction numerals on the chart on page 66 shows part of a set of names for the same fraction. Row A shows some of the names for $\frac{3}{5}$. What does Row B show?

B Now suppose you want to know which is smaller, $\frac{3}{5}$ or $\frac{2}{3}$. Look at the sets for $\frac{3}{5}$ and $\frac{2}{3}$. Find $\frac{6}{10}$ in the $\frac{3}{5}$ set. Does any fraction numeral in the $\frac{2}{3}$ set have 10 as the denominator?

C Now find $\frac{9}{15}$ in the $\frac{3}{5}$ set. Does any fraction numeral in the $\frac{2}{3}$ set have 15 as the denominator?

D Do $\frac{9}{15}$ and $\frac{10}{15}$ have denominators that are alike? 15 is a common denominator for $\frac{3}{5}$ and $\frac{2}{3}$.

E $\frac{12}{20}$ is also in the $\frac{3}{5}$ set. In the $\frac{2}{3}$ set, does any fraction numeral have 20 as the denominator? Why is 20 not a common denominator for $\frac{3}{5}$ and $\frac{2}{3}$?

F Now find $\frac{18}{30}$ in the $\frac{3}{5}$ set. In the $\frac{2}{3}$ set, is there a fraction numeral with 30 as the denominator? Why is 30 a common denominator for fifths and thirds?

G Do you think there may be other common denominators not on the chart that can be used to compare $\frac{3}{5}$ and $\frac{2}{3}$? Why do you think so?

H Of all the common denominators that can be used to compare $\frac{3}{5}$ and $\frac{2}{3}$, 15 is the lowest. 15 is the lowest common denominator for fifths and thirds. $\frac{3}{5}=\frac{\square}{15}$. $\frac{2}{3}=\frac{\square}{15}$. Which is smaller, $\frac{3}{5}$ or $\frac{2}{3}$?

I Now suppose you need to know which is largest, $\frac{3}{4}$, $\frac{5}{8}$, or $\frac{13}{16}$. On the chart find the sets of fraction numerals for these fractions. What common denominators are shown in blue for $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{13}{16}$?

J The chart shows that 48 is a common denominator for $\frac{5}{8}$ and $\frac{13}{16}$. Can $\frac{3}{4}$ be expressed by a fraction numeral with 48 for the denominator? $\frac{3}{4}=\frac{\square}{48}$. Is 48 a common denominator for fourths, eighths, and sixteenths?

K The lowest common denominator you can use to compare fourths, eighths, and sixteenths is $\square$. $\frac{3}{4}=\frac{\square}{16}$. $\frac{5}{8}=\frac{\square}{16}$. Which is largest, $\frac{3}{4}$, $\frac{5}{8}$, or $\frac{13}{16}$?

L Now find which is smallest, $\frac{2}{3}$, $\frac{1}{2}$, or $\frac{3}{4}$. Find the sets of fraction numerals for these three fractions on the chart. Is 12 a common denominator for $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$?

M Is 24 a common denominator for $\frac{2}{3}$ and $\frac{3}{4}$? Can $\frac{1}{2}$ be expressed by a fraction numeral with 24 for the denominator? $\frac{1}{2}=\frac{\square}{24}$. Is 24 a common denominator for $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$?

N The lowest common denominator you can use to compare thirds, halves, and fourths is $\square$. $\frac{2}{3}=\frac{\square}{12}$. $\frac{1}{2}=\frac{\square}{12}$. $\frac{3}{4}=\frac{\square}{12}$. Which is smallest, $\frac{2}{3}$, $\frac{1}{2}$, or $\frac{3}{4}$?

A You can find common denominators without using sets of fraction numerals. Suppose you want to find a common denominator for $\frac{5}{8}$ and $\frac{3}{10}$. Multiply 10 by 8. $8 \times 10 =$ ▒. Does the chart on page 66 show that 80 is a common denominator for eighths and tenths? Is 80 the lowest common denominator? How does the chart show that 40 is the lowest common denominator for $\frac{5}{8}$ and $\frac{3}{10}$?

B Now think about finding a common denominator for $\frac{2}{3}$ and $\frac{5}{8}$. Multiply the 8 by the 3. $3 \times 8 =$ ▒. Is 24 a common denominator for thirds and eighths? Use the chart to find the lowest common denominator for $\frac{2}{3}$ and $\frac{5}{8}$. What is it?

C You can get a common denominator for $\frac{1}{2}$ and $\frac{1}{6}$ by multiplying 2 by 6. Does the chart show a common denominator lower than 12? What is it? Can $\frac{1}{2}$ be expressed as sixths? The lowest common denominator for halves and sixths is ▒.

D To find the lowest common denominator for $\frac{5}{6}$ and $\frac{7}{8}$, first think: Can $\frac{5}{6}$ be expressed as eighths? Can $\frac{7}{8}$ be expressed as sixths? Which is the higher denominator, 6 or 8?

E Try 16 for a common denominator because 16 is the first number after 8 that can be divided by 8. You know that $\frac{7}{8}$ can be expressed as sixteenths. Can $\frac{5}{6}$ be expressed as sixteenths? Why is 16 not a common denominator for sixths and eighths?

F Now try 24, because the next number after 16 that can be divided by 8 is 24. You can express $\frac{7}{8}$ as twenty-fourths. Can $\frac{5}{6}$ also be expressed as twenty-fourths? 24 is a common denominator for $\frac{5}{6}$ and $\frac{7}{8}$. Why?

G Are there many other common denominators for sixths and eighths? Which is the lowest of all?

H Now find the lowest common denominator for $\frac{5}{6}$ and $\frac{1}{12}$. Can $\frac{5}{6}$ be expressed as twelfths? Is 12 the lowest common denominator for sixths and twelfths?

I When you find the lowest common denominator for $\frac{1}{4}$ and $\frac{1}{6}$, can you use the higher denominator, 6, as a common denominator? Why or why not?

J Try 12, the next number after 6 that can be divided by 6. How do you know that 12 is the lowest common denominator for fourths and sixths?

K When Ann compared $\frac{3}{8}$ and $\frac{5}{12}$, she used 24 for a common denominator. Here is Ann's work.

$\frac{3}{8} = \frac{9}{24}$ $\quad$ $\frac{5}{12} = \frac{10}{24}$

Which is larger, $\frac{3}{8}$ or $\frac{5}{12}$?

L When Joan compared $\frac{3}{8}$ and $\frac{5}{12}$, she used 48 as a common denominator. Here is Joan's work.

$\frac{3}{8} = \frac{18}{48}$ $\quad$ $\frac{5}{12} = \frac{20}{48}$

Which is larger, $\frac{3}{8}$ or $\frac{5}{12}$?

M Can either 24 or 48 be used as a common denominator for $\frac{3}{8}$ and $\frac{5}{12}$?

N Which girl used the lower common denominator? Is 24 the lowest common denominator for eighths and twelfths?

O Which would you use, 24 or 48, to compare eighths and twelfths? Why?

P Suppose you want to compare $\frac{7}{10}$ and $\frac{5}{6}$. Why might you first try 20 for a common denominator? Is 20 a common denominator for tenths and sixths?

Q From the numerals that follow, choose those that can be used as a common denominator to compare $\frac{7}{10}$ and $\frac{5}{6}$: 30, 40, 50, 60, 70, 80, 90, 100. Use one of these denominators to find which is smaller, $\frac{7}{10}$ or $\frac{5}{6}$.

Find the lowest common denominator for each pair of fraction numerals below. Express each pair with fraction numerals having this lowest common denominator.

A $\frac{9}{10}, \frac{4}{5}$ $\quad$ **B** $\frac{2}{3}, \frac{7}{10}$ $\quad$ **C** $\frac{1}{3}, \frac{1}{5}$ $\quad$ **D** $\frac{3}{4}, \frac{5}{8}$ $\quad$ **E** $\frac{7}{12}, \frac{3}{8}$

F $\frac{1}{3}, \frac{5}{6}$ $\quad$ **G** $\frac{5}{6}, \frac{7}{8}$ $\quad$ **H** $\frac{5}{6}, \frac{9}{10}$ $\quad$ **I** $\frac{11}{12}, \frac{2}{3}$ $\quad$ **J** $\frac{5}{6}, \frac{8}{9}$

K $\frac{4}{5}, \frac{11}{20}$ $\quad$ **L** $\frac{7}{8}, \frac{2}{3}$ $\quad$ **M** $\frac{7}{12}, \frac{3}{4}$ $\quad$ **N** $\frac{3}{8}, \frac{5}{16}$ $\quad$ **O** $\frac{5}{9}, \frac{1}{3}$

P $\frac{1}{2}, \frac{11}{16}$ $\quad$ **Q** $\frac{3}{16}, \frac{1}{4}$ $\quad$ **R** $\frac{7}{9}, \frac{1}{2}$ $\quad$ **S** $\frac{2}{3}, \frac{3}{4}$ $\quad$ **T** $\frac{1}{4}, \frac{3}{10}$

Now you should be able to find common denominators for two or more fraction numerals.

Keeping skilful

Express each pair of fraction numerals with the lowest common denominator.

A $\frac{3}{4}, \frac{1}{5}$ $\quad$ **B** $\frac{3}{8}, \frac{1}{6}$ $\quad$ **C** $\frac{4}{5}, \frac{1}{2}$ $\quad$ **D** $\frac{11}{12}, \frac{7}{8}$ $\quad$ **E** $\frac{3}{10}, \frac{5}{6}$

F $\frac{5}{6}, \frac{1}{4}$ $\quad$ **G** $\frac{8}{15}, \frac{1}{6}$ $\quad$ **H** $\frac{5}{9}, \frac{5}{6}$ $\quad$ **I** $\frac{3}{4}, \frac{7}{10}$ $\quad$ **J** $\frac{3}{4}, \frac{7}{9}$

A $8236 + a = 10000$

B $n - 7159 = 8203$

C $6245 + 3857 = c$

D $79511 - m = 54280$

E $n + 2056 = 5111$

F $11384 - 696 = d$

G $w - 4938 = 6534$

H $62065 - n = 31138$

I $z + 7508 = 9112$

J $54312 + x = 70039$

If there is a remainder, use it to make a fraction numeral in the answer.

A $3686 \div n = 97$

B $8706 \div 55 = c$

C $2598 \div 48 = y$

D $67640 \div 712 = n$

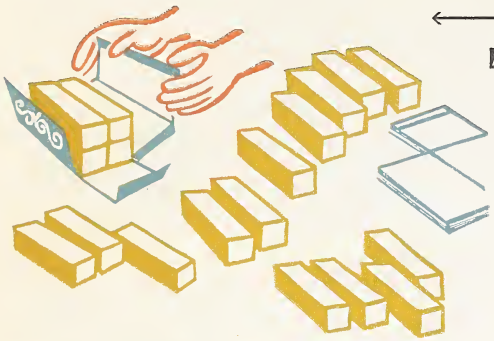
E $53940 \div e = 620$

F $5064 \div n = 64$

G $42001 \div 236 = a$

Looking back

Sometimes you will need to change improper fractions to other forms. Now you will learn how to do this.



A

Each stick of butter in Picture A is $\frac{1}{4}$ of a whole pound. The 18 fourths are being arranged in whole pounds. It takes four $\frac{1}{4}$-lb. sticks to make 1 pound.

It takes 4 fourths to make 1 whole. $\frac{4}{4} = 1$

B

The 18 fourths have been arranged as whole pounds and part of another pound. There are 4 whole pounds and 2 fourths of another pound.

You can use arithmetic to find how many wholes, or ones, there are in 18 fourths. Divide the 18 by 4.

$18 \div 4 = 4$ and 2 remainder. There are 4 ones, and 2 fourths are left over.

$\frac{18}{4} = 4$ ones and 2 fourths $\frac{18}{4} = 4\frac{2}{4}$

$4\frac{2}{4}$ in lowest terms is $4\frac{1}{2}$.

A How many sixths are there in 1?

B How many thirds are there in 1?

C How many tenths are there in 1?

D How many fifteenths are there in 1?

E Imagine that you have 21 thirds of equal-sized paper disks. To find how many whole disks can be made from the 21 thirds, first think: It takes ▒ thirds to make 1 whole. $\frac{▒}{3} = 1$

F To find how many wholes, or ones, there are in 21 thirds, divide ▒ by 3. $21 \div 3 = ▒$. There are ▒ ones. No thirds are left over. The 21 thirds equal ▒ ones. $\frac{21}{3} = ▒$

G To change $\frac{56}{5}$ to a whole number or a mixed number, first think: $\frac{\square}{5} = 1$. Now divide 56 by $\square$. $56 \div 5 = \square$ ones and $\square$ remainder. $\frac{56}{5} = 11\frac{\square}{5}$

H To change $\frac{150}{24}$ to a whole number or a mixed number, think: $\frac{\square}{24} = 1$. Divide 150 by $\square$. $150 \div 24 = \square$ ones and $\square$ remainder. $\frac{150}{24} = 6\frac{\square}{24}$. Write $6\frac{6}{24}$ in lowest terms. $\frac{150}{24} = 6\frac{1}{\square}$

Change each improper fraction below to a mixed number or a whole number. Be sure that each mixed number is expressed in lowest terms.

A	B	C	D	E	F
$\frac{19}{5}$	$\frac{86}{12}$	$\frac{39}{8}$	$\frac{40}{5}$	$\frac{11}{2}$	$\frac{52}{8}$
G	**H**	**I**	**J**	**K**	**L**
$\frac{62}{3}$	$\frac{48}{16}$	$\frac{30}{9}$	$\frac{36}{28}$	$\frac{128}{64}$	$\frac{48}{32}$
M	**N**	**O**	**P**	**Q**	**R**
$\frac{18}{4}$	$\frac{56}{8}$	$\frac{96}{64}$	$\frac{39}{14}$	$\frac{48}{3}$	$\frac{115}{30}$

Now you can change improper fractions to mixed numbers or whole numbers when you need to.

Keeping skilful

Find the sum.

A 27, 41, 80, 13
B 56, 9, 34, 78
C 15, 63, 22, 26, 40
D 689, 371, 508
E 267, 155, 438, 77
F 326, 307, 411, 193
G 692, 833, 575, 620
H 721, 548, 900, 866
I 8314, 2751, 6449
J 5030, 974, 3189, 5262
K 9657, 1664, 5032, 865
L 2851, 1563, 2490, 4134
M 4266, 9500, 5272, 8524
N 5607, 4710, 1836, 3219
O 6538, 465, 212, 3005
P 1400, 9624, 1308, 28
Q 873, 7115, 91, 6330

A $728 + n = 900$
B $t - 834 = 1096$
C $1364 - c = 687$
D $5210 - 4733 = n$
E $z + 2306 = 4008$
F $3149 + n = 6232$
G $48797 - 25289 = g$
H $a - 7380 = 10543$
I $42130 - n = 9958$
J $s + 59106 = 83341$
K $92483 - 8775 = m$
L $r - 6543 = 7020$
M $48391 + n = 55982$
N $83755 - e = 16904$
O $q + 39486 = 70020$
P $96524 - n = 2357$
Q $h - 10001 = 72999$
R $y + 35837 = 76111$

A $4 \times 509 = d$
B $73 \times 25 = n$
C $828 \times 7 = r$
D $6 \times 5080 = c$
E $49 \times 84 = f$
F $26 \times 703 = n$
G $528 \times 47 = p$
H $62 \times 189 = n$
I $434 \times 225 = w$
J $204 \times 391 = n$
K $58 \times 4090 = n$
L $37 \times 6648 = k$
M $987 \times 456 = v$
N $2736 \times 51 = c$
O $6009 \times 87 = n$
P $594 \times 602 = q$
Q $89 \times 3474 = t$
R $787 \times 653 = n$

Checking up

Test 1

Write two fraction numerals equal to each of the following. Write one with lower terms and one with higher terms.

A $\frac{8}{12}$	G $\frac{6}{9}$	M $\frac{10}{12}$	S $\frac{9}{36}$
B $\frac{15}{20}$	H $\frac{8}{10}$	N $\frac{14}{16}$	T $\frac{4}{20}$
C $\frac{12}{16}$	I $\frac{3}{12}$	O $\frac{5}{25}$	U $\frac{3}{9}$
D $\frac{25}{30}$	J $\frac{5}{15}$	P $\frac{6}{18}$	V $\frac{10}{15}$
E $\frac{6}{8}$	K $\frac{4}{32}$	Q $\frac{5}{10}$	W $\frac{4}{8}$
F $\frac{4}{6}$	L $\frac{9}{24}$	R $\frac{16}{24}$	X $\frac{3}{18}$

Test 2

Write two ratios equal to each ratio below. Write one with lower terms and one with higher terms.

A $\frac{12}{15}$	G $\frac{14}{24}$	M $\frac{14}{18}$	S $\frac{3}{15}$
B $\frac{4}{12}$	H $\frac{28}{32}$	N $\frac{9}{3}$	T $\frac{20}{25}$
C $\frac{16}{10}$	I $\frac{15}{30}$	O $\frac{30}{5}$	U $\frac{6}{30}$
D $\frac{18}{24}$	J $\frac{36}{24}$	P $\frac{46}{14}$	V $\frac{28}{21}$
E $\frac{21}{9}$	K $\frac{14}{35}$	Q $\frac{7}{21}$	W $\frac{11}{22}$
F $\frac{14}{7}$	L $\frac{18}{10}$	R $\frac{18}{12}$	X $\frac{35}{5}$

Test 3

In each exercise one ratio has a letter. Find the numeral that will replace it.

A $\frac{3}{5}=\frac{z}{25}$	E $\frac{12}{48}=\frac{2}{m}$	I $\frac{18}{27}=\frac{2}{e}$
B $\frac{10}{25}=\frac{2}{n}$	F $\frac{36}{4}=\frac{d}{1}$	J $\frac{28}{14}=\frac{14}{n}$
C $\frac{14}{2}=\frac{b}{1}$	G $\frac{10}{30}=\frac{t}{3}$	K $\frac{5}{1}=\frac{y}{7}$
D $\frac{1}{6}=\frac{r}{36}$	H $\frac{7}{8}=\frac{21}{f}$	L $\frac{17}{12}=\frac{s}{36}$

Test 4

Change each improper fraction to a mixed number or to a whole number. Write the fraction numeral in each mixed number in lowest terms.

A $\frac{40}{8}$	D $\frac{64}{30}$	G $\frac{25}{12}$	J $\frac{93}{24}$
B $\frac{56}{6}$	E $\frac{21}{3}$	H $\frac{40}{9}$	K $\frac{150}{12}$
C $\frac{29}{7}$	F $\frac{19}{8}$	I $\frac{64}{32}$	L $\frac{80}{20}$

Test 5

Write each pair of fraction numerals with the lowest common denominator.

A $\frac{1}{2}, \frac{7}{16}$	F $\frac{7}{12}, \frac{1}{2}$	K $\frac{7}{9}, \frac{5}{12}$
B $\frac{7}{10}, \frac{4}{5}$	G $\frac{9}{10}, \frac{2}{3}$	L $\frac{5}{9}, \frac{4}{5}$
C $\frac{3}{5}, \frac{1}{4}$	H $\frac{2}{3}, \frac{3}{5}$	M $\frac{3}{4}, \frac{5}{6}$
D $\frac{3}{4}, \frac{3}{10}$	I $\frac{3}{8}, \frac{1}{6}$	N $\frac{8}{15}, \frac{7}{10}$
E $\frac{9}{10}, \frac{5}{6}$	J $\frac{5}{12}, \frac{3}{4}$	O $\frac{2}{3}, \frac{5}{8}$

End-of-block tests on reduction of fraction numerals and ratios; on finding the missing term; on finding the lowest common denominator

Using arithmetic

A When Mr. Olds planned to paint the kitchen, he found that there were 675 sq. ft. of surface to paint. How many square feet would he paint in all if he gave the kitchen 2 coats of paint?

B Mr. Olds knew that 1 gal. of paint would cover 450 sq. ft. How many gallons of paint would he need for the 2 coats?

C The paint that Mr. Olds bought cost $4.29 per gallon. How much in all did he have to pay for this paint and for 1 qt. of enamel that cost $1.95?

D If Mr. Olds had bought the paint in quart cans, it would have cost $15. How much less did he pay for the paint by buying it in gallons?

E Mr. Olds bought a large paintbrush for $5.39 and a small brush for $2.98. He spent how much more for the large brush than for the small one?

F Mr. Olds spent how much in all for the paintbrushes, paint, and enamel?

G June Olds helped her mother with the housework for 50 min. on Monday, 35 min. on Tuesday, 40 min. on Thursday, and 55 min. on Saturday. She helped with the housework for how many hours that week?

H June's mother paid her 50¢ per hour for her work. June added her earnings to her savings of $9.04. How much money did she have then?

I June worked an average of how many minutes per day for the 6 days from Monday through Saturday?

J June used part of her savings to buy a sweater. After she had bought the sweater, she still had $2.56 left. How much had the sweater cost?

K Mrs. Olds sent Philip to buy some bananas and apples. He bought 2 lb. of bananas at 28¢ per pound and 3 lb. of apples at 18¢ per pound. How much did Philip pay for all the fruit?

L How much change should Philip have received from a 5-dollar bill?

M When Mrs. Olds put shelf paper on the cupboard shelves, she needed enough paper for 6 shelves that were each 4 ft. long. How many yards of shelf paper did she need?

N Mrs. Olds used 16 ft. of shelf paper from a leftover roll. How much more shelf paper did she need from a second roll to cover all 6 shelves?

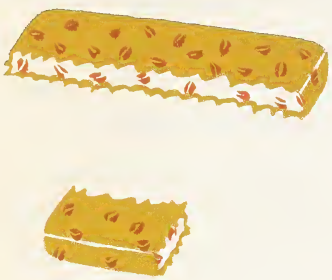
O Mr. Olds paid Philip $4.20 for washing 28 windows. How much did he pay Philip per window?

Learning how

In this lesson you will learn how to add fractions.

see

A



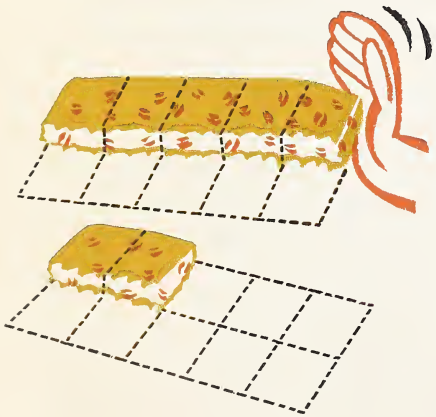
Susan gave $\frac{1}{5}$ of a candy bar to Ann and $\frac{1}{2}$ of the same candy bar to Tom. How much of the bar did she give to Ann and Tom?

$\frac{1}{5} + \frac{1}{2} = n$

Holds a place for the total amount given away

Imagine the $\frac{1}{2}$ bar put with the $\frac{1}{5}$ bar. You need a name for the part given away.

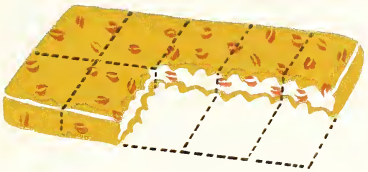
B



Picture B shows how you can find a name for the new fraction that tells how much was given away. Think of the $\frac{1}{5}$ of the bar as $\frac{2}{10}$ of the bar. Then think of the $\frac{1}{2}$ of the bar as $\frac{5}{10}$ of the bar.

You can also find the new fraction by adding the fractions $\frac{1}{5}$ and $\frac{1}{2}$. When you add, you use a common denominator. You can change $\frac{1}{5}$ to tenths. You can also change $\frac{1}{2}$ to tenths.

$\frac{1}{5} = \frac{2}{10}$ $\frac{1}{2} = \frac{5}{10}$ $\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10}$



C

You can see from Picture C that $\frac{7}{10}$ of the bar was given away.

You can also get the answer $\frac{7}{10}$ by adding the fractions $\frac{2}{10}$ and $\frac{5}{10}$.

Add the numerators. The sum is 7.

$$\frac{2}{10}+\frac{5}{10}=\frac{7}{10}$$

Write 7 as the numerator. You are adding tenths. So write 10 as the denominator.

Susan gave $\frac{7}{10}$ of the bar to Ann and Tom.

$\frac{1}{5}+\frac{1}{2}=\frac{7}{10}$

See

A

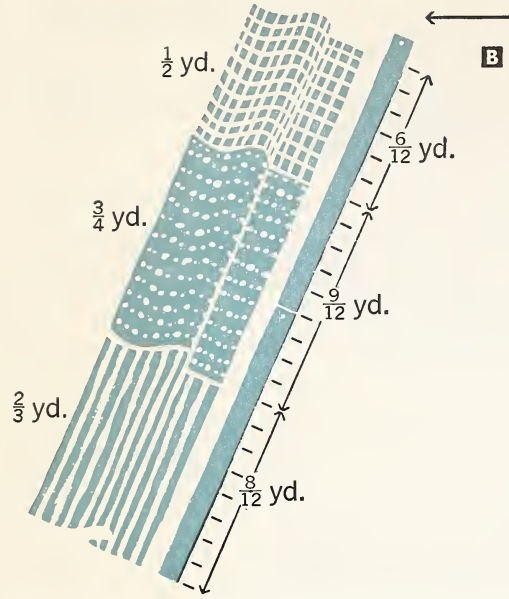
Alice bought cloth to make stuffed toys. She bought $\frac{2}{3}$ yd. of one kind of cloth, $\frac{3}{4}$ yd. of another kind, and $\frac{1}{2}$ yd. of a third kind. How many yards of cloth in all did she buy?

$$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}=t$$

Holds a place for the total number of yards bought

Imagine the $\frac{2}{3}$ yd., the $\frac{3}{4}$ yd., and the $\frac{1}{2}$ yd. as being put together. You need a name for the new fraction that tells how many yards there are in all.

Now turn the page.



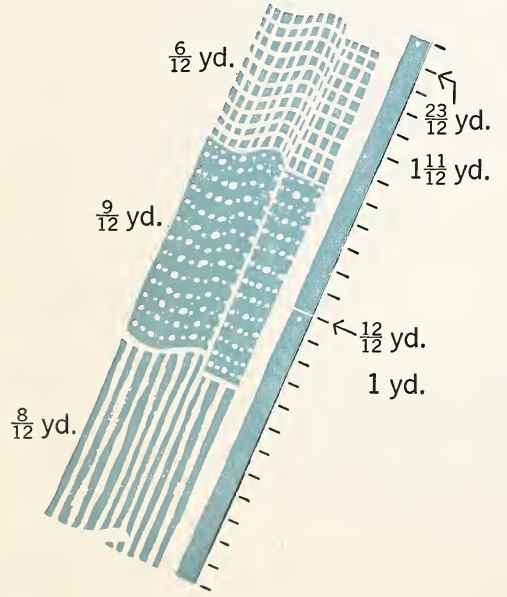
Picture B shows how to find the name for this new fraction. Think of the $\frac{2}{3}$ yd. as $\frac{8}{12}$ yd. Think of the $\frac{3}{4}$ yd. as $\frac{9}{12}$ yd. Think of the $\frac{1}{2}$ yd. as $\frac{6}{12}$ yd. You know that the new fraction will be a certain number of twelfths.

You can also find the new fraction by adding the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$. You can use 12 as a common denominator and change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to twelfths.

$\frac{2}{3}=\frac{\square}{12}$ $\frac{3}{4}=\frac{\square}{12}$ $\frac{1}{2}=\frac{\square}{12}$

$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}=\frac{8}{12}+\frac{9}{12}+\frac{6}{12}$

Picture C shows that Alice bought a total of $\frac{23}{12}$ yd.



You can also get the answer by adding.

Add the numerators. The sum is 23.

$$\frac{8}{12}+\frac{9}{12}+\frac{6}{12}=\frac{23}{12}$$

Write 23 as the numerator. You are adding twelfths. So write 12 as the denominator.

Picture C shows that $\frac{23}{12}$ yd. is more than one whole yard. You can see that Alice bought 1 whole yard and $\frac{11}{12}$ of another yard.

$\frac{23}{12}=1\frac{11}{12}$

Alice bought $1\frac{11}{12}$ yd. of cloth in all.

$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}=1\frac{11}{12}$

think

A

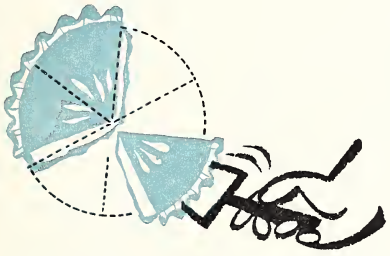
Mrs. Cook gave $\frac{1}{3}$ of a pie to Mrs. Page. She gave Jim $\frac{1}{6}$ of the same pie for his lunch. How much of the pie did she give away?

$\frac{1}{3} + \frac{1}{6} = n$ ← Holds a place for the amount of pie given away

If you put $\frac{1}{6}$ of the pie with $\frac{1}{3}$ of the pie, would you know the name of the new fraction of the pie?

B

← Picture B shows how to find the name for this new fraction. Can you think of both $\frac{1}{3}$ and $\frac{1}{6}$ as a number of sixths? Think of one third as ▒ sixths.



When you compute to find the new fraction, you must use a common denominator. You can change $\frac{1}{3}$ to $\frac{▒}{6}$.

$\frac{1}{3} = \frac{▒}{6}$ $\quad \frac{1}{3} + \frac{1}{6} = \frac{▒}{6} + \frac{▒}{6}$

C

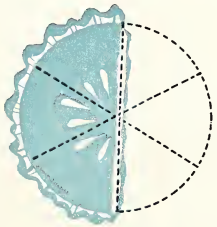
← The new fraction is ▒ sixths of the pie.

What fractions can you add to get the answer?

What do you do with the numerators?

$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

Why is 3 written in the numerator?
Why is 6 written in the denominator?



Is $\frac{3}{6}$ in lowest terms?

$\frac{3}{6} = \frac{▒}{2}$

Mrs. Cook gave $\frac{▒}{▒}$ of the pie away.

$\frac{1}{3} + \frac{1}{6} = n$

try

A $\frac{1}{10}+\frac{7}{10}=x$

Do you need to change the fraction numerals?

$\frac{1}{10}+\frac{7}{10}=\frac{8}{10}$ or $\frac{4}{5}$

How do you get the 8 in $\frac{8}{10}$?

How do you get $\frac{4}{5}$?

You may write your work this way if you want to.

$$\begin{array}{r} \frac{1}{10} \\ +\ \frac{7}{10} \\ \hline \frac{8}{10} \text{ or } \frac{4}{5} \end{array}$$

B $\frac{2}{3}+\frac{7}{8}+\frac{5}{6}=f$

What common denominator do you use?

$\frac{16}{24}+\frac{21}{24}+\frac{20}{24}=f$

$\frac{16}{24}+\frac{21}{24}+\frac{20}{24}=\frac{57}{24}$ or $2\frac{9}{24}$ or $2\frac{3}{8}$

How do you get the 57 in $\frac{57}{24}$?

How do you know that $2\frac{3}{8}$ is in lowest terms?

You can also write your work for Example B in this way.

$$\begin{array}{r} \frac{2}{3}=\frac{16}{24} \\ \frac{7}{8}=\frac{21}{24} \\ +\ \frac{5}{6}=\frac{20}{24} \\ \hline \frac{57}{24} \text{ or } 2\frac{9}{24} \text{ or } 2\frac{3}{8} \end{array}$$

C $\frac{2}{5}+\frac{2}{3}+\frac{4}{5}=t$

What common denominator do you use?

$\frac{6}{15}+\frac{10}{15}+\frac{12}{15}=t$

$\frac{6}{15}+\frac{10}{15}+\frac{12}{15}=\frac{28}{15}$ or $1\frac{13}{15}$

How do you know that $1\frac{13}{15}$ is in lowest terms?

$$\begin{array}{r} \frac{2}{5}=\frac{6}{15} \\ \frac{2}{3}=\frac{10}{15} \\ +\ \frac{4}{5}=\frac{12}{15} \\ \hline \frac{28}{15} \text{ or } 1\frac{13}{15} \end{array}$$

D $\frac{1}{4}+\frac{11}{12}+\frac{5}{6}=d$

What common denominator do you use?

$\frac{3}{12}+\frac{11}{12}+\frac{10}{12}=d$

$\frac{3}{12}+\frac{11}{12}+\frac{10}{12}=\frac{24}{12}$ or 2

$$\begin{array}{r} \frac{1}{4}=\frac{3}{12} \\ \frac{11}{12}=\frac{11}{12} \\ +\ \frac{5}{6}=\frac{10}{12} \\ \hline \frac{24}{12} \text{ or } 2 \end{array}$$

do

A Tom bought $\frac{5}{8}$ lb. of fudge and $\frac{1}{4}$ lb. of caramels. What fraction of a pound of candy did Tom buy in all?
$\frac{5}{8}+\frac{1}{4}=p$

B Bill sold $\frac{1}{2}$ page of advertising space in the yearbook, Ellen sold $\frac{3}{8}$ of a page, and Mary sold $\frac{3}{4}$ of a page. How much space did they sell in all?
$\frac{1}{2}+\frac{3}{8}+\frac{3}{4}=h$

Show your answers in lowest terms.

C $\frac{2}{3}+\frac{1}{6}=n$
D $\frac{7}{10}+\frac{3}{10}=z$
E $\frac{3}{8}+\frac{1}{4}+\frac{7}{12}=n$
F $\frac{3}{5}+\frac{1}{2}=c$
G $\frac{3}{16}+\frac{5}{8}=p$
H $\frac{1}{12}+\frac{5}{12}=n$
I $\frac{5}{6}+\frac{7}{8}+\frac{3}{4}=a$

J $\frac{2}{3}+\frac{3}{5}=n$
K $\frac{3}{10}+\frac{1}{2}=y$
L $\frac{7}{12}+\frac{3}{4}+\frac{2}{3}=n$
M $\frac{4}{5}+\frac{4}{5}=d$
N $\frac{2}{9}+\frac{1}{6}+\frac{1}{2}=n$
O $\frac{5}{12}+\frac{3}{8}=r$
P $\frac{1}{7}+\frac{1}{2}+\frac{3}{7}=n$

A $\frac{5}{16}+\frac{1}{2}=n$
B $\frac{5}{7}+\frac{2}{3}+\frac{3}{7}=n$
C $\frac{2}{5}+\frac{1}{4}=t$
D $\frac{2}{5}+\frac{3}{10}=b$
E $\frac{1}{6}+\frac{3}{4}+\frac{5}{12}=w$
F $\frac{2}{3}+\frac{2}{3}=n$
G $\frac{5}{6}+\frac{3}{10}+\frac{1}{5}=q$

H $\frac{4}{9}+\frac{1}{4}+\frac{7}{12}=f$
I $\frac{7}{8}+\frac{1}{2}=n$
J $\frac{1}{3}+\frac{5}{16}=w$
K $\frac{7}{10}+\frac{3}{4}=n$
L $\frac{1}{6}+\frac{1}{8}=x$
M $\frac{4}{5}+\frac{7}{12}+\frac{1}{3}=n$
N $\frac{1}{9}+\frac{5}{6}=s$

Now you should know how to add fractions and give the answers in different forms.

Keeping skilful

A $15732+n=23600$
B $4379-208=x$
C $6519+13929=n$
D $3788-a=928$
E $n+3502=7100$
F $q-1918=1620$
G $5440-1066=b$
H $38412+96755=n$
I $5639-n=2487$
J $71861-51492=w$
K $h+9375=12022$
L $87339+16504=n$
M $27000-6139=e$
N $n+6398=32003$

A $135\times86=n$
B $724\times903=t$
C $38\times156=m$
D $553\times19=n$
E $88\times88=d$
F $614\times93=s$
G $85\times975=n$
H $600\times23=w$
I $111\times586=q$
J $199\times73=n$
K $414\times643=n$
L $529\times780=y$
M $219\times79=n$
N $4342\times66=n$

If there is a remainder, write a fraction numeral in the answer.

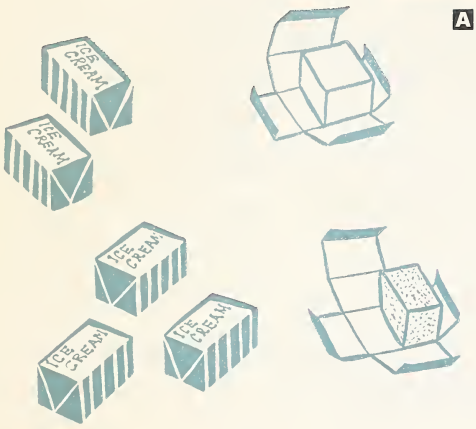
A $1894\div62=n$
B $2492\div b=356$
C $783728\div803=n$
D $98680\div25=h$
E $333\div k=11$
F $15625\div125=s$
G $27679\div38=n$
H $97902\div441=a$
I $3419\div c=17$
J $9604\div n=98$
K $46368\div322=f$

Learning how

Now you will learn how to add mixed numbers and fractions.

see

A



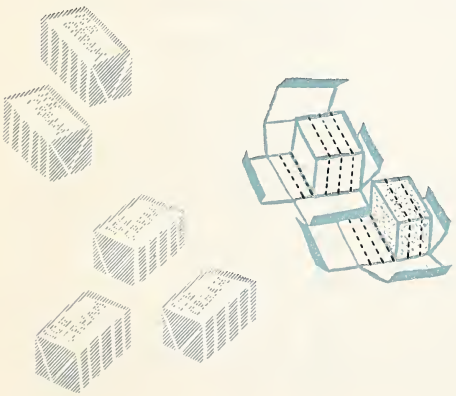
Mrs. Newton has $2\frac{2}{3}$ pt. of vanilla ice cream and $3\frac{1}{2}$ pt. of blueberry ice cream in her freezer. She has how much ice cream in her freezer?

$2\frac{2}{3} + 3\frac{1}{2} = n$

Holds a place for the total number of pints

To find the total number of pints of ice cream, put the $\frac{2}{3}$ pt. and the $\frac{1}{2}$ pt. together before you put the whole pints together. You need a new name for this fractional part of the ice cream.

B



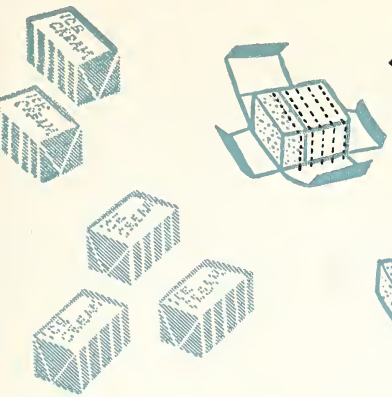
Picture B shows how to find a name for the new fraction that tells how much ice cream there is when you put $\frac{2}{3}$ pt. and $\frac{1}{2}$ pt. together. Think of the $\frac{2}{3}$ pt. as $\frac{4}{6}$ pt. and the $\frac{1}{2}$ pt. as $\frac{3}{6}$ pt. There are $\frac{7}{6}$ pt. in all.

You can use arithmetic to find the total number of pints. You add the numbers $2\frac{2}{3}$ and $3\frac{1}{2}$. First you add $\frac{2}{3}$ and $\frac{1}{2}$. You must find a common denominator. You can change $\frac{2}{3}$ to $\frac{4}{6}$, and you can change $\frac{1}{2}$ to $\frac{3}{6}$.

The sum of $\frac{4}{6}$ and $\frac{3}{6}$ is $\frac{7}{6}$.

$2\frac{2}{3} = 2\frac{4}{6}$

$3\frac{1}{2} = 3\frac{3}{6}$



← $\frac{7}{6}$ pt. will make 1 whole pint and $\frac{1}{6}$ of a pint.

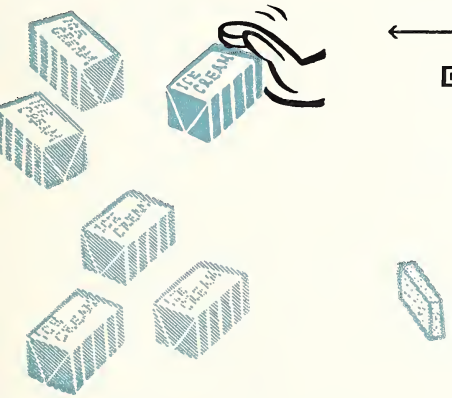
C When you compute, change $\frac{7}{6}$ to a mixed number.

The sum is $1\frac{1}{6}$.

$$2\frac{2}{3} = 2\frac{4}{6}$$
$$3\frac{1}{2} = 3\frac{3}{6}$$

Remember the 1.

$\frac{1}{6}$ ← Write $\frac{1}{6}$ here.



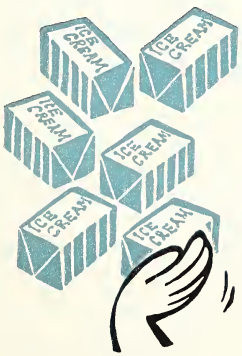
← Put the new whole pint with the other whole pints.

D

In your computation, you put the new 1 with the whole numbers.

1 ← Write 1 in the ones' column to show that there is 1 more whole.

$$2\frac{2}{3} = 2\frac{4}{6}$$
$$3\frac{1}{2} = 3\frac{3}{6}$$
$$\frac{1}{6}$$



← Put the whole pints together.

E When you compute, you add the whole numbers.

1 ← Add the ones. The sum is 6.

$$2\frac{2}{3} = 2\frac{4}{6}$$
$$3\frac{1}{2} = 3\frac{3}{6}$$
$$6\frac{1}{6}$$

Write 6 here. →

$6\frac{1}{6}$ pt. of ice cream are in the freezer.

$2\frac{2}{3} + 3\frac{1}{2} = 6\frac{1}{6}$

think

A

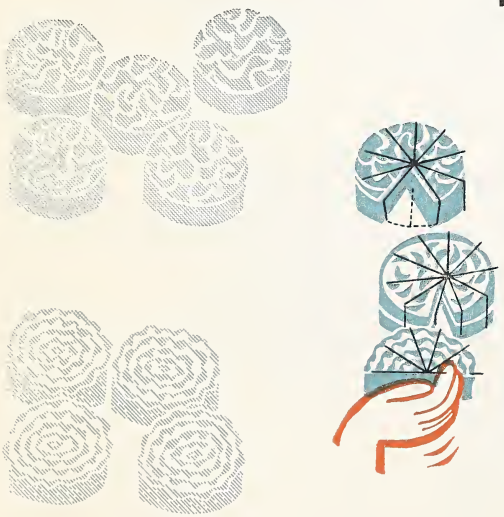
One day in his restaurant Mr. Miner sold $5\frac{4}{5}$ chocolate cakes, $\frac{9}{10}$ of a banana cake, and $4\frac{1}{2}$ spice cakes. Mr. Miner sold how much cake in all that day?

$5\frac{4}{5} + \frac{9}{10} + 4\frac{1}{2} = n$

Holds a place for the number that tells how much cake was sold

First put together the parts of cakes. Do you know what name to use for the fractional part of the cakes that you will have?

Picture B shows how to find a name for the new fraction that the parts of cakes will make. Why can you think of each fraction of cake as marked off to show tenths of a whole cake?



B

Think of $\frac{4}{5}$ of a cake as $\frac{\square}{10}$ of a cake, and $\frac{1}{2}$ of a cake as $\frac{\square}{10}$. There will be $\frac{\square}{10}$ in all.

When you use arithmetic, what will you add first? What common denominator will you use?

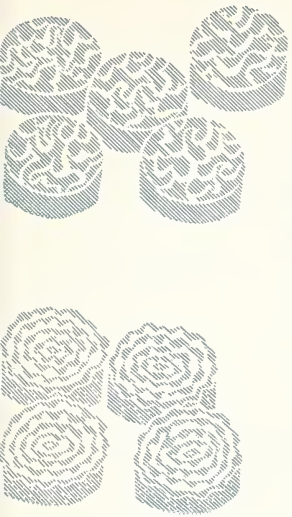
$\frac{4}{5} = \frac{\square}{10}$ $\frac{1}{2} = \frac{\square}{10}$

$5\frac{4}{5} = 5\frac{8}{10}$ The sum of the fractions is $\frac{\square}{10}$.

$\frac{9}{10} = \frac{9}{10}$

$4\frac{1}{2} = 4\frac{5}{10}$

$\frac{22}{10}$ cakes are more than 1 cake.



C

There are ▒ new whole cakes. There is also $\frac{▒}{10}$ of another cake.

When you compute, change $\frac{22}{10}$ to a mixed number.

The sum is $2\frac{▒}{10}$.

$$5\frac{4}{5} = 5\frac{8}{10}$$
$$\frac{9}{10} = \frac{9}{10}$$
$$4\frac{1}{2} = 4\frac{5}{10}$$

$\frac{2}{10}$ ← Why is $\frac{2}{10}$ written here? What must you remember?

D

Think of the ▒ new cakes as being put with the other whole cakes.

In your computation, what should you do with the 2 that you were to remember?

2 ← Why may 2 be written here?

$$5\frac{4}{5} = 5\frac{8}{10}$$
$$\frac{9}{10} = \frac{9}{10}$$
$$4\frac{1}{2} = 4\frac{5}{10}$$
$$\frac{2}{10}$$

Now turn the page.

What do you put together now? There are ▩ whole cakes.

E

When you compute, what do you add now?

What numbers do you add? The sum is ▩.

$$5\frac{4}{5} = 5\frac{8}{10}$$
$$\frac{9}{10} = \frac{9}{10}$$
$$4\frac{1}{2} = 4\frac{5}{10}$$
$$11\frac{2}{10}$$

Why do you write 11 here?

F

How much cake in all did Mr. Miner sell?

$5\frac{4}{5} + \frac{9}{10} + 4\frac{1}{2} = n$

Is the answer in lowest terms? $\frac{2}{10} = \frac{▩}{5}$

Mr. Miner sold ▩ cakes.

---

try

A Mrs. Philips bought $2\frac{5}{8}$ lb. of hamburger and a steak that weighed $1\frac{2}{3}$ lb. How much in all did the hamburger and the steak weigh?

$2\frac{5}{8} + 1\frac{2}{3} = n$

Either remember this 1 or write it.

$$\overset{1}{}$$
$$2\frac{5}{8} = 2\frac{15}{24}$$
$$1\frac{2}{3} = 1\frac{16}{24}$$
$$4\frac{7}{24}$$

B $3\frac{3}{8}+6\frac{1}{6}=y$

$3\frac{3}{8}=3\frac{9}{24}$
$6\frac{1}{6}=6\frac{4}{24}$
$9\frac{13}{24}$

C $1\frac{1}{5}+\frac{2}{3}=a$

$1\frac{1}{5}=1\frac{3}{15}$
$\frac{2}{3}=\frac{10}{15}$
$1\frac{13}{15}$

D $9\frac{1}{2}+1\frac{3}{4}=m$

$9\frac{1}{2}=9\frac{2}{4}$
$1\frac{3}{4}=1\frac{3}{4}$
$11\frac{1}{4}$

Why do you write 11 here instead of 10?

E $4\frac{1}{9}+2\frac{5}{9}=z$

$4\frac{1}{9}$
$2\frac{5}{9}$
$6\frac{6}{9}$ or $6\frac{2}{3}$

F $\frac{3}{10}+7\frac{4}{5}+5\frac{3}{4}=c$

$\frac{3}{10}=\frac{6}{20}$
$7\frac{4}{5}=7\frac{16}{20}$
$5\frac{3}{4}=5\frac{15}{20}$
$13\frac{17}{20}$

G $11\frac{5}{12}+23\frac{1}{4}+8\frac{1}{3}=n$

$11\frac{5}{12}=11\frac{5}{12}$
$23\frac{1}{4}=23\frac{3}{12}$
$8\frac{1}{3}=8\frac{4}{12}$
43

Why is there no fraction numeral in the answer?

do

A One Saturday Tony spent $2\frac{3}{4}$ hours working in the yard, $1\frac{1}{3}$ hours on his homework, and $1\frac{1}{2}$ hours on his hobby. How much time did he spend on these three things?
$2\frac{3}{4}+1\frac{1}{3}+1\frac{1}{2}=n$

B The distance from Don's home to Bill's is $7\frac{3}{10}$ miles. Jim's home is $12\frac{3}{4}$ miles beyond Bill's on the same road. How far is it from Don's home to Jim's home?
$7\frac{3}{10}+12\frac{3}{4}=v$

Show any fractions in your answers in lowest terms.

C $6\frac{1}{8}+3\frac{2}{3}=d$
D $5\frac{1}{6}+\frac{9}{10}=y$
E $2\frac{4}{5}+9\frac{3}{5}=n$
F $1\frac{7}{12}+3\frac{5}{6}=k$
G $8\frac{3}{4}+7\frac{2}{3}=a$
H $6\frac{1}{7}+\frac{4}{5}=z$
I $\frac{13}{16}+2\frac{3}{8}=n$

J $3\frac{1}{4}+5\frac{1}{4}=p$
K $8\frac{5}{9}+15\frac{1}{3}=e$
L $13\frac{7}{12}+26\frac{5}{8}=n$
M $\frac{1}{2}+19\frac{3}{5}=t$
N $7\frac{1}{4}+20\frac{5}{6}=b$
O $28\frac{3}{10}+5\frac{2}{3}=n$
P $17\frac{5}{7}+3\frac{1}{2}=x$

Q $\frac{11}{12}+17\frac{3}{4}=k$
R $22\frac{4}{5}+19\frac{1}{5}=n$
S $3\frac{1}{3}+\frac{5}{12}+4\frac{3}{8}=r$
T $15\frac{1}{2}+\frac{3}{4}+6\frac{3}{4}=c$
U $9\frac{1}{16}+2\frac{5}{8}+1\frac{7}{16}=n$
V $\frac{2}{5}+6\frac{5}{6}+3\frac{9}{10}=w$
W $8\frac{2}{3}+5\frac{1}{7}+11\frac{3}{7}=t$

A $16\frac{1}{2}+5\frac{1}{2}=n$
B $23\frac{2}{3}+12\frac{5}{6}=s$
C $9\frac{3}{4}+2\frac{1}{5}=f$
D $\frac{5}{9}+15\frac{7}{12}=n$
E $13\frac{5}{8}+\frac{7}{8}=v$
F $4\frac{9}{10}+6\frac{1}{15}=g$
G $7\frac{13}{16}+7\frac{1}{4}=n$

Now you should be able to add mixed numbers and fractions.

Using arithmetic

A On a trip to the Trent Canal, John used $\frac{3}{4}$ of a roll of film on the way, $2\frac{1}{8}$ rolls when he went through the locks, and $1\frac{3}{4}$ rolls on the way home. He had used how much film?

B When John arrived home, he had $\frac{3}{8}$ of a roll of unused film in his camera and 1 full roll of film that he had not used. How many rolls of film had John taken with him?

C When Tony's Boy Scout troop made Indian headdresses, they first cut leather into strips and then sewed the ends of the strips together. They used one strip $\frac{1}{6}$ yd. long, one strip $\frac{5}{8}$ yd. long, one strip $\frac{2}{3}$ yd. long, and one strip $\frac{3}{4}$ yd. long. About how long was the piece of leather after all the strips had been sewn together?

D George made a carrier for his knapsack. He cut 3 pieces from a board. One piece of board was $10\frac{1}{2}$ in. long, one piece was $9\frac{1}{2}$ in. long, and the third piece was $8\frac{3}{4}$ in. long. A piece of board $3\frac{5}{16}$ in. long was left over. How long was the board before George cut it up?

E George also needed 3 pieces of webbing. He needed one piece $3\frac{1}{3}$ ft. long, one piece $2\frac{3}{8}$ ft. long, and a third piece $1\frac{1}{2}$ ft. long. How many feet of webbing did he need in all?

THINK What would be a sensible amount of webbing for him to buy?

F George began to pack his knapsack for a camping trip. His blanket weighed $3\frac{3}{4}$ lb. He had $7\frac{5}{8}$ lb. of food to pack. His flashlight and first aid kit together weighed $2\frac{7}{8}$ lb. His canteen weighed $\frac{1}{3}$ lb. How much in all did these things weigh?

G He then put in his mess kit, which weighed $1\frac{1}{2}$ lb., extra shoes that weighed $2\frac{3}{8}$ lb., and other small things that weighed $2\frac{5}{8}$ lb. How much in all did these things weigh?

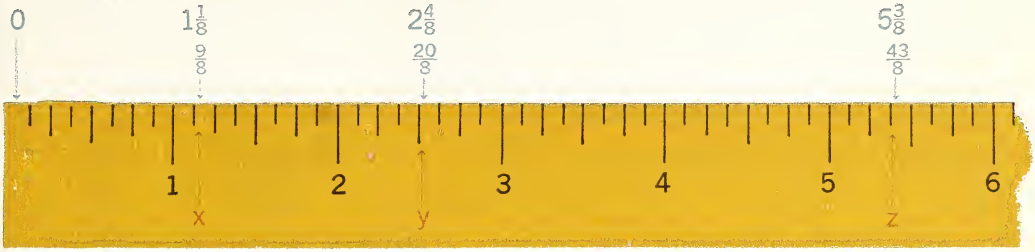
H George's knapsack, with the carrier, weighed $2\frac{1}{8}$ lb. The total weight of the knapsack and all the things George put into it was how much?

I Sue had a bag of potting soil. After she had put $5\frac{2}{3}$ cups of the soil in a flower pot, she had $2\frac{2}{3}$ cups left. How much soil did she have to begin with?

J Ruth mixed different kinds of seed to put in her bird feeder. She used $6\frac{1}{2}$ lb. of wheat, $9\frac{3}{4}$ lb. of corn, $11\frac{5}{8}$ lb. of sunflower seed, and $2\frac{1}{8}$ lb. of nuts. How many pounds of mixed seed did she have?

Looking back

Sometimes you will need to change mixed numbers to other forms. Now you will learn how to do this.



A The picture above shows part of a ruler. Each inch on this ruler is marked off into how many equal parts?

B Why can zero be used at the beginning point of the ruler? Find the point that is marked X on the ruler. How far is it from 0 to Point X?

C The mixed number $1\frac{1}{8}$ expresses a length of 1 whole inch and $\frac{1}{8}$ in. more. If you want to, you can express this length by using an improper fraction. In 1 inch there are ▭ eighths. In $1\frac{1}{8}$ in. there are 8 eighths + 1 eighth, or ▭ eighths. You can think of $1\frac{1}{8}$ as how many eighths?

$1 = \frac{8}{8}$ $\quad 1\frac{1}{8} = \frac{9}{8}$

D The length from 0 to Point Y on the ruler is $2\frac{▭}{8}$ in. You can express this length by using an improper fraction. There are ▭ eighths in each inch. In the 2 inches there are 2×8, or ▭ eighths. In the $2\frac{4}{8}$ inches there are 16 eighths + 4 eighths, or ▭ eighths. You can think of $2\frac{4}{8}$ as how many eighths?

$1 = \frac{▭}{8}$ $\quad 2 = \frac{▭}{8}$ $\quad 2\frac{4}{8} = \frac{▭}{8}$

E The length from 0 to Point Z on the ruler is $5\frac{▭}{8}$ in. You can express this length, too, by using an improper fraction. Each whole inch = ▭ eighths. Then 5 inches $= 5 \times$ ▭, or ▭ eighths. $5\frac{3}{8}$ in. = ▭ eighths + 3 eighths, or ▭ eighths.

$5\frac{3}{8} = \frac{▭}{8}$

F Change $11\frac{4}{5}$ to an improper fraction.

$1 = \frac{5}{5}$ $\quad 11 = \frac{▭}{5}$ $\quad 11\frac{4}{5} = \frac{▭}{5}$

Change each mixed number below to an improper fraction.

A	$9\frac{2}{3}$	F	$4\frac{1}{2}$	K	$5\frac{2}{5}$	P	$35\frac{3}{4}$
B	$1\frac{5}{6}$	G	$7\frac{9}{10}$	L	$6\frac{7}{16}$	Q	$19\frac{7}{10}$
C	$6\frac{7}{8}$	H	$10\frac{1}{3}$	M	$21\frac{11}{12}$	R	$48\frac{3}{24}$
D	$7\frac{1}{4}$	I	$6\frac{8}{9}$	N	$13\frac{3}{5}$	S	$28\frac{5}{16}$
E	$4\frac{5}{12}$	J	$2\frac{3}{16}$	O	$30\frac{1}{2}$	T	$14\frac{7}{12}$

A Look again at the ruler on page 87. The distance from 0 to Point Y is $2\frac{4}{8}$ in. You can also think of part of this length as 1 in. Then think of the rest of it as eighths of an inch. If you begin at 1 in. on the ruler and count the eighths to Point Y, you can think of the distance as 1 in. and ▭ eighths in.

$2\frac{4}{8} = 1\frac{12}{8}$

B To change $2\frac{4}{8}$ to $1\frac{12}{8}$, you use 1 of the 2 wholes and think of it as $\frac{8}{8}$. Then you add the $\frac{8}{8}$ and the $\frac{4}{8}$. Why do you write 1 for the whole number instead of 2?

C The length from 0 to Point Z on the ruler is $5\frac{▭}{8}$ in. Does $4\frac{11}{8}$ in. also express this length?

D To change $5\frac{3}{8}$ to $4\frac{11}{8}$, first use 1 of the 5 wholes and think of it as $\frac{▭}{8}$. Why do you next add $\frac{3}{8}$ and $\frac{8}{8}$? Why do you write 4 instead of 5 as the whole number?

$5\frac{3}{8} = 4\frac{▭}{8}$

In each exercise below, what is the missing numerator?

A $7\frac{2}{8} = 6\frac{▭}{8}$

B $2\frac{1}{6} = 1\frac{▭}{6}$

C $5\frac{2}{3} = 4\frac{▭}{3}$

D $8\frac{10}{16} = 7\frac{▭}{16}$

E $2\frac{7}{10} = 1\frac{▭}{10}$

F $4\frac{3}{4} = 3\frac{▭}{4}$

G $3\frac{1}{5} = 2\frac{▭}{5}$

H $9\frac{5}{6} = 8\frac{▭}{6}$

I $7\frac{7}{8} = 6\frac{▭}{8}$

J $8\frac{5}{12} = 7\frac{▭}{12}$

K $5\frac{8}{10} = 4\frac{▭}{10}$

L $2\frac{2}{4} = 1\frac{▭}{4}$

M $6\frac{15}{16} = 5\frac{▭}{16}$

N $9\frac{1}{4} = 8\frac{▭}{4}$

O $8\frac{3}{10} = 7\frac{▭}{10}$

P $10\frac{11}{20} = 9\frac{▭}{20}$

Now you should be able to change mixed numbers to other forms.

Keeping skilful

A $\frac{7}{8} + \frac{3}{8} = x$

B $\frac{1}{2} + \frac{3}{5} + \frac{2}{3} = a$

C $\frac{7}{8} + \frac{5}{6} + \frac{1}{4} = r$

D $\frac{9}{10} + \frac{5}{8} = b$

E $\frac{3}{8} + \frac{3}{4} + \frac{11}{12} = s$

F $\frac{7}{10} + \frac{5}{6} + \frac{4}{5} = t$

G $\frac{13}{15} + \frac{1}{5} + \frac{2}{3} = n$

H $\frac{11}{12} + \frac{3}{8} = m$

I $\frac{2}{3} + \frac{5}{8} + \frac{5}{6} = g$

A $6\frac{3}{4} + \frac{5}{8} + 1\frac{2}{3} = n$

B $5\frac{1}{12} + 2\frac{5}{6} = a$

C $3\frac{1}{2} + 1\frac{1}{4} + \frac{1}{6} = t$

D $\frac{7}{8} + 2\frac{3}{4} + 7\frac{1}{2} = c$

E $4\frac{11}{12} + 8\frac{5}{6} + 2\frac{1}{3} = x$

F $1\frac{9}{10} + \frac{1}{2} + 9\frac{4}{5} = d$

G $17\frac{1}{2} + 21\frac{29}{32} = s$

H $24\frac{9}{16} + 13\frac{11}{12} = b$

I $76\frac{7}{12} + 30\frac{5}{8} + \frac{19}{24} = m$

A $7643 - n = 5240$

B $m + 13824 = 27615$

C $20130 \div r = 66$

D $a - 57211 = 68433$

E $50212 \div c = 842$

F $948 \times 762 = f$

G $3749 + d = 11087$

H $3403 \div 36 = x$

I $68470 - 59699 = t$

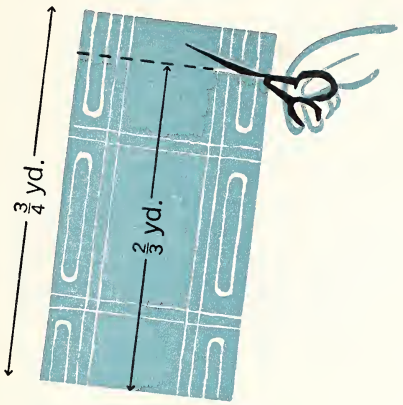
J $98325 + 4778 = n$

Learning how

Now you will learn how to subtract fractions.

see

A



Joan has $\frac{3}{4}$ yd. of material. She is cutting $\frac{2}{3}$ yd. from the piece to make place mats. What fraction of a yard will be left?

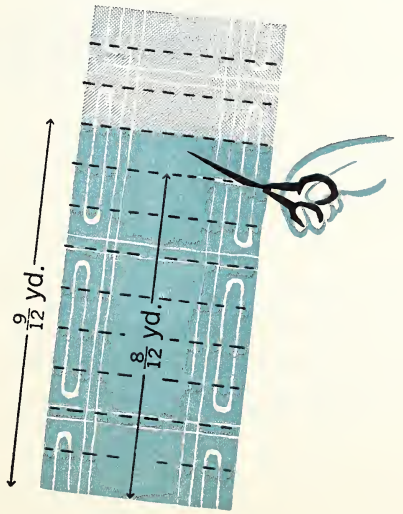
$\frac{3}{4} - \frac{2}{3} = d$

Holds a place for the fraction of a yard left

Think of the $\frac{2}{3}$ yd. as taken away from the $\frac{3}{4}$ yd. You need a name for the fraction of a yard that will be left.

Picture B shows how you must think about the $\frac{3}{4}$ yd. and the $\frac{2}{3}$ yd. before you can find a name for the fraction of a yard that will be left.

B



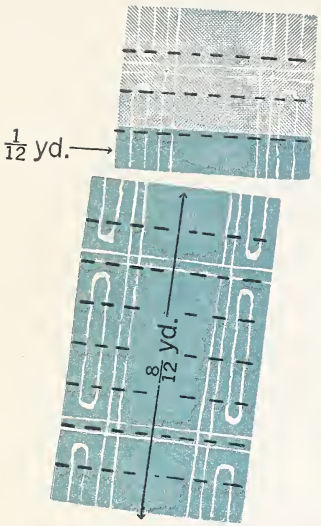
Think of the $\frac{3}{4}$ yd. that Joan had as $\frac{9}{12}$ of the whole yard. Think of the $\frac{2}{3}$ yd. that is being cut off as $\frac{8}{12}$ of the whole yard.

When you use arithmetic to find what fraction of a yard will be left, you subtract $\frac{2}{3}$ from $\frac{3}{4}$. But before you can do this, you must replace the fraction numerals $\frac{2}{3}$ and $\frac{3}{4}$ by other fraction numerals with a common denominator. You will use 12 as a common denominator.

$\frac{3}{4} = \frac{9}{12} \qquad \frac{2}{3} = \frac{8}{12}$

$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12}$

Now turn the page.



C Picture C shows that Joan will have $\frac{1}{12}$ yd. of material left when she has cut off $\frac{8}{12}$ yd., or $\frac{2}{3}$ yd.

You can also get the answer by subtracting $\frac{8}{12}$ from $\frac{9}{12}$.

$$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

Subtract the numerators. $9 - 8 = 1$

Write 1 as the numerator. You are subtracting twelfths. So write 12 as the denominator.

Joan will have $\frac{1}{12}$ yd. of material left.

$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

think

A After breakfast Mrs. Peters had $\frac{7}{8}$ of a coffee cake left. She used $\frac{1}{2}$ of the whole coffee cake for supper. How much of the whole coffee cake did she have left then?

$$\frac{7}{8} - \frac{1}{2} = a$$

Holds a place for the amount of coffee cake left

You know that $\frac{\square}{2}$ of the coffee cake was taken away from $\frac{\square}{8}$ of the coffee cake. Do you know what name to use for the fraction of the coffee cake that was left?

Before you can find a name for the fraction of the coffee cake that was left, think of the $\frac{1}{2}$ taken away as $\frac{4}{8}$ of the whole coffee cake.

When $\frac{4}{8}$ of the coffee cake is taken away from $\frac{7}{8}$ of the coffee cake, $\frac{\square}{8}$ is left.

When you use arithmetic to get the answer, you subtract $\frac{1}{2}$ from $\frac{7}{8}$. You must use a common denominator. Why can you use 8 as a common denominator? $\frac{1}{2}=\frac{\square}{8}$

$\frac{7}{8}-\frac{1}{2}=\frac{7}{8}-\frac{4}{8}$

Subtract $\square$ from $\square$. What do you write as the numerator of the answer? What do you write as the denominator?

$\frac{\square}{8}$ of the coffee cake was left.

$\frac{7}{8}-\frac{1}{2}=\frac{\square}{8}$

---

try

A $\frac{5}{6}-\frac{1}{8}=t$

Why do you use 24 as a common denominator when you subtract?

$\frac{20}{24}-\frac{3}{24}=t$

$\frac{20}{24}-\frac{3}{24}=\frac{17}{24}$

You may write your work this way if you want to.

$\frac{5}{6}=\frac{20}{24}$

$\underline{\frac{1}{8}=\frac{3}{24}}$

$\frac{17}{24}$

B $\frac{15}{16}-\frac{3}{16}=b$

Do you need to change the fraction numerals? Why?

$\frac{15}{16}-\frac{3}{16}=\frac{12}{16}$ or $\frac{3}{4}$

Is $\frac{3}{4}$ in lowest terms?

You may write your work this way.

$\frac{15}{16}$

$\underline{\frac{3}{16}}$

$\frac{12}{16}$ or $\frac{3}{4}$

C $\frac{9}{10}-\frac{3}{4}=m$

What common denominator do you use?

$\frac{18}{20}-\frac{15}{20}=m$

$\frac{18}{20}-\frac{15}{20}=\frac{3}{20}$

Your work may be written this way.

$\frac{9}{10}=\frac{18}{20}$

$\underline{\frac{3}{4}=\frac{15}{20}}$

$\frac{3}{20}$

D $\frac{11}{24}-\frac{1}{12}=c$

What common denominator do you use?

$\frac{11}{24}-\frac{2}{24}=c$

$\frac{11}{24}-\frac{2}{24}=\frac{9}{24}$ or $\frac{3}{8}$

You may write your work this way.

$\frac{11}{24}=\frac{11}{24}$

$\underline{\frac{1}{12}=\frac{2}{24}}$

$\frac{9}{24}$ or $\frac{3}{8}$

E $\frac{5}{6}-\frac{3}{10}=n$

What common denominator do you use?

$\frac{25}{30}-\frac{9}{30}=n$

$\frac{25}{30}-\frac{9}{30}=\frac{16}{30}$ or $\frac{8}{15}$

Your work may be written this way.

$\frac{5}{6}=\frac{25}{30}$

$\underline{\frac{3}{10}=\frac{9}{30}}$

$\frac{16}{30}$ or $\frac{8}{15}$

---

do

A June bought $\frac{5}{8}$ lb. of candy. She gave away $\frac{1}{4}$ lb. of the candy. How much candy was left?

$\frac{5}{8}-\frac{1}{4}=r$

B Paul sawed $\frac{3}{16}$ in. from one end of a piece of metal that was $\frac{3}{4}$ in. long. How long was the piece of metal then?

$\frac{3}{4}-\frac{3}{16}=x$

Be sure your answers are in lowest terms.

C $\frac{1}{2}-\frac{5}{12}=n$

D $\frac{3}{4}-\frac{3}{8}=t$

E $\frac{3}{4}-\frac{3}{10}=a$

F $\frac{2}{3}-\frac{1}{6}=g$

G $\frac{7}{8}-\frac{1}{8}=m$

H $\frac{9}{16}-\frac{3}{8}=b$

I $\frac{2}{3}-\frac{3}{5}=d$

J $\frac{11}{16}-\frac{3}{8}=k$

K $\frac{3}{4}-\frac{1}{4}=e$

L $\frac{23}{24}-\frac{1}{3}=x$

M $\frac{5}{8}-\frac{3}{16}=r$

N $\frac{1}{3}-\frac{1}{8}=g$

O $\frac{9}{16}-\frac{3}{16}=m$

P $\frac{3}{4}-\frac{2}{5}=h$

Q $\frac{11}{12}-\frac{1}{12}=b$

R $\frac{3}{4}-\frac{1}{6}=s$

S $\frac{7}{8}-\frac{4}{5}=w$

T $\frac{2}{3}-\frac{1}{2}=x$

A $\frac{11}{24}-\frac{1}{3}=s$

B $\frac{9}{10}-\frac{2}{5}=t$

C $\frac{2}{3}-\frac{3}{8}=v$

D $\frac{3}{4}-\frac{7}{10}=x$

E $\frac{3}{5}-\frac{1}{3}=a$

F $\frac{7}{8}-\frac{2}{3}=b$

G $\frac{19}{24}-\frac{5}{12}=c$

H $\frac{11}{12}-\frac{5}{6}=r$

I $\frac{5}{8}-\frac{1}{6}=m$

J $\frac{4}{5}-\frac{1}{2}=d$

K $\frac{17}{20}-\frac{1}{10}=e$

L $\frac{14}{15}-\frac{2}{3}=n$

Now you should know how to subtract one fraction from another.

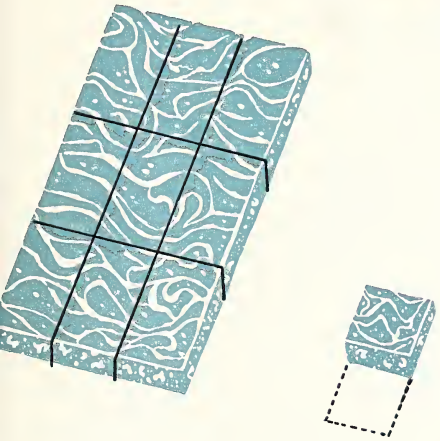
Learning how

This lesson will show you how to subtract when you use mixed numbers.

see

A

Mr. Paul, a candy maker, made $9\frac{1}{2}$ lb. of fudge one morning. He sold $4\frac{7}{8}$ lb. of the fudge in the afternoon. How much fudge did he have left?

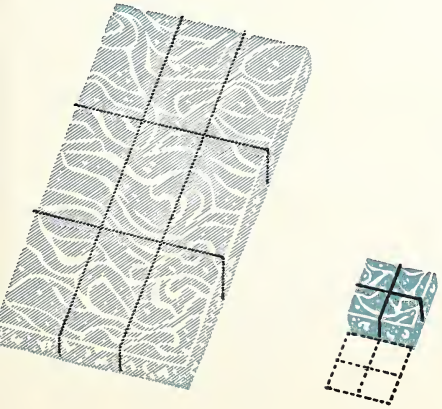


$9\frac{1}{2} - 4\frac{7}{8} = r$ ← Holds a place for the number that tells how much was left

B

← The first thing to do is to see if $\frac{7}{8}$ lb. can be taken from $\frac{1}{2}$ lb. Think of $\frac{1}{2}$ lb. as $\frac{4}{8}$ lb. You cannot take $\frac{7}{8}$ lb. from $\frac{4}{8}$ lb.

When you use arithmetic and compute, you subtract $4\frac{7}{8}$ from $9\frac{1}{2}$. First you will subtract the fractions. You must use a common denominator. You can change $\frac{1}{2}$ to eighths.



$9\frac{1}{2} = 9\frac{4}{8}$ ← You cannot subtract $\frac{7}{8}$ from $\frac{4}{8}$.

$4\frac{7}{8} = 4\frac{7}{8}$

Now turn the page.

C

You need more eighths before you can take away $\frac{7}{8}$ lb. So think of 1 of the whole pounds as cut up into eighths. You can think of this pound as $\frac{8}{8}$ lb. Put the $\frac{8}{8}$ lb. with the $\frac{4}{8}$ lb. Now you have 8 whole pounds and $\frac{12}{8}$ lb. more. You can take $\frac{7}{8}$ lb. from $\frac{12}{8}$ lb.

When you compute, you take 1 from the 9 ones and think of it as $\frac{8}{8}$. Put $\frac{8}{8}$ with $\frac{4}{8}$. You now have $\frac{12}{8}$.

$$\begin{array}{r} 8\ \frac{12}{8} \\ 9\frac{1}{2} = 9\frac{4}{8} \\ 4\frac{7}{8} = 4\frac{7}{8} \end{array}$$

You have changed $9\frac{1}{2}$ to $8\frac{12}{8}$. You can subtract $\frac{7}{8}$ from $\frac{12}{8}$.

D

When you take $\frac{7}{8}$ lb. from $\frac{12}{8}$ lb., the amount left is $\frac{5}{8}$ lb.

In your computation you subtract the fractions.

$$\begin{array}{r} 8\ \frac{12}{8} \\ 9\frac{1}{2} = 9\frac{4}{8} \\ 4\frac{7}{8} = 4\frac{7}{8} \\ \hline \frac{5}{8} \end{array}$$

Subtract $\frac{7}{8}$ from $\frac{12}{8}$. $\frac{5}{8}$ is left.

Write $\frac{5}{8}$ here.

Now the 4 whole pounds can be taken from the 8 whole pounds. 4 lb. will be left.

E

In your computation the next step is to subtract the whole numbers.

Subtract 4 from 8.

$$\begin{array}{r} 8\frac{12}{8} \\ 9\frac{1}{2} = 9\frac{4}{8} \\ 4\frac{7}{8} = 4\frac{7}{8} \\ \hline 4\frac{5}{8} \end{array}$$

Write the 4 here.

Mr. Paul had $4\frac{5}{8}$ lb. of fudge left.

$9\frac{1}{2} - 4\frac{7}{8} = 4\frac{5}{8}$

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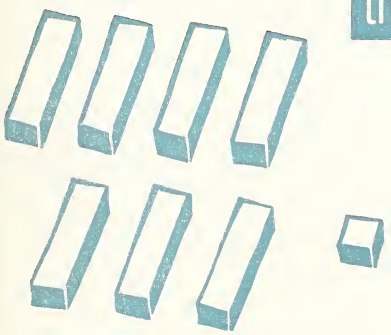
think

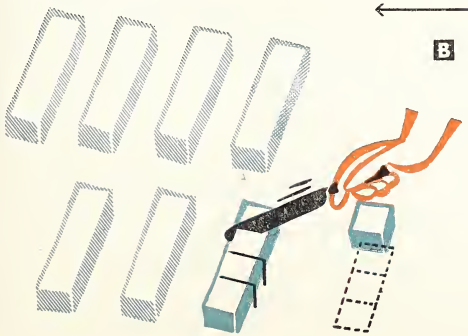


A

Mrs. Price had $7\frac{1}{4}$ sticks of butter. She used $6\frac{3}{4}$ sticks for baking. How much butter did she have left?

$7\frac{1}{4} - 6\frac{3}{4} = S$ Holds a place for the number of sticks left

Can the $\frac{3}{4}$ stick be taken from the $\frac{1}{4}$ stick? How do you know right away that you will have to think of cutting up another stick?



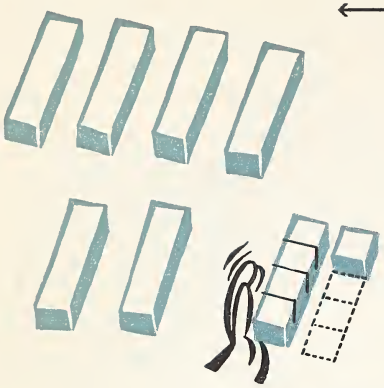
B

When you compute, do you have to find a common denominator? Why?

$$\begin{array}{r} 7\frac{1}{4} \\ 6\frac{3}{4} \\ \hline \end{array}$$

How do you know you need more fourths before you can subtract?

Now turn the page.



C

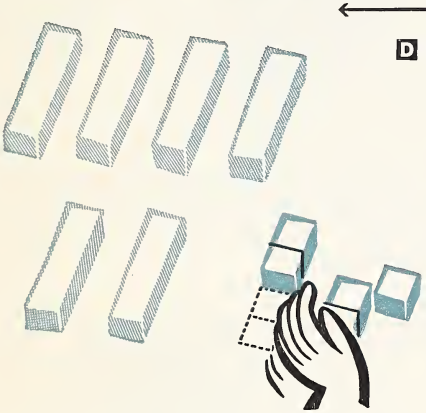
Why is the stick thought of as cut into fourths? How many one fourths of a stick will there be in all? How many whole uncut sticks will there be?

$6\frac{5}{4}$

$7\frac{1}{4}$

$6\frac{3}{4}$

When you compute, how do you get $6\frac{5}{4}$? What do you subtract first?



D

When you take $\frac{3}{4}$ stick from $\frac{5}{4}$ stick, $\frac{\square}{4}$ of a stick is left.

$6\frac{5}{4}$

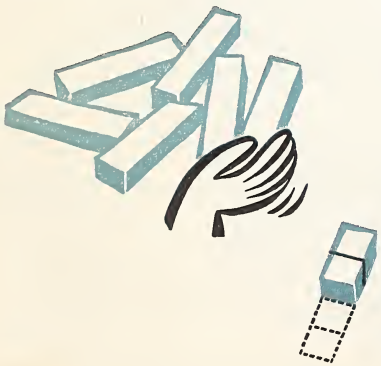
$7\frac{1}{4}$

$6\frac{3}{4}$

$\frac{2}{4}$

In your computation, you subtract $\frac{3}{4}$ from $\frac{\square}{4}$. $\frac{\square}{4}$ is left.

Why do you write $\frac{2}{4}$ here?



E

Now take away the 6 whole sticks. How many whole sticks will be left?

$6\frac{5}{4}$

$7\frac{1}{4}$

$6\frac{3}{4}$

$\frac{2}{4}$

In your computation, you subtract $\square$ from $\square$. What is left?

Why do you not write a numeral here?

Mrs. Price had $\frac{2}{4}$, or $\frac{1}{2}$, of a stick left.

$7\frac{1}{4} - 6\frac{3}{4} = \frac{\square}{4}$ or $\frac{\square}{2}$

try

A $5 - 1\frac{11}{16} = t$

$5 = 4\frac{16}{16}$

$1\frac{11}{16} = 1\frac{11}{16}$

$3\frac{5}{16}$

Why do you change 5 to $4\frac{16}{16}$?

B $3\frac{4}{5} - \frac{9}{10} = d$

$3\frac{4}{5} = 2\frac{18}{10}$

$\frac{9}{10} = \frac{9}{10}$

$2\frac{9}{10}$

Why is it unnecessary to subtract from the 2?

C $11\frac{7}{10} - 5\frac{7}{10} = x$

$11\frac{7}{10}$

$5\frac{7}{10}$

6

Why is there no fraction numeral in the answer?

D $9\frac{7}{8} - 7\frac{2}{3} = r$

$9\frac{7}{8} = 9\frac{21}{24}$

$7\frac{2}{3} = 7\frac{16}{24}$

$2\frac{5}{24}$

Why do you not have to change $9\frac{21}{24}$ before you subtract?

E $9\frac{5}{8} - 2 = c$

$9\frac{5}{8}$

2

$7\frac{5}{8}$

F $13\frac{1}{6} - 12\frac{5}{6} = n$

$13\frac{1}{6} = 12\frac{7}{6}$

$12\frac{5}{6} = 12\frac{5}{6}$

$\frac{2}{6}$ or $\frac{1}{3}$

G $4 - \frac{7}{12} = k$

$4 = 3\frac{12}{12}$

$\frac{7}{12} = \frac{7}{12}$

$3\frac{5}{12}$

H $8\frac{5}{6} - 3\frac{1}{6} = s$

$8\frac{5}{6}$

$3\frac{1}{6}$

$5\frac{4}{6}$ or $5\frac{2}{3}$

do

A Don bought $1\frac{1}{4}$ lb. of nails. He used $\frac{7}{8}$ lb. How many pounds of nails did he have left?

$1\frac{1}{4} - \frac{7}{8} = m$

B Mrs. Peters made $4\frac{1}{2}$ doz. cookies. She gave $2\frac{2}{3}$ doz. to friends. How many dozen cookies did she have left?

$4\frac{1}{2} - 2\frac{2}{3} = a$

Show any fraction numerals in your answers in lowest terms.

C $5\frac{7}{10} - 4\frac{3}{10} = d$

D $3\frac{1}{6} - \frac{7}{8} = x$

E $7\frac{3}{5} - 5\frac{2}{3} = m$

F $9\frac{9}{10} - 6\frac{3}{10} = c$

G $6\frac{1}{2} - \frac{11}{12} = a$

H $14\frac{3}{16} - 7\frac{5}{8} = e$

I $11\frac{1}{3} - 2\frac{4}{5} = g$

J $13\frac{3}{4} - 6\frac{2}{3} = h$

K $37\frac{1}{2} - 32\frac{1}{5} = d$

L $50 - 18\frac{7}{12} = n$

A $6\frac{1}{4} - 5\frac{1}{4} = z$

B $7\frac{5}{12} - 2\frac{1}{6} = f$

C $4\frac{1}{8} - 3\frac{5}{12} = d$

D $9\frac{1}{6} - 3\frac{2}{3} = a$

E $5\frac{3}{4} - 2\frac{11}{16} = b$

F $19\frac{1}{8} - 11\frac{3}{4} = m$

G $31\frac{7}{12} - 20 = r$

H $52\frac{11}{16} - 43\frac{1}{16} = n$

I $20 - 15\frac{5}{6} = s$

J $42\frac{1}{4} - 17\frac{5}{6} = x$

Now you should know how to subtract mixed numbers.

Using arithmetic

A Mrs. May used $\frac{3}{4}$ c. sugar to make lemon drops and $\frac{2}{3}$ c. sugar to make spice bars. Did she use more or less sugar for the lemon drops than for the spice bars? How much more sugar or less sugar did she use?

B Mrs. May made $3\frac{2}{3}$ doz. lemon drops and $1\frac{1}{2}$ doz. spice bars. She made how many more dozen lemon drops than spice bars?

C Jane bought $2\frac{1}{3}$ yd. of cloth. After she had made an apron, she had $1\frac{5}{6}$ yd. left. How much cloth had she used to make the apron?

D Then Jane made aprons for her two sisters. She used $\frac{3}{4}$ yd. of the cloth for one of the aprons and $\frac{2}{3}$ yd. of the cloth for the other. How much of the cloth was left then?

E When Ed made a shelf, he used $3\frac{2}{3}$ ft. from a board that was $5\frac{1}{4}$ ft. long. How long was the leftover piece of board?

F Before her holidays, Ann weighed $68\frac{1}{2}$ lb. After the holidays, she weighed 71 lb. How much had she gained?

G Mr. Row had $\frac{3}{4}$ gal. of green paint and $3\frac{1}{2}$ gal. of white paint. He had how much more white paint than green paint?

H Mr. Row mixed $2\frac{1}{4}$ gal. of paint by putting some green paint in a pail and adding $1\frac{3}{4}$ gal. of white paint. Later he wanted to match this color. How much green paint should he add to $1\frac{3}{4}$ gal. of white paint?

I Mrs. Row put $\frac{5}{8}$ gal. of paint in a pail. After she had painted some furniture, she still had $\frac{1}{4}$ gal. of paint in the pail. How much paint had she used on the furniture?

J Mrs. Row used $\frac{1}{2}$ gal. of paint to paint a fence. How much less paint did she use on the furniture than on the fence?

K Jim's cat would not stay on the scales when Jim tried to weigh him. Jim knew that his own weight was 97 lb. He weighed himself holding the cat and found the total weight was $105\frac{1}{2}$ lb. How much did the cat weigh?

L When Pat's father filled the almost empty tank of his car with gasoline, he noticed that he had driven a total of $8056\frac{6}{10}$ mi. By the time this gasoline was used up, Pat's father had driven $8345\frac{3}{10}$ mi. in all. How many miles had he driven on the tank of gasoline?

M Sally bought $1\frac{2}{3}$ yd. of ribbon. She used $\frac{7}{8}$ yd. How much did she have left?

Checking up

Test 1

A $\frac{5}{6}+\frac{5}{6}=n$

B $\frac{3}{8}+\frac{1}{2}=r$

C $\frac{1}{2}+\frac{3}{8}+\frac{5}{8}=s$

D $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}=a$

E $\frac{7}{10}+\frac{5}{6}=d$

F $\frac{5}{8}+\frac{3}{4}+\frac{11}{16}=r$

G $\frac{5}{12}+\frac{7}{8}+\frac{1}{4}=m$

H $\frac{9}{10}+\frac{3}{5}+\frac{1}{2}=t$

I $\frac{7}{16}+\frac{3}{4}+\frac{1}{2}=b$

Test 2

A $\frac{7}{16}+8\frac{13}{16}=b$

B $3\frac{4}{5}+2\frac{1}{2}=d$

C $1\frac{5}{6}+\frac{11}{12}+\frac{2}{3}=x$

D $14\frac{1}{4}+6\frac{4}{5}=t$

E $9\frac{7}{10}+\frac{4}{5}=a$

F $26\frac{1}{2}+14\frac{1}{4}=g$

G $18\frac{9}{10}+6\frac{3}{4}=k$

H $11\frac{7}{8}+\frac{2}{3}+7\frac{5}{6}=z$

I $\frac{11}{12}+6\frac{3}{4}+4\frac{1}{3}=m$

Test 3

A $\frac{7}{10}-\frac{1}{4}=s$

B $\frac{15}{16}-\frac{3}{4}=b$

C $\frac{4}{5}-\frac{1}{2}=n$

D $\frac{7}{8}-\frac{3}{8}=a$

E $\frac{5}{8}-\frac{1}{6}=d$

F $\frac{11}{16}-\frac{5}{8}=h$

G $\frac{3}{4}-\frac{1}{5}=c$

H $\frac{15}{16}-\frac{13}{16}=m$

I $\frac{5}{6}-\frac{3}{8}=r$

Test 4

A $7\frac{1}{6}-3\frac{1}{2}=z$

B $12-4\frac{5}{8}=t$

C $6\frac{2}{3}-\frac{2}{3}=b$

D $18\frac{3}{16}-17\frac{7}{8}=a$

E $20\frac{4}{5}-16\frac{1}{3}=k$

F $9\frac{11}{12}-6\frac{3}{8}=n$

G $15\frac{3}{10}-4\frac{1}{2}=d$

H $31\frac{13}{16}-23\frac{5}{8}=m$

I $11\frac{1}{6}-10\frac{11}{12}=r$

Keeping skilful

A $1573\div 86=m$

B $c+38214=61033$

C $74163-68219=a$

D $r-4286=11302$

E $927\times 84=t$

F $s+38624=40000$

G $53036-y=48626$

H $81310\div z=865$

I $17432+k=31748$

J $96004-79866=n$

K $s+42617=83025$

L $496\times 8301=r$

M $w-65632=101375$

N $b+48520=95601$

O $94370\div 704=n$

P $33406-z=7432$

Q $30492\div r=63$

A 6 fl. oz. = ▭ tbs.

B 180 in. = ▭ yd.

C 17 bu. = ▭ pk.

D 120 mo. = ▭ yr.

E 3 mi. = ▭ ft.

F 3600 sec. = ▭ min.

G 24 qt. = ▭ gal.

H 72 pt. = ▭ gal.

I 99 ft. = ▭ yd.

J 4 yr. = ▭ da.

K 192 hr. = ▭ da.

L 24 c. = ▭ fl. oz.

M 64 pk. = ▭ bu.

N 18 ft. = ▭ in.

O 8 mi. = ▭ yd.

P 192 oz. = ▭ lb.

Q 9 T. = ▭ lb.

A $s+1\frac{1}{2}=4$

B $\frac{3}{4}+m=1\frac{1}{3}$

C $14\frac{7}{10}-12\frac{3}{10}=t$

D $6\frac{5}{16}+\frac{7}{8}+\frac{1}{4}=d$

E $5\frac{11}{12}-a=\frac{2}{3}$

F $r-4\frac{9}{16}=3\frac{3}{8}$

G $7\frac{4}{5}+10\frac{1}{5}=x$

H $20\frac{1}{3}-19\frac{5}{6}=s$

I $a+15\frac{3}{4}=30\frac{11}{12}$

J $14\frac{1}{8}-n=6\frac{3}{4}$

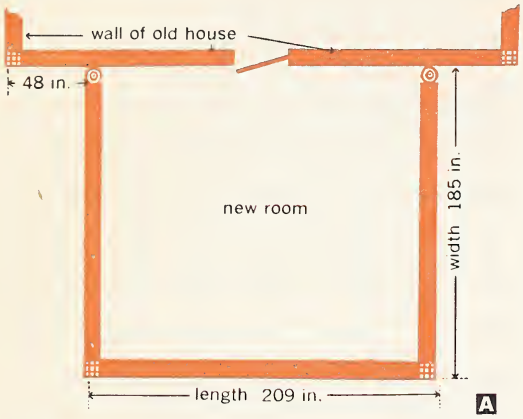
K $16\frac{5}{6}+d=20\frac{1}{10}$

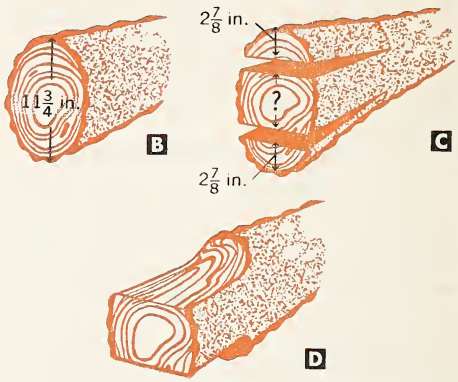
L $m-11\frac{3}{8}=\frac{7}{12}$

M $34-29\frac{9}{16}=y$

N $g-8\frac{3}{4}=11\frac{7}{10}$

Using arithmetic





A John Price and his father built a new room on the family summer cabin. Picture A shows the floor plan of this room. The outside length of the room, including the end walls, was 209 inches. Picture B shows a log like those the walls were made of. The diameter of each log was $11\frac{3}{4}$ in. What was the inside length of the room from wall to wall?

B At the point where each side wall of the new room was to be attached to the old room, an upright log was placed against the outside wall of the old cabin. The outer side of each of these logs was 48 inches in from the corner of the old wall. How long was the old wall on the outside?

C What was the inside width of the new room in inches? Study the plan before you answer this question.

D Picture C shows how John's father planned to cut a peg at the end of each log so the logs would fit together at the corners of the room. Picture D shows the finished peg. After the $2\frac{7}{8}$-inch pieces had been cut from opposite sides of the log, how wide was the centre piece that was left?

E The walls of the new room were to be $8\frac{1}{2}$ ft. high. John measured the walls one day when he and his father were working. The walls were then $5\frac{3}{4}$ ft. high. How much higher did they need to make the walls?

F Mr. Price said the ridge pole should be $19\frac{3}{4}$ ft. long. He chose a log $22\frac{2}{3}$ ft. long. How many feet should he have cut from this log to make the ridge pole?

G When John and his father began working one morning, the temperature was $60\frac{8}{10}°$. By noon the temperature was $69\frac{6}{10}°$. The temperature had gone up how many degrees?

THINK Why do you not need to change the $\frac{8}{10}$ in your answer to $\frac{4}{5}$?

H One Monday there were $3\frac{1}{2}$ gal. of white gasoline for the stove. By Sunday night $1\frac{3}{4}$ gal. were left. How much gasoline had been used that week?

I There were two ways to get from the cabin to the nearest town. By boat across the lake, the distance was $5\frac{7}{10}$ mi. By car on the road around the lake, the distance was $6\frac{1}{4}$ mi. longer. The distance by road from the cabin to the town was how many miles?

J Mrs. Price needed $36\frac{3}{4}$ yd. of cloth for slipcovers and 24 yd. for curtains for the new room. How many yards of cloth did she need in all for curtains and slipcovers?

K When Mrs. Price bought the cloth, she also bought $9\frac{1}{4}$ yd. of another color for pillow covers. She paid $1.19 a yard for all the cloth. How much did she spend for all the cloth?

L Mrs. Price brought 25 lb. of flour to the cabin. She used all of this flour in six weeks. She had used an average of how many pounds of flour a week?

M One day Mrs. Price bought $2\frac{1}{2}$ doz. eggs from a farmer and put them with the eggs she had. Then she found that she had $3\frac{1}{6}$ doz. eggs. What fraction of a dozen eggs did Mrs. Price have before she bought the $2\frac{1}{2}$ doz.?

N John's father worked 475 hours in building the new room. A man from the town would have charged $2.69 per hour to do this work. How much money would Mr. Price have had to pay this man for 475 hours of work?

Keeping skilful

A $6302 + 975 = a$
B $s - 8245 = 11153$
C $m + 25490 = 50000$
D $74812 - 68905 = r$
E $40306 + t = 71584$

F $9526 + d = 9875$
G $56400 - 47983 = s$
H $m - 48391 = 61000$
I $42737 + 58663 = n$
J $w + 74891 = 100000$

A $89642 \div 149 = c$
B $56110 \div r = 246$
C $9900 \div 132 = x$
D $48009 \div t = 827$
E $391313 \div a = 971$

Checking up

Test 1

A John wants to buy a cornet so that he can play in the school band. He looked at two cornets. One was priced at \$54.95, and the other at \$129.50. What was the difference in price of these two cornets?

B Bill is saving money to buy a drum that costs \$22.50. He needs \$8.83 more before he can buy the drum. How much money has he saved?

C The school bought 5 clarinets for a total price of \$495. If the clarinets had been bought separately, each one would have cost \$110. How much money was saved by buying 5 of them at one time?

D The school also bought two cornets at \$54.95 each and one trombone for \$129.50. What was the total cost of these three instruments?

E Sue bought an instrument case that cost \$19.95. Then she had \$5.05 left. How much money did she have before she bought the instrument case?

F The school bought 57 music stands at \$3.79 each. How much did all the music stands cost?

G Alice is supposed to practise on her trombone for 7 hours each week. This week, from Monday to Wednesday, she has practised a total of $3\frac{3}{4}$ hr. How many more hours should she practise this week?

H The Band Mothers' Club raised \$300. They used most of this money to buy jackets for the band. Then they had \$15 left. How much did they spend for the jackets?

I Philip plays two instruments. He weighed both instruments in their cases to see how many pounds he had to carry when he took both instruments home. One instrument weighed $18\frac{2}{3}$ lb., and the other weighed $7\frac{1}{2}$ lb. How much did Philip's two instruments weigh together?

J In one 10-month school term, the school collected \$1900 from renting instruments. Each pupil who rented an instrument paid \$5 per month for the term. How many pupils had rented instruments that term?

K 35 of the pupils bought music that cost \$43.75 altogether. What was the average cost per pupil for this music?

L 22 other pupils paid \$1.75 apiece for music that they needed. How much did the music for these 22 pupils cost?

Test 2

A $483 + 697 = t$
B $69 + 42 + 18 + 55 = r$
C $862 + 444 + 909 = a$
D $6347 + 523 + 200 = x$
E $70059 + 93941 = b$
F $758 + 346 + 1125 = s$
G $156386 + 847644 = d$
H $114 + 968 + 593 = w$
I $460095 + 79906 = g$
J $87355 + 680075 = f$

Test 3

A $900 - 114 = n$
B $743 - 685 = k$
C $814 - 96 = c$
D $501 - 433 = a$
E $4768 - 3919 = x$
F $6310 - 1274 = t$
G $18742 - 8596 = b$
H $40036 - 28757 = m$
I $92147 - 86338 = d$
J $47086 - 3298 = r$

Test 4

A $9 \times 765 = w$
B $86 \times 8 = d$
C $47 \times 63 = t$
D $956 \times 38 = r$
E $58 \times 416 = g$
F $69 \times 7803 = b$
G $746 \times 909 = z$
H $508 \times 130 = c$
I $872 \times 647 = h$
J $2473 \times 86 = s$

Test 5

A $2436 \div 29 = d$
B $1327 \div c = 76$
C $10586 \div m = 67$
D $96250 \div 124 = s$
E $8256 \div t = 129$
F $38964 \div 408 = n$
G $49181 \div r = 68$
H $39236 \div 523 = w$
I $13858 \div d = 82$
J $53682 \div 198 = e$

Test 6

A $\frac{3}{4} + \frac{7}{10} = a$
B $1\frac{1}{2} + \frac{3}{8} + 5\frac{1}{4} = x$
C $\frac{5}{6} + \frac{2}{3} + 1\frac{7}{10} = t$
D $8\frac{1}{5} + \frac{9}{10} + \frac{7}{10} = d$
E $6\frac{1}{4} + 5 + 4\frac{3}{4} = w$
F $21\frac{2}{3} + 9\frac{4}{5} = b$
G $5\frac{7}{12} + \frac{3}{8} + 9\frac{1}{4} = k$
H $6\frac{15}{16} + 1\frac{3}{4} + \frac{5}{8} = m$

Test 7

A $\frac{7}{8} - \frac{1}{8} = c$
B $4\frac{1}{2} - 3\frac{5}{6} = x$
C $11 - 2\frac{9}{10} = a$
D $15\frac{7}{12} - \frac{5}{8} = r$
E $40\frac{5}{6} - 21\frac{5}{6} = w$
F $20\frac{3}{4} - 19\frac{15}{16} = b$
G $17\frac{9}{16} - 11\frac{1}{16} = n$
H $4\frac{3}{10} - \frac{5}{6} = t$

Test 8

Change to mixed numbers in lowest terms.

A $\frac{29}{8}$
B $\frac{30}{5}$
C $\frac{72}{16}$
D $\frac{58}{24}$
E $\frac{18}{8}$
F $\frac{46}{4}$
G $\frac{65}{10}$
H $\frac{27}{3}$
I $\frac{84}{32}$
J $\frac{49}{6}$
K $\frac{81}{12}$
L $\frac{28}{6}$
M $\frac{63}{28}$
N $\frac{43}{15}$
O $\frac{55}{30}$
P $\frac{56}{25}$
Q $\frac{60}{15}$
R $\frac{72}{10}$

Test 9

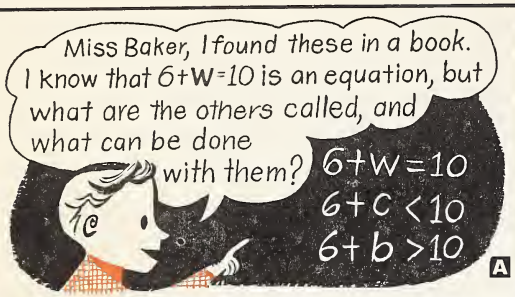
Change to improper fractions.

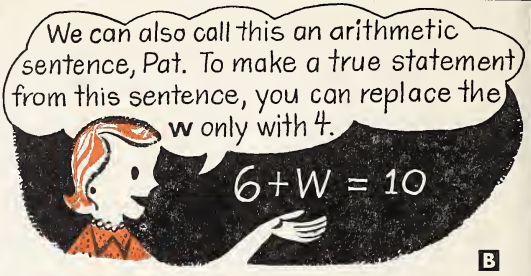
A $3\frac{3}{4}$
B $5\frac{1}{2}$
C $7\frac{5}{6}$
D $9\frac{5}{12}$
E $4\frac{1}{4}$
F $6\frac{9}{10}$
G $4\frac{7}{8}$
H $8\frac{7}{12}$
I $7\frac{1}{4}$
J $6\frac{1}{12}$
K $9\frac{1}{3}$
L $4\frac{7}{10}$
M $13\frac{2}{3}$
N $18\frac{1}{2}$
O $30\frac{5}{8}$
P $10\frac{4}{5}$
Q $11\frac{1}{16}$
R $42\frac{7}{8}$

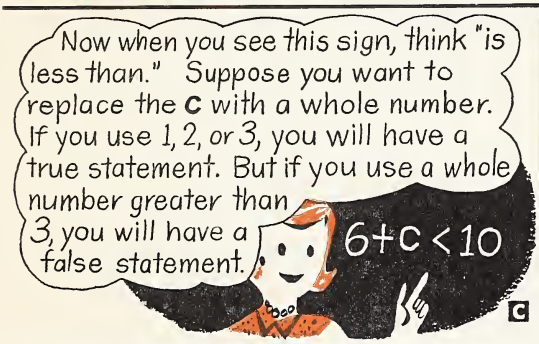
Test 10

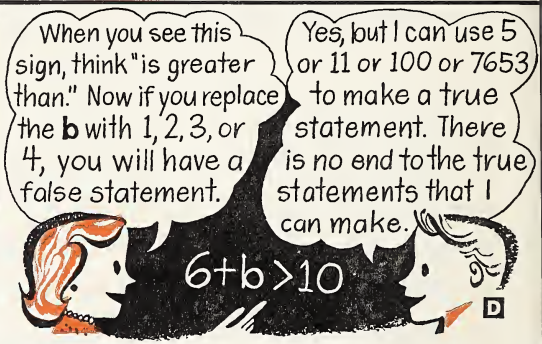
A 7 yr. = ▒ mo.
B 80 pt. = ▒ gal.
C 120 hr. = ▒ da.
D 40 fl. oz. = ▒ c.
E 17 gal. = ▒ qt.
F 7 hr. = ▒ min.
G 84 in. = ▒ ft.
H 5 yr. = ▒ da.
I 3 mi. = ▒ yd.

Thinking straight


Miss Baker, I found these in a book. I know that 6+W=10 is an equation, but what are the others called, and what can be done with them?
6+W=10
6+c<10
6+b>10
A


We can also call this an arithmetic sentence, Pat. To make a true statement from this sentence, you can replace the w only with 4.
6+W = 10
B


Now when you see this sign, think "is less than." Suppose you want to replace the c with a whole number. If you use 1, 2, or 3, you will have a true statement. But if you use a whole number greater than 3, you will have a false statement.
6+c<10
C


When you see this sign, think "is greater than." Now if you replace the b with 1, 2, 3, or 4, you will have a false statement.
Yes, but I can use 5 or 11 or 100 or 7653 to make a true statement. There is no end to the true statements that I can make.
6+b>10
D

A In Picture A look at the signs for "is less than" and "is greater than." How are the signs different?

For each exercise below, make a true statement by putting the sign for "is less than," "is greater than," or "is equal to" where it belongs.

B $16+5 \sim\sim 21$
C $4\times15 \sim\sim 12\times5$
D $13+10 \sim\sim 9+12$
E $17 \sim\sim 56\div4$
F $71+4 \sim\sim 4\times19$
G $21-8 \sim\sim 12+2$
H $10\div5 \sim\sim 10\div2$

I Jim said, "I have 9 records. I have more records than you have, Don." Either of the sentences shown below expresses what Jim said.

$9 > k$ $\qquad$ $k < 9$

Holds a place for the numeral that tells how many records Don has

Could Don have 9 records? Could he have 8? 7? 6? What other numbers of records could he have? Could he have none?

J In the sentences $9 > k$ and $k < 9$, you can replace k with any numeral, but with some numbers you will get false statements. For example, if you write $9 > 11$, you have a false statement. If you want to make true statements, there is a set of nine whole numbers that you can use. What are the nine whole numbers in the set?

K Now study the arithmetic sentence $5 + c < 11$. Can you use 1 to replace c to make a true statement? Can you use 2? Can you use 0? Can you use 6? What other whole numbers can you use to make true statements?

L There is a set of six whole numbers you can use to make true statements of the sentence $5 + c < 11$. Each number in this set is a solution. The set of six whole numbers is a solution set for $5 + c < 11$. Write this solution set.

M Is 13 in the solution set for the sentence $13 - d > 1$? Is 12 in the solution set? Is 0 in the solution set?

N Write the solution set of twelve whole numbers for $13 - d > 1$.

O What is the least whole number in the solution set for $4 + 7 < b$? Is there a greatest whole number in the solution set?

P Write ten of the whole numbers in the solution set for $4 + 7 < b$.

For each sentence below, write a short description of the solution set of whole numbers. For example, for Sentence A you might write, "All whole numbers greater than 14." For Sentence B you might write, "0, 1, 2, and 3."

A $8 + s > 22$
B $t + 1 < 5$
C $r + 5 > 7$
D $9 < d - 3$
E $10 + b < 16$
F $18 - k > 4$
G $14 < a + 7$
H $8 > 16 - n$
I $12 - f < 6$
J $b - 6 = 10$
K $13 < g + 7$
L $a + 19 = 26$

Make a true statement for each exercise below, using the sign $<$, $>$, or $=$.

A $\frac{1}{3} + \frac{1}{3} \rightsquigarrow \frac{1}{2}$
B $1\frac{1}{4} - \frac{2}{3} \rightsquigarrow 2$
C $7 \rightsquigarrow \frac{3}{4} + \frac{9}{10}$
D $\frac{1}{2} - \frac{1}{2} \rightsquigarrow 1$
E $\frac{15}{16} \rightsquigarrow \frac{1}{2} + \frac{7}{16}$
F $\frac{3}{4} + \frac{3}{4} \rightsquigarrow 1$
G $13\frac{1}{4} - 1\frac{3}{4} \rightsquigarrow 12$
H $119 \rightsquigarrow 2 \times 59$
I $36 - 35\frac{1}{3} \rightsquigarrow \frac{2}{3}$
J $45 \div 3 \rightsquigarrow 15\frac{1}{2}$
K $29\frac{5}{6} - 11 \rightsquigarrow 15$
L $17 + 2\frac{1}{10} \rightsquigarrow 20$

M There is only one whole number in the solution set for $5 > n + 4$. What is this whole number?

N Now see if there are any proper fractions in the solution set for $5 > n + 4$. If you use $\frac{11}{12}$, will you have a true statement? If you use $\frac{99}{100}$, will you have a true statement?

O Does the solution set for $5 > n + 4$ contain proper fractions? Is there any end to the number of proper fractions in the solution set?

P Does the solution set for $5 > n + 4$ contain any mixed numbers? For example, can you replace n with $1\frac{1}{200}$?

Q There is only one whole number in the solution set for $3 - d < 1$. What is this whole number?

R Now see if there are any mixed numbers in the solution set for $3 - d < 1$. Will you have a true statement if you use $2\frac{1}{100}$? If you use $2\frac{24}{25}$? Are all mixed numbers greater than 2 but less than 3 in the solution set?

S Is there any end to the number of mixed numbers in the solution set for $3 - d < 1$?

For each sentence below, name three solutions from the solution set of whole numbers, mixed numbers, and proper fractions. For the first sentence you could use any number greater than $1\frac{1}{2}$. For example, you could use $1\frac{3}{4}$, 2, $3\frac{1}{10}$. Each of these numbers is a solution for the sentence.

A $8\frac{1}{2} + t > 10$

B $r - \frac{1}{3} < 5$

C $6 > a + 4\frac{1}{4}$

D $\frac{15}{16} - g < \frac{1}{4}$

E $\frac{4}{5} + m > 1\frac{1}{2}$

F $k - \frac{7}{10} < 1$

G $b - 1\frac{2}{3} < 8$

H $17 - c < 15\frac{5}{6}$

I $d + 1\frac{11}{12} > 5$

J $21 < m + 16\frac{1}{2}$

K $w - 8\frac{4}{5} > 4$

L $25 + r > 26$

M $d - \frac{5}{8} < 2$

N $18\frac{1}{4} + m > 19$

O $16 + 2\frac{1}{3} < p$

P $41 - k < 2\frac{3}{4}$

Q $z + 4\frac{3}{8} > 6\frac{1}{2}$

R $n - 9\frac{4}{5} > 10$

S $26\frac{1}{3} + e < 30$

T $40\frac{1}{2} - 29\frac{3}{4} > y$

U $2\frac{7}{10} + 4\frac{3}{5} < f$

V $n + 5\frac{9}{16} > 5\frac{1}{4}$

Keeping skilful

Find the sum.

A $\frac{1}{2}, \frac{3}{8}, \frac{5}{6}, \frac{11}{12}$

B $1\frac{3}{5}, \frac{7}{10}, 3\frac{1}{15}$

C $9\frac{7}{16}, \frac{3}{4}, 21\frac{3}{8}, \frac{1}{2}$

D $\frac{9}{10}, 2\frac{7}{15}, 3\frac{1}{6}$

E $60\frac{5}{6}, \frac{3}{8}, 11\frac{2}{3}, 15\frac{1}{12}$

F $4\frac{1}{3}, 17\frac{5}{9}, \frac{2}{3}$

G $\frac{7}{12}, \frac{19}{24}, \frac{3}{8}, \frac{1}{4}, \frac{1}{2}$

H $84\frac{3}{5}, 119\frac{2}{3}, 75\frac{1}{5}$

A $s - 5\frac{1}{4} = 3\frac{7}{12}$

B $16\frac{1}{2} - r = \frac{5}{6}$

C $d + 10\frac{3}{4} = 11\frac{1}{2}$

D $16\frac{7}{10} - 8\frac{3}{5} = m$

E $8\frac{7}{9} + a = 9\frac{1}{3}$

F $20\frac{1}{3} - 17 = s$

G $b + 9\frac{7}{12} = 18\frac{1}{4}$

H $c - \frac{5}{6} = 11$

I $32 - w = 12\frac{9}{10}$

A 1188 in. = ▩ ft.

B 7 T. = ▩ lb.

C 48 gal. = ▩ qt.

D 216 hr. = ▩ da.

E 1095 da. = ▩ yr.

F 19 yd. = ▩ in.

G 4800 sec. = ▩ min.

H 60 tbs. = ▩ fl. oz.

I 208 wk. = ▩ yr.

J 9 c. = ▩ fl. oz.

Thinking straight

A $13 \times (9+30) = d$

B $13(9+30) = d$

C $13(39) = d$

D $13(9)+13(30) = d$

A In Example A above, how would you find the numeral that replaces d? If the equation were $(9+30) \times 13 = d$, how would you find the numeral that replaces d? Would the answer be the same as the answer in Example A?

B Now look at Example B. This equation shows one way to rewrite the equation in Example A. Notice that when you use parentheses, you can leave out the multiplication sign. Now rewrite $5 \times (8+16) = n$ without using the multiplication sign.

C Example C shows another way to rewrite the equation in Example A. In Example C, part of the computation called for in Examples A and B has been done. What part has been done?

D Rewrite the equation in Example C, using a multiplication sign. When you do this, you may leave out the parentheses if you want to.

E Whenever you multiply, the answer is called the product. What is the product of 13 times 39?

F Now find the numeral that replaces d in Example D. Is the answer the same as the answer to the equation in Example C? When you find the numeral that replaces the letter in an equation, you have solved the equation.

G Examine the equations in Examples A and D again. Notice that you can add 30 and 9 and then multiply the sum by 13. Or you can multiply 9 by 13 and 30 by 13 and then add the products. Does the way you solve the equation make a difference in the answer that you get?

Solve each equation below in two ways. For each way, rewrite the equation in the way that shows how you solved it.

H $15(6+19) = q$

I $24(62+9) = c$

J $(16+15)7 = k$

K $22(31+16) = n$

L $9(106+53) = d$

M $125(5+10) = t$

N $200(3+9) = m$

O $550(12+31) = b$

E $(45-17)\times 11=s$

F $11(45-17)=s$

G $11(28)=s$

H $11(45)-11(17)=s$

A How would you solve the equation in Example E above?

B The equation in Example F shows the equation in Example E rewritten without the multiplication sign. Do you think it matters whether the equation is rewritten as $11(45-17)=s$ or as $(45-17)11=s$? Why?

C Now rewrite the equation in Example G, using the multiplication sign. Leave out the parentheses if you want to. What is the product of 11 and 28?

D Solve the equation in Example H. Is the answer the same as the product you found in Example G?

E Examine the equation in Example E again. Does it make any difference in the answer whether you subtract before you multiply or multiply before you subtract?

Solve each equation below in two ways. For each way, rewrite the equation in the way that shows how you solved it.

F $33(37-28)=f$

G $(13-7)12=n$

H $25(38-12)=m$

I $8(29-17)=a$

J $5(26-12)=s$

K $30(18-16)=z$

L $200(50-25)=c$

M $400(100-60)=e$

A Solve the equation in Example I at the top of the next page. Did you have 50 before you divided? How did you get it? Whenever you divide, the answer is called the quotient. What is the quotient when you divide 50 by 5?

B Look at Example J. This equation shows one way to rewrite the equation in Example I. Notice that the division sign (÷) is not used. Where is the divisor, 5, written?

C The equation shown in Example I is rewritten again in Example K. What part of the computation called for in Example I has been done now?

D Earlier in your work, you learned that expressions like 50 over 5 may be fraction numerals or ratios. In Example K, what does the expression 50 over 5 mean? What is the quotient?

I $(15+35)\div 5=n$

J $\frac{15+35}{5}=n$

K $\frac{50}{5}=n$

L $(15\div 5)+(35\div 5)=n$

M $\frac{15}{5}+\frac{35}{5}=n$

N $(68-36)\div 4=t$

O $\frac{32}{4}=t$

P $(68\div 4)-(36\div 4)=t$

E Solve the equation in Example L. Is your answer the same as the answer you found for Example K?

F The equation in Example M shows another way to write the equation in Example L. Here, too, you first divide 15 by 5 and 35 by 5. Then what do you do with the two quotients?

G How would you solve the equation in Example N?

H Rewrite the equation in Example N using the form shown in Example J.

I Explain how Example O shows another way to write the equation in Example N. What is the quotient?

J Solve the equation in Example P. What do you notice about the answer?

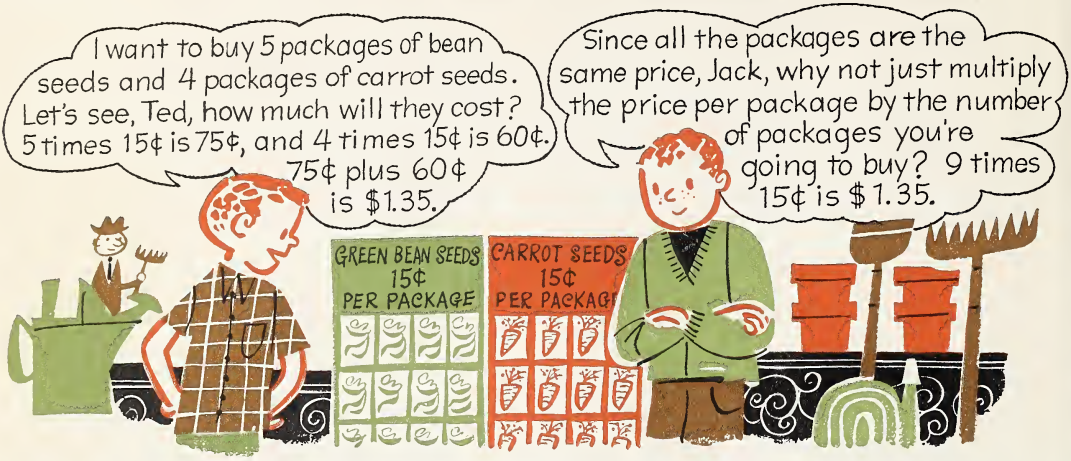
K Rewrite the equation in Example P, using the form shown in Example M.

L Now think about the equations that you have been solving. Sometimes you have first added or subtracted and then divided. Sometimes you have divided first and then added or subtracted. Has the way you solved these equations made any difference in the answer?

Rewrite each equation below in two ways without using a division sign (÷). Then solve your equations.

M $(48+108)\div 12=n$
N $(92-76)\div 4=p$
O $(49+84)\div 7=b$
P $(75-30)\div 15=v$
Q $(54+135)\div 27=h$
R $(184+69)\div 23=x$
S $(132-84)\div 12=d$
T $(318-106)\div 53=z$
U $(410+492)\div 41=a$

Thinking straight


I want to buy 5 packages of bean seeds and 4 packages of carrot seeds. Let's see, Ted, how much will they cost? 5 times 15¢ is 75¢, and 4 times 15¢ is 60¢. 75¢ plus 60¢ is $1.35.
Since all the packages are the same price, Jack, why not just multiply the price per package by the number of packages you're going to buy? 9 times 15¢ is $1.35.
GREEN BEAN SEEDS 15¢ PER PACKAGE
CARROT SEEDS 15¢ PER PACKAGE

A In the picture, why did Jack add 60¢ and 75¢?

B Why did Ted multiply 15¢ by 9?

C Write an equation to show how each boy solved the problem. Which of these two ways would you use to solve the problem? Why?

D To find the product of 33 and 8 in your head, you can think of 33 as $30+3$. Then multiply 30 by 8 and 3 by 8. Now what do you do with these two products? What is the answer?

E Or you can multiply 33 by 8 by thinking of 33 as $40-7$. You then multiply ▭ by 8 and ▭ by 8. What do you do with these two products?

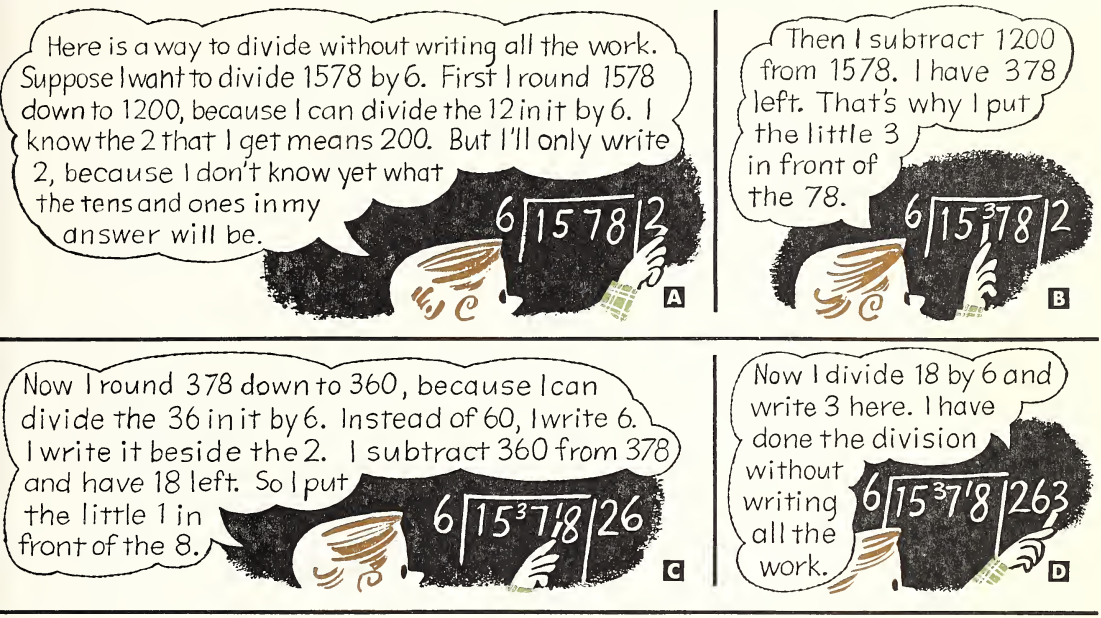
F Which way of multiplying 33 by 8 is easier for you, the way shown in Exercise D or that shown in Exercise E?

G Suppose you want to multiply 45 by 6. You can think of 45 as ▭ -5 or as ▭ $+5$. If you think of 45 as $40+5$, what do you do with the product of 40 and 6 and the product of 5 and 6? What is the answer?

Multiply in your head. Use the way that is easier for you.

A $7\times28=t$
B $3\times98=p$
C $6\times71=n$
D $9\times16=c$
E $4\times35=m$
F $8\times22=b$
G $5\times63=e$
H $(5\times9)+(3\times9)=s$
I $(8\times7)+(8\times3)=c$
J $8\times53=n$
K $(4\times6)+(4\times9)=a$
L $3\times77=g$
M $(7\times8)+(7\times9)=r$
N $(9\times3)+(9\times8)=f$

Thinking straight


Here is a way to divide without writing all the work. Suppose I want to divide 1578 by 6. First I round 1578 down to 1200, because I can divide the 12 in it by 6. I know the 2 that I get means 200. But I'll only write 2, because I don't know yet what the tens and ones in my answer will be.
6 | 15 78 | 2
A
Then I subtract 1200 from 1578. I have 378 left. That's why I put the little 3 in front of the 78.
6 | 15 3 78 | 2
B
Now I round 378 down to 360, because I can divide the 36 in it by 6. Instead of 60, I write 6. I write it beside the 2. I subtract 360 from 378 and have 18 left. So I put the little 1 in front of the 8.
6 | 15 3 7 1 8 | 26
C
Now I divide 18 by 6 and write 3 here. I have done the division without writing all the work.
6 | 15 3 7 1 8 | 263
D

A Divide 930 by 5.

$5\overline{)9^{4}30}\,|\,1$

What does the 1 represent? Why is the small 4 written at the left of 30? What do you divide next?

$5\overline{)9^{4}3^{3}0}\,|\,18$

What does the 8 represent? Why is the small 3 written at the left of the 0? What do you divide now?

$5\overline{)9^{4}3^{3}0}\,|\,186$

Does the 6 represent ones or tens? Is the division finished?

B Divide 721 by 3.

$3\overline{)7^{1}2\,1}\,|\,240$ and 1 rm.

Why is the small 1 written at the left of the 21? How do you know that the answer is 240 and 1 remainder, and not 24 and 1 remainder?

Divide without writing all the work.

A	$776 \div 8 = p$	**G**	$3008 \div 6 = n$
B	$3756 \div 7 = b$	**H**	$2997 \div 9 = c$
C	$2960 \div 4 = n$	**I**	$2903 \div 7 = z$
D	$1073 \div 9 = k$	**J**	$648 \div 6 = t$
E	$362 \div 5 = x$	**K**	$1487 \div 3 = m$
F	$2472 \div 8 = s$	**L**	$960 \div 5 = a$

Exploring problems

see

A

Mr. Castle bought 12 rosebushes for $28. At this rate, he paid how much for 3 rosebushes?

$\frac{12}{28}$ ← This ratio shows that the rosebushes were bought at the rate of 12 for $28.

You do not know how much 3 rosebushes cost. Look at Picture B.

B

Mr. Castle paid for 3 bushes at the same rate as for 12 bushes, but now the rate is expressed by a different ratio.

$\frac{3}{x}$ ← Number of rosebushes

← Number of dollars 3 rosebushes cost

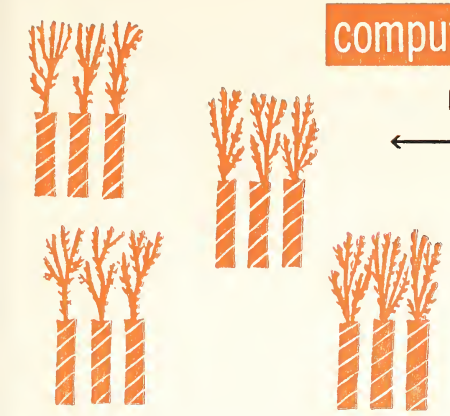
$\frac{12}{28} = \frac{3}{x}$

The first term in each ratio stands for the number of rosebushes.

The second term in each ratio stands for the number of dollars. You must find the numeral that replaces x.

The ratio 3 per x expresses the same rate as the ratio 12 per 28.

compute



C

You can think of the 12 rosebushes as divided into groups of 3. There are 4 groups.

3 can be found by dividing 12 by 4.

$$\frac{12}{28} = \frac{3}{x}$$

D

It takes $28 to pay for 4 groups of 3 bushes. So divide $28 into 4 equal groups to find how much a group of 3 bushes cost.

You can compute to find the numeral that replaces x.

$$\frac{12}{28} = \frac{3}{x}$$

3 can be found by dividing 12 by 4. So the numeral that replaces x can be found by dividing 28 by 4. 7 replaces x.

3 rosebushes cost $7.

see

A

Mr. Castle also bought 150 flower bulbs priced at $2.30 for 25 bulbs. How much did he pay for the 150 bulbs?

This ratio shows that the bulbs were sold at the rate of $2.30, or 230¢, per 25 bulbs.

$$\frac{230}{25}$$

Now turn the page.

You do not know how much 150 bulbs cost.

B

The rate for 150 bulbs is the same as the rate for 25 bulbs, but now the rate is expressed by a different ratio.

$\frac{m}{150}$ ← Number of cents 150 bulbs cost

← Number of bulbs

$$\frac{230}{25} = \frac{m}{150}$$

Do these terms stand for number of cents or for number of bulbs? You must find the numeral that replaces m.

What do these terms stand for?

compute

It takes 6 groups of 25 bulbs to make the group of 150 bulbs.

C

$$\frac{230}{25} = \frac{m}{150}$$

150 can be found by multiplying 25 by 6.

It takes $2.30 to pay for each group of 25 bulbs. It takes 6 groups of $2.30 to pay for 6 groups of 25 bulbs.

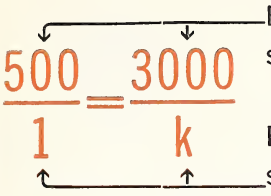
D

$$\frac{230}{25} = \frac{m}{150}$$

150 can be found by multiplying 25 by 6. How do you find the numeral that replaces m? 1380 replaces m.

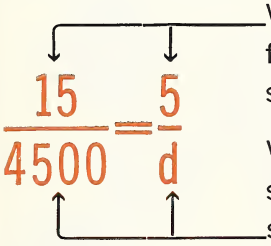
The 150 bulbs cost 1380¢, or $13.80.

A Mr. Castle knows that he uses 1 lb. of grass seed for 500 sq. ft. of lawn. How many pounds of grass seed will he need for 3000 sq. ft. of lawn?



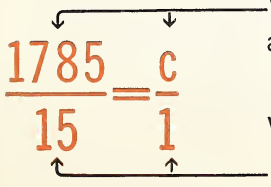
Why must you multiply 1 by 6 to find the numeral that replaces k?

B Mr. Bills used 15 lb. of grass seed to make 4500 sq. ft. of new lawn. He used grass seed at a rate of 5 lb. per how many square feet of lawn?



Why must you divide 4500 by 3 to find the numeral that replaces d?

C Mr. Bills paid $17.85 for the 15 lb. of grass seed. He bought the seed at a rate of how much per pound?



D Mr. Castle knows that he should use 8 fl. oz. of weed killer for 1500 sq. ft. of lawn. How many fluid ounces of weed killer should he use for his 3000 sq. ft. of lawn?

$$\frac{8}{1500}=\frac{b}{3000} \text{ or } \frac{1500}{8}=\frac{3000}{b}$$

Does it make any difference which equation you use? Why is b in a different place in the two equations?

E How many square feet of lawn will 1 32 oz. qt. of weed killer cover?

$$\frac{8}{1500}=\frac{32}{n} \text{ or } \frac{1500}{8}=\frac{n}{32}$$

F Mrs. Castle buys about 3 lb. of chicken to serve 4 persons. About how many pounds of chicken should she buy to serve 12 persons?

$$\frac{3}{4}=\frac{x}{12} \text{ or } \frac{4}{3}=\frac{12}{x}$$

G A meat loaf made with 3 lb. of ground meat will serve 10 persons. How many pounds of ground meat would be needed to make meat loaf to serve 50 persons?

H Alice paid $1.70 for 5 lb. of seed to feed her parakeet. She bought the seed at a rate of how many cents per pound?

I Alice has saved 75¢ in 4 weeks. At this rate, how much money can she expect to save in 16 weeks?

J Mrs. Castle paid 80¢ for buttons that were priced at 40¢ for 6 buttons. How many buttons did she buy?

K Mrs. Castle bought 6 dishcloths for 99¢. The dishcloths were selling at the rate of how many for 33¢?

L Philip measured the distance between two towns on a road map and found it was 6 inches. 1 inch on the map represented a distance of 4 miles. What was the actual distance between the two towns?

M Philip's father drove 237 mi. going from his home to Silver City. In the first 2 hr. he drove 79 mi. If he travelled at the same rate for the whole distance, how long did it take him to drive from his home to Silver City?

N Mr. Castle drove 294 mi. on 15 gal. of gasoline. At this rate, how far did he drive on 5 gal. of gasoline?

O In a typing test, Betty typed 765 words in 15 minutes. Betty typed at a rate of how many words per minute?

P Bill typed 735 words in 15 minutes. At this rate, how many words would he type in 1 hour?

Now you should be able to make ratios that express rates and to use these ratios to solve problems.

Keeping skilful

A 15 lb. = ▒ oz.
B 80 pk. = ▒ bu.
C 78 ft. = ▒ yd.
D 3 min. = ▒ sec.
E 12 tbs. = ▒ fl. oz.
F 6 mi. = ▒ ft.
G 50 pt. = ▒ qt.
H 91 da. = ▒ wk.
I 8 T. = ▒ lb.
J 9 yd. = ▒ in.
K 19 fl. oz. = ▒ tbs.

A $8314 + z = 9660$
B $7565 - a = 3008$
C $n - 5067 = 4129$
D $c + 4898 = 10713$
E $36710 - 18495 = w$
F $64450 + 9212 = b$
G $80040 - d = 63241$
H $f + 7983 = 27342$
I $t - 57248 = 46011$
J $28050 - 19336 = x$
K $36450 + r = 70093$

A $6 \times 5093 = m$
B $728 \div 35 = x$
C $3000 \div r = 62$
D $37 \times 419 = v$
E $508 \times 634 = y$
F $2070 \div a = 59$
G $45 \times 6007 = n$
H $8587 \div 160 = c$
I $9021 \div k = 372$
J $867 \times 509 = w$
K $73 \times 8016 = b$

Exploring problems

see

A

Philip and Paul go to different schools. Philip spends 50¢ for round-trip bus fare each day, and Paul spends 30¢ a day. When Philip has spent $2.50 for bus fare, how much will Paul have spent?

$\frac{50}{30}$ ← This ratio compares the number of cents Philip spends in a day with the number of cents Paul spends in a day.

B

Imagine that Philip has spent the $2.50. You can make a ratio that compares what he has spent with the amount that Paul will have spent.

$\frac{250}{a}$ ← Number of cents Philip has spent

← Number of cents Paul will have spent

This new ratio is equal to the ratio 50 to 30.

$\frac{50}{30} = \frac{250}{a}$

Amount Philip spends in cents

Amount Paul spends in cents. You are to find the numeral that replaces a.

Now turn the page.

compute

C

Every time Philip pays 50¢ for bus fare, Paul pays 30¢. Philip has to pay 50¢ five times before he has spent $2.50 or 250¢. So when Philip has spent 5 groups of 50¢, Paul will have spent 5 groups of 30¢, or 150¢.

You can compute to find the numeral that replaces a.

$$\frac{50}{30} = \frac{250}{a}$$

You can multiply 50 by 5 to get 250. So you multiply 30 by 5 to find the numeral that replaces a. 150 replaces a.

When Philip has spent $2.50 for bus fare, Paul will have spent $1.50.

see

A

May and Ann have been weaving squares. May wove 16 while Ann wove 12. How many squares did May weave for each 3 that Ann wove?

$$\frac{16}{12}$$

This ratio compares the number of squares that May wove with the number that Ann wove.

B

You do not know how many squares May wove for each group of 3 that Ann wove.

But you can make a ratio that compares the unknown number of May's squares with Ann's 3 squares.

$\frac{n}{3}$ ← Number of squares May wove

3 ← Number of squares Ann wove

This ratio, n to 3, expresses the same comparison as the ratio 16 to 12.

Number of May's squares

$$\frac{16}{12} = \frac{n}{3}$$

Number of Ann's squares

compute

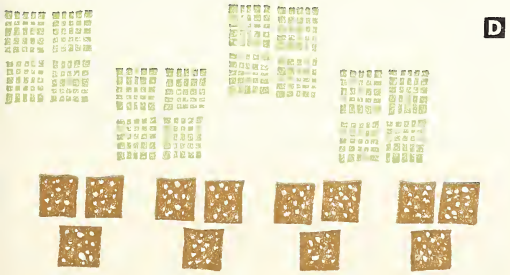
C

Think of Ann's 12 squares as divided into groups of 3. There are 4 groups.

$$\frac{16}{12} = \frac{n}{3}$$

3 can be found by dividing 12 by 4.

Think of May's squares also as divided into 4 groups. 4 squares are in each group.



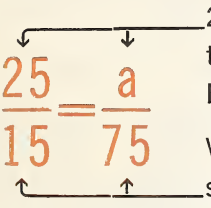
D

3 can be found by dividing 12 by 4. How do you find the numeral that replaces n?

$$\frac{16}{12} = \frac{n}{3}$$

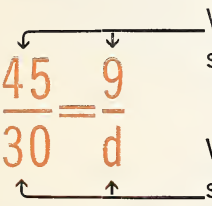
May wove 4 squares for each 3 squares that Ann wove.

A Don saves 25¢ a week, and George saves 15¢ a week. How much will Don have saved when George has saved 75¢?



Why is 75 in the second term of the new ratio? Why do you multiply 25 by 5 to find the numeral that replaces a?

B Ann wrote 45 invitations while Sue wrote 30. How many invitations did Sue write for each 9 that Ann wrote?



Why is d in the second term of the new ratio? Why do you divide 30 by 5?

C Two planes left Winnipeg at the same time to go to Medicine Hat, 600 mi. away. One plane flew at 300 mi. per hour. The other plane flew at 250 mi. per hour. How far had the slower plane gone when the faster plane had arrived at Medicine Hat?

$\frac{300}{250}=\frac{600}{n}$ or $\frac{250}{300}=\frac{n}{600}$

THINK Each ratio in the first equation compares the distance travelled by the faster plane with the distance travelled by the slower plane. What does each ratio in the second equation compare? Why is n in a different place in this equation?

D Sally skated 15 times around the rink while Jane skated 12 times. When Sally had gone around 5 times, how many times had Jane gone around?

$\frac{15}{12}=\frac{5}{t}$ or $\frac{12}{15}=\frac{t}{5}$

THINK What does t stand for in the first equation? Does t stand for the same thing in the second equation? Why is t in a different place in the second equation?

E 24 in. of snow fell in Elmwood on Wednesday. During the same time, 16 in. of snow fell in Hometown. How much snow fell in Hometown for each 3 in. of snow that fell in Elmwood?

$\frac{24}{16}=\frac{3}{s}$ or $\frac{16}{24}=\frac{s}{3}$

F A large wheel on a machine turns 20 times while a small wheel on the same machine turns 120 times. How many times has the large wheel turned when the small wheel has turned 720 times?

$\frac{20}{120}=\frac{m}{720}$ or $\frac{120}{20}=\frac{720}{m}$

G One man made 300 machine parts while another man made 250. How many parts did the first man make for each 25 parts the second man made?

H Mr. Philips said, "I have 150 bu. of corn. I have 3 bu. for every 2 bu. that Mr. Price has." How many bushels of corn has Mr. Price ?

I Bob delivers morning papers, and Jack delivers afternoon papers. Bob says that he delivers 4 papers for every 3 papers that Jack delivers. Jack delivers 60 papers. How many papers does Bob deliver?

J Mrs. Farmer says she spends about $40 a week for groceries. Mrs. Bell says she spends about $30. How much does Mrs. Bell spend for every $8 that Mrs. Farmer spends?

K Mrs. Johnson says that each month she spends $150 for groceries and $100 for rent. How much money has she spent for groceries when she has spent $1200 for rent?

L One month Mr. Cook worked 216 hr. on his job, and Mr. Hill worked 192 hr. How many hours did Mr. Hill work for each 54 hr. that Mr. Cook worked?

M Park City built 240 blocks of new streets one summer. That summer Deepwater built 160 blocks of new streets. How many blocks of new streets did Park City build for each 20 blocks that Deepwater built?

N Jim said, "During the holidays, I travelled 6 mi. to your 5 mi., Bob." Bob had travelled 380 mi. How many miles had Jim travelled?

Now you should know how to use ratios that express comparisons in solving problems.

Keeping skilful

A $8\frac{2}{3}+5\frac{1}{8}=t$

B $7\frac{3}{4}-a=2\frac{1}{5}$

C $16\frac{5}{6}-4\frac{1}{2}=x$

D $g-3\frac{1}{3}=5\frac{4}{5}$

E $9\frac{3}{8}+n=12$

F $6\frac{1}{12}-c=2\frac{2}{9}$

G $25\frac{1}{7}+6\frac{2}{3}+18\frac{3}{7}=n$

H $14\frac{3}{4}-7\frac{1}{4}=b$

I $s-6\frac{1}{2}=3\frac{1}{5}$

J $22\frac{1}{10}+g=35\frac{1}{10}$

K $5\frac{2}{3}+7\frac{5}{12}=p$

L $d+8\frac{2}{7}=31$

M $n-9\frac{3}{10}=16\frac{1}{2}$

N $33\frac{1}{3}-16\frac{7}{8}=e$

O $1\frac{3}{5}+w=2\frac{1}{4}$

P $5\frac{11}{12}+13\frac{1}{6}+3\frac{7}{12}=m$

Q $h+2\frac{5}{9}=17\frac{2}{3}$

R $4\frac{3}{4}+5\frac{1}{4}+6\frac{5}{8}=n$

Exploring problems

A Don and Pat have a small printing press. One day Don printed 48 cards on the press, and Pat printed 16 cards. Don printed how many times as many cards as Pat that day?

To answer the question, you must find how many cards Don printed for each card that Pat printed.

The picture shows that Don printed 3 cards for each card that Pat printed.

You can also compute to find the answer. First you make a ratio that compares the total number of cards Don printed with the total number of cards Pat printed.

$\frac{48}{16}$ ← Number Don printed / Number Pat printed

Next you make a ratio that compares the number of cards Don printed with one card that Pat printed. Look at the ratio below.

$\frac{x}{1}$ ← Number of cards that Don printed / One card that Pat printed

The ratios 48 over 16 and x over 1 express the same comparison. So you can write the equation shown below.

$$\frac{48}{16} = \frac{x}{1}$$

1 can be found by dividing 16 by ▩. So to find the numeral that replaces x, you divide ▩ by 16.

$$\frac{48}{16} = \frac{3}{1}$$

3 replaces x. Don printed 3 cards for each card that Pat printed.

Don printed 3 times as many cards as Pat printed.

B Sue has 19 ducklings. She also has 2 times as many chicks as ducklings. How many chicks has she?

You are told that Sue has 2 times as many chicks as ducklings. So you know that she has 2 chicks for each duckling.

You can use the ratio 2 over 1 to compare chicks with ducklings. Can you also use the ratio m over 19 to compare chicks with ducklings?

Now you can make an equation of these equal ratios and compute.

$$\frac{2}{1} = \frac{m}{19}$$

19 can be found by multiplying ▒ by ▒. How do you find the numeral that replaces m?

$$\frac{2}{1} = \frac{38}{19}$$

38 replaces m.
Sue has ▒ chicks.

C Mr. Peters said, "I have 70 lambs this spring. I have 5 times as many lambs as Mr. Cook." How many lambs did Mr. Cook have?

You know that Mr. Peters had 5 lambs for each lamb that Mr. Cook had.

You can use the ratio 5 over 1 to compare Mr. Peters' lambs with Mr. Cook's lambs. Does the ratio 70 over d also express the comparison of Mr. Peters' lambs with Mr. Cook's?

When you solve the equation made from these equal ratios, you will know how many lambs Mr. Cook had.

$$\frac{5}{1} = \frac{70}{d}$$

How do you find the numeral that replaces d?

$$\frac{5}{1} = \frac{70}{14}$$

14 replaces d.
Mr. Cook had ▒ lambs.

D Jack is raising 13 pigs for a 4-H Club project. Paul is raising twice as many pigs as Jack. How many pigs is Paul raising?

"Twice as many" means "2 times as many." So Paul is raising 2 pigs for each pig that Jack is raising.

$$\frac{2}{1} = \frac{a}{13}$$

Number of pigs Paul is raising

Number of pigs Jack is raising

What numeral replaces a?

E On a trip, Ann sent 45 postcards, and May sent 15. Ann sent how many times as many postcards as May?

$$\frac{45}{15} = \frac{x}{1}$$

F Patty sold 35 cups of lemonade at the school fair. Kathy sold 2 times as much lemonade as Patty did. How many cups of lemonade did Kathy sell?

$\frac{2}{1} = \frac{g}{35}$

G Jim has a collection of 56 science pictures. He has 4 times as many science pictures as Henry has. How many science pictures has Henry?

$\frac{4}{1} = \frac{56}{t}$

H Peter brought 48 lb. of paper for the paper drive. He said, "I brought twice as much paper as George did." How much paper did George bring?

I Jim brought 96 lb. of paper. He brought how many times as much paper as George did?

J Mr. Parker's garden is 240 sq. ft. in size. Mr. Cook's garden is twice as large as Mr. Parker's. Mr. Cook's garden is how many square feet in size?

K In one summer Jane saved $7.60. Her sister saved 6 times as much as Jane did. How much money did Jane's sister save that summer?

L Jane weighs 80 lb. Her baby brother weighs 16 lb. Jane weighs how many times as much as her baby brother?

M Jane's father said, "You weigh 10 times as much as the cat, Jane." How much did the cat weigh?

N George skated 300 yd., and Henry skated 75 yd. George skated how many times as far as Henry?

Now you should be able to use ratios in solving problems like those in this lesson.

Keeping skilful

A $\frac{1}{4} + \frac{3}{8} + 9\frac{11}{16} = x$

B $a - 14\frac{2}{3} = 16\frac{1}{8}$

C $20\frac{4}{5} - 6\frac{9}{10} = t$

D $s + 42\frac{1}{6} = 60$

E $11\frac{1}{8} - r = 10\frac{5}{8}$

F $80\frac{1}{2} + b = 100$

G $17\frac{4}{5} + 9\frac{5}{6} = c$

H $w - 8\frac{11}{12} = 22\frac{1}{6}$

I $49\frac{1}{8} - 48\frac{11}{12} = s$

J $x + 14\frac{3}{5} = 21\frac{1}{4}$

K $33\frac{9}{10} - d = 21$

L $19\frac{3}{4} - m = 8\frac{3}{4}$

M $r - 6\frac{4}{5} = 8\frac{1}{2}$

N $21\frac{11}{12} - 20\frac{23}{24} = f$

O $10\frac{9}{16} - n = 5\frac{3}{8}$

P $d + 14\frac{11}{24} = 16\frac{1}{3}$

Q $25\frac{3}{10} + 17\frac{7}{10} = x$

R $\frac{7}{8} + \frac{11}{12} + 1\frac{17}{24} = n$

Find the sum.

A $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$

B $\frac{1}{2}, \frac{4}{5}, \frac{1}{4}, \frac{3}{10}$

C $5\frac{7}{16}, 1\frac{1}{2}, \frac{3}{8}$

D $2\frac{1}{10}, \frac{3}{5}, 4\frac{1}{6}$

E $\frac{1}{3}, 6\frac{7}{8}, 2\frac{3}{4}$

Thinking straight

A Carol has picked 10 roses. 8 are white, and 2 are red. To compare the red roses with all the roses, you can say, "2 out of 10 are red." You are comparing part of the group with the whole group. Write a ratio that shows this comparison.

B But if you want to compare the red roses with the white roses, you can say, "2 red roses to 8 white roses." Now you are comparing one group with another group. Write a ratio that shows this comparison.

C Imagine that you have a pile of cards. Some of them are blue, and some are yellow. Suppose you say, "5 in each 6 are blue." Are you comparing the blue cards with all the cards, or are you comparing the blue cards with the yellow cards?

D In Exercise C, are you comparing part of the group with the whole group, or are you comparing two groups? Write a ratio that shows the comparison made in Exercise C.

E Now imagine that you have another pile of cards. Some of them are white, and some of them are green. Suppose you say, "I have 3 white cards for each 7 green cards." Are you comparing the white cards with all the cards, or are you comparing the white cards with the green cards?

F In Exercise E, are you comparing part of the group with the whole group, or are you comparing two groups? Write a ratio showing the comparison made in Exercise E.

G Bob said, "I have 2 science books to 5 story books." Was he comparing two groups, or was he comparing part of one group with the whole group?

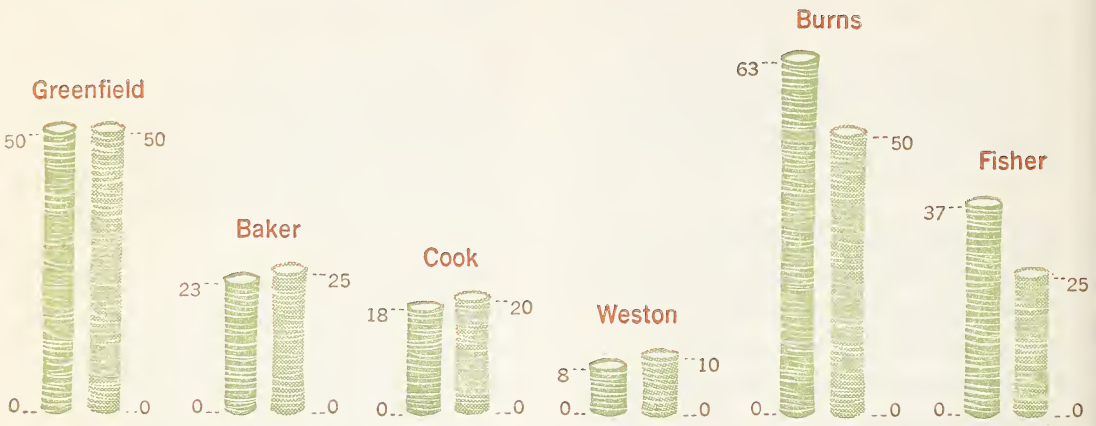
H Suppose Bob had said, "2 of these 7 books are science books." In this case, would he be comparing part of a group with the whole group, or would he be comparing two groups?

Each exercise below expresses a comparison. For each one, decide what is meant, the comparison of a part of a group with the whole group or the comparison of two groups.

A 5 in each 7
B 3 of the 4
C 11 to 15
D 2 in each 10
E 6 of the 9
F 14 out of 16
G 9 for each 12
H 3 to 8
I 6 out of 11
J 25 to 15
K 13 for each 26
L 14 of the 15
M 37 to 100
N 19 of the 40
O 80 to 45
P 18 of the 50

Moving forward

Now you will study a new way to express some comparisons.



A The schools of Silver City raised money to buy some new equipment for a park playground. Look at the picture above. For each school the pile of silver dollars on the right shows the goal, which is the total amount the school wanted to raise. The pile of silver dollars on the left shows how much money the school really raised. How much money did each of the schools raise?

B You can tell from the picture that the Greenfield School exactly reached its goal. Which schools passed their goals? Which schools did not reach their goals? You cannot tell from the picture which school did the best.

C To compare some of these schools to see how well they did, you can start with the Baker and Weston schools. The Baker School raised $23 of its goal of $25. The Weston School raised ▤ of its goal of ▤. Do the ratios 23 over 25 and 8 over 10 show the comparisons? You still cannot easily tell which school did better.

D Now express each comparison by a new ratio with 50 in its second term. Can 46 over 50 be a ratio for the Baker School? A ratio for the Weston School can be ▤ over 50. How do these new ratios make it easy to tell that the Baker School did better than the Weston School?

E To decide which did better, the Burns School or the Fisher School, look at Column A below. Does the ratio 63 over 50 express the comparison for the Burns School? The comparison for the Fisher School is expressed by the ratio 37 over ▒. You cannot easily tell which school did better. Why?

F Can a ratio for the Fisher School be expressed with 50 in its second term? Does the ratio 74 over 50 express the same comparison as the ratio 37 over 25? Now how can you tell that the Fisher School did better than the Burns School?

G It is easy to decide which school did better when both ratios have the same second term. Why?

H Now suppose you want to find which school did better, the Baker School or the Cook School. What ratios express the comparisons for the two schools? Why is it not convenient to replace both of these ratios with new ratios that have 50 in the second term?

I People often make comparisons by using ratios that have 100 in the second term. Look at Column B in the table. Can the comparison expressed by the ratio 23 over 25 for the Baker School also be expressed as 92 over 100? The ratio 18 over 20 for the Cook School can be replaced by ▒ over 100. Which of these two schools did better? How do you know?

J Notice that Column B shows, for each school, a ratio with 100 in its second term. Each ratio compares the money raised with the school's goal. How was each ratio in Column B found?

	A	B	C	D
Greenfield	$\frac{50}{50}$	$\frac{100}{100}$	100 per cent	100%
Baker	$\frac{23}{25}$	$\frac{92}{100}$	92 per cent	92%
Cook	$\frac{18}{20}$	$\frac{90}{100}$	90 per cent	90%
Weston	$\frac{8}{10}$	$\frac{80}{100}$	80 per cent	80%
Burns	$\frac{63}{50}$	$\frac{126}{100}$	126 per cent	126%
Fisher	$\frac{37}{25}$	$\frac{148}{100}$	148 per cent	148%

K Now use the ratios in Column B to make a table listing the schools in the order of how well they did. If you plan to end with the Fisher School, with which school should you begin?

L When a ratio that expresses a comparison has 100 in its second term, you can read the ratio as a per cent. Look at Column C. When you say "92 per cent," you mean that the ratio you are using has 100 in its second term. What is the second term of a ratio that is expressed as 90 per cent? As 100 per cent? As 126 per cent?

M The sign at the right is called the per cent sign. You read it "per cent." Column D on page 127 shows how to write the per cents in Column C with the per cent sign. From the per cents in Column D decide which school exactly reached its goal. How do the per cents show that two schools passed their goals? How do the per cents show that three schools failed to reach their goals?

%

Each of the ratios below expresses a comparison. Write each ratio as a per cent. Use the per cent sign.

A $\frac{15}{100}$ **B** $\frac{99}{100}$ **C** $\frac{134}{100}$ **D** $\frac{6}{100}$

E $\frac{24}{100}$ **F** $\frac{2}{100}$ **G** $\frac{43}{100}$ **H** $\frac{312}{100}$

Write each of the following as a ratio with 100 in the second term.

I 44% **J** 11% **K** 82% **L** 101%

M 19% **N** 6% **O** 23% **P** 140%

For each per cent given below, write as many equal ratios as you can without using the per cent sign. The work for 120% might look like this.

A $120\% = \frac{120}{100} = \frac{60}{50} = \frac{6}{5}$

A 120% **B** 50% **C** 48% **D** 31%

E 110% **F** 80% **G** 75% **H** 90%

I 250% **J** 5% **K** 12% **L** 36%

A A farmer said that 50% of his corn crop was lost because of a storm. Do you know how many acres of corn were lost? Do you know how many acres of corn he had planted?

B Could he have planted 100 acres? Could he have planted 10 acres?

C Can you say that he lost 50 acres out of each 100 acres he planted? Can you say that he lost 5 acres out of every 10 acres?

D Carol worked all the examples in a test on arithmetic. Her grade was 75%. Did she make any mistakes in her work?

E Is there any way to tell how many examples Carol worked? Is there any way to tell how many examples she worked incorrectly?

F Could Carol have worked more than 100% of the examples correctly? Why or why not?

	Goal	Money saved
Peter	$20	$18
Don	$25	$11
Jack	$10	$9
Pat	$50	$27
George	$10	$14

G The table above shows how much money five boys planned to save and how much money each of them really did save. For each boy, first write a ratio that compares the amount he saved with his goal. Next write an equal ratio with 100 as its second term. Then write each ratio as a per cent.

H Which boy did best in reaching or passing his goal? Which boy saved the smallest per cent of his goal? Write the names of the boys in an order that shows how well they did. Begin with the boy who did the best.

For each statement below, first write a ratio to show the comparison. Then write an equal ratio with 100 in its second term. Finally, express the ratio as a per cent.

A 4 out of 5
B 1 to 10
C 6 for each 5
D 8 in each 20
E 11 in each 50
F 19 in each 25
G 40 to 20
H 7 out of 50
I 15 to 5
J 13 of the 25
K 9 out of 50
L 7 of the 20
M 66 for each 50
N 14 out of 25
O 6 of the 10
P 8 in each 50

Now you can read and write per cents and can express a comparison by a per cent.

Keeping skilful

A $17\frac{7}{8} - 9\frac{9}{16} = f$
B $\frac{11}{18} + 11\frac{1}{9} = h$
C $d + 14\frac{2}{3} = 21\frac{5}{6}$
D $k - 23\frac{3}{4} = 57\frac{5}{8}$
E $34\frac{1}{6} - h = 6\frac{1}{2}$
F $\frac{3}{7} + \frac{3}{4} + \frac{1}{2} = w$
G $7\frac{1}{12} + d = 9\frac{3}{8}$

Solve each equation below.

A $\frac{18}{7} = \frac{f}{21}$
B $\frac{44}{64} = \frac{11}{h}$
C $\frac{9}{27} = \frac{1}{t}$
D $\frac{15}{90} = \frac{k}{18}$
E $\frac{12}{8} = \frac{36}{t}$
F $\frac{17}{9} = \frac{68}{s}$

A 7 mi. = ▒ ft.
B 420 sec. = ▒ min.
C 19 c. = ▒ fl. oz.
D 128 oz. = ▒ lb.
E 56 pk. = ▒ bu.
F 105 da. = ▒ wk.
G 25 da. = ▒ hr.
H 32 tbs. = ▒ fl. oz.

Exploring problems

A 700 pupils are enrolled in the Jackson School. 420 of the pupils usually walk to school. The pupils who usually walk to school are what per cent of all the pupils?

You can use an equation of ratios to find the answer. Start with what the problem tells you and make a ratio that compares the number of pupils who walk to school with the total number of pupils enrolled.

$\frac{420}{700}$ ← Number of pupils who walk to school / Number of pupils enrolled

To find the per cent who walk to school, you need another ratio. This ratio must express a comparison for 100 pupils that is the same as the comparison of 420 pupils with 700 pupils. You do not know how many pupils walk for each 100 pupils enrolled. But you can use a letter in your ratio to hold a place for the numeral that tells how many.

$\frac{f}{100}$ ← Number of pupils who walk to school / Number of pupils enrolled

The two ratios are equal because they make the same comparison.

$\frac{420}{700} = \frac{f}{100}$

You can divide 700 by 7 to get 100. So you divide 420 by 7 to find the numeral to replace f. 60 replaces f.

The comparison of 420 pupils to 700 pupils is the same as the comparison of 60 pupils to 100 pupils. 60 to 100 can be expressed as 60%. The 420 pupils who walk to school are 60% of the 700 pupils enrolled.

B One day 14% of the 50 sixth graders enrolled in the Jackson School were absent. How many of the sixth graders were absent that day?

To find this answer, you again can use an equation of ratios. You are told that 14% of the sixth graders were absent. You know that 14% can mean 14 out of 100. So you can make a ratio that compares 14 absent sixth graders with 100 sixth graders enrolled.

$\frac{14}{100}$ ← Number of sixth graders absent / Number of sixth graders enrolled

The other ratio you make compares the number of sixth graders who really were absent with the 50 sixth graders who were really enrolled.

$\frac{t}{50}$ ← t: Number of sixth graders absent

50: Number of sixth graders enrolled

The comparison for the 50 sixth graders is the same as for 100 sixth graders. So the two ratios are equal.

$\frac{14}{100} = \frac{t}{50}$

You can divide 100 by 2 to get 50. So you divide 14 by 2 to find the numeral to replace t. 7 replaces t.

7 sixth graders were absent that day.

C One day 95% of the pupils in the fourth, fifth, and sixth grades went to a movie in the auditorium. 190 pupils went to the movie. How many pupils are enrolled in these grades?

You are told that 95% of the pupils in the three grades went to the movie. So you know that you can make a ratio that compares 95 pupils who went to the movie with 100 pupils enrolled.

$\frac{95}{100}$ ← 95: Number of pupils who went to the move

100: Number of pupils enrolled

You know that 190 pupils went to the movie. So the other ratio will compare these 190 pupils with the number of pupils really enrolled.

$\frac{190}{d}$ ← 190: Number of pupils who went to the movie

d: Number of pupils enrolled

The two ratios are equal because they make the same comparison.

$\frac{95}{100} = \frac{190}{d}$

You can multiply 95 by 2 to get 190. So you multiply 100 by 2 to find the numeral to replace d. 200 replaces d.

200 pupils are enrolled in the three grades.

D About 75% of the 700 pupils at the school eat lunch in the school lunchroom. About how many pupils eat lunch in the school lunchroom?

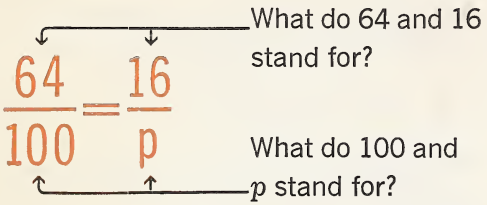
$\frac{75}{100} = \frac{s}{700}$

s: Number of pupils who eat in the school lunchroom

700: Number of pupils enrolled

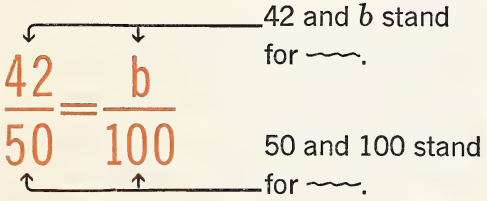
Why is 100 in the second term of the ratio 75 over 100? How do you find the numeral to replace s?

E The Jackson School basketball team has won 64% of its games. The team has won 16 games. How many games has the basketball team played?



Why is 100 in the second term of the ratio 64 over 100? Why is p in the second term of the ratio 16 over p? Why do you divide to find the numeral to replace p?

F 42 of the 50 sixth graders have pets. What per cent of the sixth graders have pets?



What numeral will replace b? Why can you express the second ratio as 84%?

G The sixth-grade class collected $20 for a class party. They spent 65% of the money for food for the party. How many dollars did they spend for food?

$$\frac{65}{100} = \frac{m}{20}$$

H Cal's allowance is $2.00 per week. He spends $1.30 per week of his allowance for lunch at school. What per cent of his allowance does Cal spend for lunch at school?

$$\frac{130}{200} = \frac{h}{100}$$

THINK Do the 130, 200, and 100 represent dollars or cents?

I 70% of the pupils in the school band rent their instruments. 35 pupils rent their instruments. How many pupils are there in the school band?

$$\frac{70}{100} = \frac{35}{a}$$

J 46% of the 50 sixth graders are boys. How many of the sixth graders are boys?

K 364 of the 700 pupils in the Jackson School are girls. What per cent of the pupils are girls?

L 15 sixth graders are in the Jackson School Science Club. This is 30% of the total membership of the club. How many pupils in all belong to the Science Club?

M The Science Club had $100 to spend for supplies in one year. By the end of November, the club had used 60% of the money. How many dollars had the club used?

N One day Ted worked 88% of the exercises on an arithmetic test correctly. He worked 22 exercises correctly. How many exercises were there on the arithmetic test?

O On the same test, Jean worked 23 of the exercises correctly. What per cent of the exercises did she work correctly?

P 40% of the 700 pupils at the Jackson School have younger brothers and sisters who are not yet enrolled in school. How many of the pupils have brothers and sisters not yet in school?

Q 140 of the pupils have older brothers and sisters who have already finished high school. What per cent of the pupils have brothers and sisters who have finished high school?

R 40 of the 50 sixth graders are 11 years old. What per cent of the sixth graders are 11 years old?

Now you should be able to solve problems about per cents.

Keeping skilful

A $n + 1870 = 11745$
B $32497 + 6755 = a$
C $563 \times 14 = t$
D $7841 - 6868 = b$
E $4361 \div 19 = c$
F $72563 - n = 9787$
G $25648 \div w = 526$
H $62710 + z = 80009$
I $k - 53987 = 92043$
J $27 \times 6058 = g$
K $436 \times 781 = e$
L $52004 \div 67 = y$
M $833120 - 54929 = a$
N $d + 9263 = 100410$
O $57637 - f = 20499$
P $18246 \div m = 48$

A $863 + n = 1000$
B $w - 6305 = 4795$
C $424 + 963 + 82 = d$
D $e + 437 = 2001$
E $6198 + n = 8222$
F $13404 - k = 2910$
G $c - 5230 = 4874$
H $m + 24630 = 109332$
I $41203 \div 80 = z$
J $640 \times 917 = h$
K $25501 \div d = 335$
L $82346 - 49790 = v$
M $7930 \times 55 = e$
N $1865 \div 52 = s$
O $r + 38692 = 60000$
P $473 \times 931 = w$

A $11\frac{9}{10} - n = 9\frac{1}{2}$
B $\frac{5}{12} + \frac{3}{4} + \frac{1}{3} = a$
C $72\frac{4}{5} + n = 86\frac{1}{10}$
D $y - 18\frac{1}{4} = 3\frac{3}{5}$
E $33\frac{1}{3} - \frac{5}{6} = f$
F $g + 4\frac{7}{8} = 10\frac{2}{3}$
G $5\frac{9}{16} + 7\frac{3}{4} = w$
H $9\frac{2}{3} - 4\frac{3}{5} = h$
I $17 - 8\frac{3}{10} = c$
J $14\frac{5}{6} - d = 12\frac{1}{4}$
K $e + 9\frac{7}{8} = 17\frac{7}{8}$
L $12\frac{1}{4} - m = 6\frac{9}{10}$
M $20\frac{2}{5} + 18\frac{1}{6} = v$

Exploring problems

A The sixth-grade class gave a play at the Cook School Friday afternoon and Friday evening. 50 tickets were sold for the afternoon performance, and 150 tickets were sold for the evening performance. The number of tickets that were sold for the evening performance was what per cent of the number of tickets that were sold for the afternoon performance?

THINK To solve the problem, you must find what per cent 150 is of 50. Will the per cent be more than 100% or less than 100%? How do you know?

$$\frac{150}{50} = \frac{b}{100}$$ ← Number of evening tickets to 100 afternoon tickets

150 is ▒% of 50.

B Of the 200 tickets sold for the play, the sixth graders sold 75% before the first performance. How many tickets did the sixth graders sell before the first performance?

THINK You must find 75% of 200.

$$\frac{75}{100} = \frac{m}{200}$$ ← Number of tickets sold by the sixth graders

▒ is 75% of 200.

C 40% of the money that the sixth graders raised by selling tickets was used for expenses. $20 was used for expenses. How much money did the sixth graders raise selling tickets for the class play?

THINK 20 is 40% of what number?

$$\frac{40}{100} = \frac{20}{t}$$ ← Number of dollars raised in all

20 is 40% of ▒.

D The $50 that the sixth graders raised selling the tickets was what per cent of the $20 that they used for expenses?

THINK How do you know that the per cent will be greater than 100%? What equation will you make to find what per cent 50 is of 20?

Which of these two sentences is correct for Problem D?

1 50 is ▒% of 20.

2 20 is ▒% of 50.

E The sixth graders wanted to buy some classroom equipment. The $30 they made from the play was 60% of the money they needed for equipment. How much money did they need for equipment?

THINK 30 is 60% of what number? What equation will you make for Problem E?

Which of these two sentences is correct for Problem E?

1 60% of 30 is ▢.
2 30 is 60% of ▢.

F There are 50 sixth graders in the Cook School. 40% of the sixth graders were in the class play. How many of the sixth graders were in the play?

THINK To solve this problem, you must find the number that is 40% of 50. How do you know that the number will be smaller than 50? What equation will you make?

Which of these two sentences is correct for Problem F?

1 40% of 50 is ▢.
2 50 is 40% of ▢.

G The sixth graders sold refreshments at the play. The money they raised by selling refreshments was 175% of the $20 still needed for the equipment. How much money did they raise selling refreshments?

THINK Which of these two sentences is correct for Problem G?

1 175% of 20 is ▢.
2 20 is 175% of ▢.

H 12 of the 50 sixth graders helped make the scenery for the play. What per cent of the sixth graders helped make the scenery?

THINK Which of these two sentences is correct for Problem H?

1 12 is ▢% of 50.
2 50 is ▢% of 12.

I The number of tickets Sue sold for the play was 125% of the number of tickets Ann sold. Sue sold 5 tickets. How many tickets did Ann sell?

THINK Which of these two sentences is correct for Problem I?

1 125% of 5 is ▢.
2 5 is 125% of ▢.

For each exercise below, first make an equation. Then find the answer.

A 42 is 84% of ▢.
B 65% of 20 is ▢.
C 100 is ▢% of 25.
D 36 is 180% of ▢.
E 14 is ▢% of 200.
F 210% of 100 is ▢.
G 92 is ▢% of 50.
H 38% of 300 is ▢.
I 27 is 54% of ▢.
J 16% of 25 is ▢.
K 75 is 150% of ▢.
L 5 is ▢% of 10.

Now you should be able to make equations for problems and sentences that use per cents.

Thinking straight

$2400 \times 1600 = r$

A

$$\begin{array}{r} 1600 \\ 2400 \\ \hline 384 \end{array}$$

B

$$\begin{array}{r} 1600 \\ 2400 \\ \hline 38400 \end{array}$$

C

$$\begin{array}{r} 1600 \\ 2400 \\ \hline 3840000 \end{array}$$

A You can use a short cut to find the product of 1600 and 2400. Study the three steps shown above. Look at Step A. First multiply 16 and 24.

B Look at Step B. You have found the product of 16 and 24. But you should have multiplied 1600 instead of 16. 1600 is 100 times ▒. So write two zeros after 384 to make a new product that is 100 times the product in Step A.

C Look at Step C. The 38400 in Step B is the product of 1600 and 24. But you should have multiplied 1600 by 2400. 2400 is 100 times ▒. You write two more zeros after 38400 to make a new product that is ▒ times the product in Step B.

D 1600 ends in ▒ zeros. 2400 ends in ▒ zeros. Since the product of 16 and 24 is 384, you can put ▒ zeros after 384 to get the product of 1600 and 2400.

E To find the product of 240 and 70, first multiply 24 and 7.

F 240 is ▒ times 24. Why do you now write a zero after 168?

G 70 is ▒ times 7. Why do you write another zero after 1680?

H How do you know that there will be two zeros at the end of the product of 70 and 240?

I To multiply 8000 by 850, first multiply 85 by ▒. Why do you write four zeros after 680?

J When you multiply 27000 by 90, why do you write four zeros after 243?

Solve these equations. Use the short cut to make your work easier.

A $300 \times 110 = r$

B $320 \times 100 = g$

C $4500 \times 10 = a$

D $160 \times 330 = d$

E $59 \times 7400 = s$

F $1810 \times 70 = n$

G $870 \times 500 = w$

H $400 \times 18000 = m$

I $920 \times 1800 = t$

J $6500 \times 300 = b$

K $9800 \times 410 = r$

L $750 \times 1000 = h$

M $1100 \times 5000 = c$

N $430 \times 7000 = v$

108000 ÷ 1800 = s

A $\frac{108000}{1800} = \frac{108000 \div 100}{1800 \div 100} = \frac{1080}{18} = \frac{1080 \div 18}{18 \div 18} = \frac{60}{1} = 60$

B $\frac{108000}{1800} = \frac{108000}{1800} = \frac{1080}{18} = \frac{1080}{18} = \frac{60}{1} = 60$

A You learned on pages 108 and 109 that you can write division exercises in the way shown above. Where do you write the divisor?

B Look at the work in Row A. How do you know that both 108000 and 1800 can be divided by 100? Will 1080 divided by 18 give the same quotient as 108000 divided by 1800?

C Is 60 the quotient for 108000 divided by 1800? Is 60 also the quotient for 1080 divided by 18? For 60 divided by 1?

D Row B shows another way to do the work. Notice that two zeros have been crossed out in 108000 and in 1800. This is done to show that both 108000 and 1800 have been divided by ▤.

E Now look at the fourth example in Row B. Both 1080 and 18 are divided by 18. Why is 60 written above 1080? Why is 1 written below 18? 1080 and 18 are crossed out to show that they are replaced by 60 and 1.

F 1950 ÷ 250 = a

$\frac{1950}{250} = \frac{195}{25} = \frac{39}{5} = 7\frac{4}{5}$

Why was one zero crossed out in 1950 and in 250? What divisor was used when 195 and 25 were crossed out? Is $7\frac{4}{5}$ the correct quotient for 1950 ÷ 250?

Solve these equations. Write your division in the way shown in Row B.

A 42000 ÷ 700 = x
B 9000 ÷ 60 = d
C 180000 ÷ 4500 = a
D 67500 ÷ 250 = w
E 1596000 ÷ 1900 = p
F 1323000 ÷ 63000 = t
G 7612500 ÷ 17500 = r
H 570000 ÷ 190000 = g
I 1050000 ÷ 35000 = z
J 5880000 ÷ 8400 = e

Using arithmetic

Use two ratios in your equation for each of the following problems.

A The diameter of the planet Neptune is about 10 times as great as the diameter of the planet Mercury. Mercury has a diameter of about 3100 mi. Neptune has a diameter of about how many miles?

B Mercury travels around the sun at a speed of about 30 mi. per second. About how far does Mercury travel in 1 hour?

THINK Why will the second term of the second ratio in your equation be 60×60?

$$\frac{30}{1} = \frac{a}{60 \times 60}$$

C The earth travels around the sun at a speed of about 19 mi. per second. About how far does the earth travel in 1 hour?

D The distance around the earth at the equator is about 25,000 mi. If an airplane travelled at an average speed of 500 mi. per hour, in how many hours could it travel around the earth at the equator?

THINK Why do you use 1 in one ratio?

$$\frac{500}{1} = \frac{25000}{n}$$

E The distance around the planet Mars at its equator is about 13,000 mi. About how many hours would it take the airplane in Problem D to travel around Mars at the equator?

THINK Is this problem like Problem D?

F The earth travels once around the sun in 365 da. The planet Jupiter takes 12 times as many days to travel once around the sun. How many days does it take Jupiter to travel once around the sun?

THINK One ratio can be 12 over 1. What does the 1 represent?

G For each trip that the earth makes around the sun, the moon makes 12 trips around the earth. When the earth has made 25 trips around the sun, how many trips will the moon have made around the earth?

THINK One ratio can be 1 over 12. What does the 1 represent?

H Sound travels through the air at a speed of about 1100 ft. per second. In about how many seconds will sound travel 1 mi.?

THINK To make your work easier, use 5500 as a round number for the number of feet in 1 mi. You can get the answer by dividing 55 by ▭

I About how many feet does sound travel in 1 hour?

THINK You will use 60×60 in one ratio. Why?

J The speed of sound through the air is about how many miles per hour?

K Sound travelled from one point to another point in 10 seconds. The two points were about how many feet apart?

THINK Is it correct to say that the two points were about 2 mi. apart?

L An airplane travelled at a speed of 1200 mi. per hour. About how many hours would it take this airplane to travel the 240,000 mi. from the earth to the moon?

M About how many hours would it take the airplane in Problem L to travel around the earth at the equator?

THINK To make your work easier, use 24,000 mi. as a round number for the distance around the earth at the equator.

Keeping skilful

A $600 \times 1700 = b$
B $1800 \times 360 = r$
C $1550 \times 2100 = m$
D $800 \times 765000 = d$
E $960 \times 48360 = t$
F $31000 \times 1800 = n$
G $4870 \times 98000 = a$
H $57000 \times 7100 = c$
I $7800 \times 9300 = f$
J $1960 \times 43000 = d$
K $9000 \times 82000 = p$
L $434000 \times 570 = a$
M $2400 \times 7630 = n$
N $82000 \times 640 = b$
O $3000 \times 12000 = y$
P $9800 \times 48700 = d$
Q $4800 \times 99700 = g$
R $5900 \times 28300 = n$

A $18000 \div 6000 = t$
B $48000 \div 1200 = a$
C $49760 \div 2600 = z$
D $57000 \div 11000 = r$
E $92100 \div 7000 = b$
F $42130 \div 160 = m$
G $86000 \div 4300 = d$
H $78500 \div 350 = c$
I $111000 \div 1100 = n$
J $24000 \div 8000 = p$
K $39000 \div 130 = f$
L $4750 \div 900 = y$
M $63300 \div 3000 = h$
N $80000 \div 20000 = r$
O $57000 \div 190 = n$
P $94000 \div 6200 = k$
Q $6500 \div 5000 = m$
R $78000 \div 13000 = w$

A $\frac{7800}{2750} = \frac{x}{550}$

B $\frac{3000}{1200} = \frac{1000}{m}$

C $\frac{6300}{4590} = \frac{d}{51}$

D $\frac{16000}{2800} = \frac{40}{t}$

E $\frac{5400}{180} = \frac{r}{10}$

F $\frac{2500}{300} = \frac{25}{n}$

G $\frac{81000}{2700} = \frac{c}{9}$

H $\frac{90000}{1800} = \frac{450}{a}$

Thinking straight

A $(8 \times 32) \div 4 = w$

B $(8 \times 32) \div 4 = 256 \div 4 = 64$

C $\frac{8 \times 32}{4} = \frac{256}{4} = 64$

D $\frac{\overset{2}{\cancel{8}} \times 32}{\underset{1}{\cancel{4}}} = \frac{2 \times 32}{1} = \frac{64}{1} = 64$

E $(8 + 32) \div 4 = w$

F $(8 + 32) \div 4 = 40 \div 4 = 10$

G $\frac{8 + 32}{4} = \frac{40}{4} = 10$

H $\frac{\overset{2}{\cancel{8}} + \overset{8}{\cancel{32}}}{\underset{1}{\cancel{4}}} = \frac{2 + 8}{1} = \frac{10}{1} = 10$

A How does the work in Example C differ from that in Example B?

B In Example D, 8 and 4 are crossed out to show they are divided by ▤.

C Another way to do the work is to divide both 32 and 4 by 4. Do you still get 64 as the quotient?

D How does Equation E differ from Equation A?

E In Example H, when you divide 4 by 4, you must also divide both 8 and 32 by 4. How do you get 10?

F $(78 - 36) \div 6 = t$

$$\frac{\overset{13}{\cancel{78}} - \overset{6}{\cancel{36}}}{\underset{1}{\cancel{6}}} = \frac{13 - 6}{1} = \frac{7}{1} = 7$$

Why were both 78 and 36 crossed out?

G $(11 \times 30) \div 10 = m$

$$\frac{11 \times \overset{3}{\cancel{30}}}{\underset{1}{\cancel{10}}} = \frac{11 \times 3}{1} = \frac{33}{1} = 33$$

Is the division correct?

When you solve the equations below, cross out numerals to show division.

A $(45 - 30) \div 15 = d$

B $(7 \times 24) \div 4 = r$

C $(117 + 78) \div 39 = s$

D $(50 + 60) \div 10 = a$

E $(21 \times 56) \div 7 = n$

F $(91 - 65) \div 13 = w$

G $(125 - 100) \div 25 = t$

H $(49 \times 84) \div 7 = y$

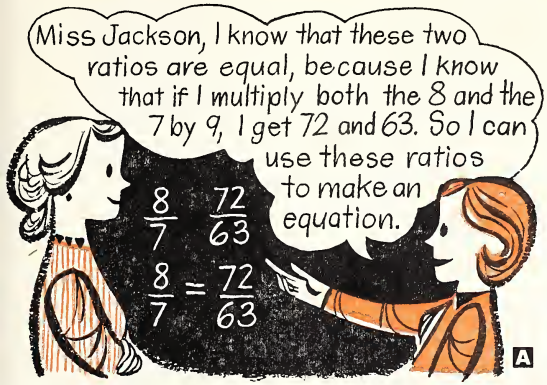
I $(65 + 70) \div 5 = b$

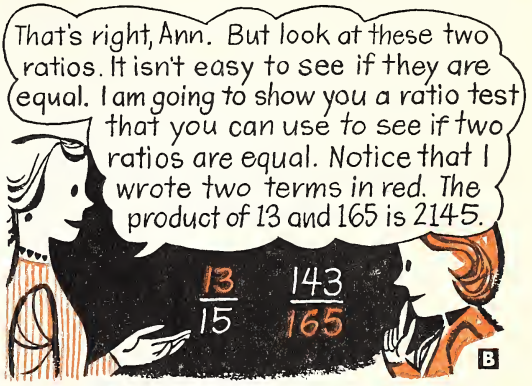
J $(57 \times 19) \div 19 = m$

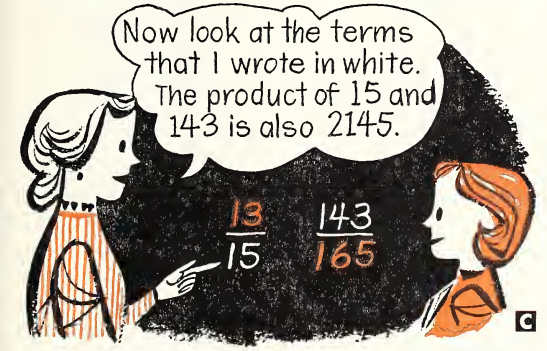
K $(111 - 74) \div 37 = k$

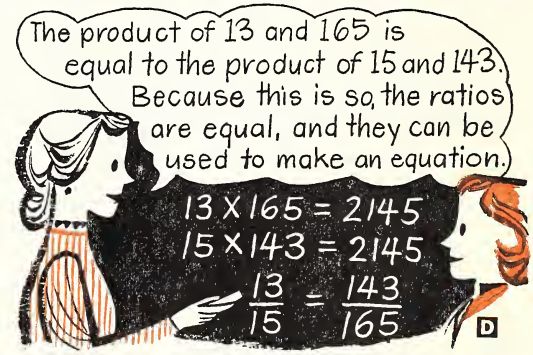
L $(1400 + 900) \div 50 = f$

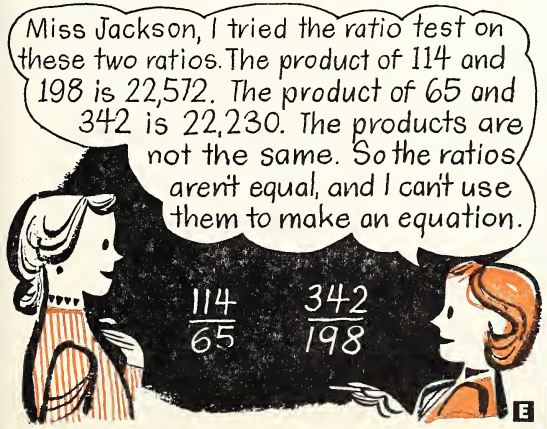
Thinking straight

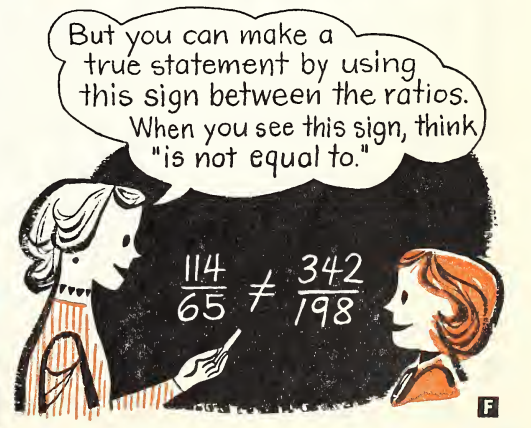

Miss Jackson, I know that these two ratios are equal, because I know that if I multiply both the 8 and the 7 by 9, I get 72 and 63. So I can use these ratios to make an equation.
8/7 72/63
8/7 = 72/63
A


That's right, Ann. But look at these two ratios. It isn't easy to see if they are equal. I am going to show you a ratio test that you can use to see if two ratios are equal. Notice that I wrote two terms in red. The product of 13 and 165 is 2145.
13/15 143/165
B




The product of 13 and 165 is equal to the product of 15 and 143. Because this is so, the ratios are equal, and they can be used to make an equation.
13 × 165 = 2145
15 × 143 = 2145
13/15 = 143/165
D


Miss Jackson, I tried the ratio test on these two ratios. The product of 114 and 198 is 22,572. The product of 65 and 342 is 22,230. The products are not the same. So the ratios aren't equal, and I can't use them to make an equation.
114/65 342/198
E


But you can make a true statement by using this sign between the ratios. When you see this sign, think "is not equal to."
114/65 ≠ 342/198
F

A To use the ratio test for the ratios at the right, first find the product of 21 and ▭. Then find the product of ▭ and ▭. Make a true statement about the products.

$\frac{21}{8}$ $\frac{84}{24}$

B Now write the ratios with the sign that makes a true statement.

C Use the ratio test to see whether or not the ratios at the right are equal. Then write a true statement about these two ratios.

$\frac{75}{18}$ $\frac{25}{6}$

In Exercises D to S, use the ratio test to see whether or not the ratios are equal. Write each exercise so that it makes a true statement. Use the sign = or the sign ≠.

D $\frac{13}{26}, \frac{10}{20}$

E $\frac{16}{42}, \frac{4}{12}$

F $\frac{47}{92}, \frac{94}{174}$

G $\frac{30}{50}, \frac{12}{20}$

H $\frac{99}{115}, \frac{198}{230}$

I $\frac{650}{125}, \frac{26}{5}$

J $\frac{29}{51}, \frac{9}{17}$

K $\frac{100}{75}, \frac{120}{90}$

L $\frac{625}{500}, \frac{125}{100}$

M $\frac{41}{65}, \frac{200}{325}$

N $\frac{70}{35}, \frac{100}{50}$

O $\frac{98}{24}, \frac{588}{168}$

P $\frac{99}{77}, \frac{198}{154}$

Q $\frac{24}{6}, \frac{80}{20}$

R $\frac{240}{80}, \frac{10}{3}$

S $\frac{77}{11}, \frac{1}{7}$

Keeping skilful

A 20 is ▭% of 5.
B 18% of 300 is ▭.
C 36 is 9% of ▭.
D 80 is 160% of ▭.
E 500 is ▭% of 50.
F 48% of 700 is ▭.
G 21 is ▭% of 300.
H 152% of 25 is ▭.
I 80 is ▭% of 500.
J 30 is 5% of ▭.
K 60% of 50 is ▭.
L 900 is ▭% of 400.
M 64 is 32% of ▭.

A 8 mi. = ▭ yd.
B 4800 min. = ▭ hr.
C 360 in. = ▭ yd.
D 12 gal. = ▭ pt.
E 9 T. = ▭ lb.
F 432 hr. = ▭ da.
G 70 lb. = ▭ oz.
H 96 fl. oz. = ▭ c.
I 14 yr. = ▭ da.
J 48 tbs. = ▭ fl. oz.
K 3520 yd. = ▭ mi.
L 19 gal. = ▭ pt.
M 192 oz. = ▭ lb.

A $897 \times 1046 = d$
B $94872 + 36549 = a$
C $96\frac{3}{5} + 24\frac{7}{10} = r$
D $360382 \div 594 = w$
E $57000 \times 14926 = g$
F $65\frac{1}{4} - 36\frac{11}{12} = c$
G $1100 \times 489 = r$
H $403\frac{5}{6} + 6\frac{4}{5} = t$
I $724 + 99 + 8215 = m$
J $242179 \div 29 = b$
K $118\frac{1}{3} - 46\frac{14}{15} = f$

Moving forward

Now you will learn to use the ratio test when you solve certain kinds of equations.

A $\frac{18}{3} = \frac{n}{2}$

$3 \times n = 18 \times 2$

$n \times 3 = 2 \times 18$

$18 \times 2 = 3 \times n$

$2 \times 18 = n \times 3$

Look at Example A. To find the numeral that replaces n, you can use the ratio test. The work in green shows different ways of writing the ratio test. All of these ways are correct.

B $3n = 18 \times 2$

$3n = 36$ To find the numeral that replaces n, divide 36 by 3.

$\frac{36}{3} = \square$

$3 \times 12 = 36$ n is replaced with 12.

You can often show, without using the multiplication sign, that you are to multiply. Look at Example B. $3n$ means $3 \times n$. Why would it be unwise to write 18×2 in the same way?

Now look at the work in brown. How can you find the numeral that replaces n? What is this numeral?

C $3n = 18 \times 2$ To find the numeral that replaces n, you can divide 18×2 by 3.

6

$\frac{\cancel{18} \times 2}{\cancel{3}\ 1} = \frac{6 \times 2}{1} = \frac{12}{1} = 12$

$3 \times 12 = 18 \times 2$ n is replaced with 12.

Example C and the work in red show another way to find the numeral that replaces n. 18 and 3 are crossed out to show that each number is divided by $\square$. What is done next? What numeral replaces n?

D $\frac{12}{36}=\frac{100}{k}$

$12 \times k = 36 \times 100$

$12k = 36 \times 100$

$$\frac{\overset{3}{\cancel{36}} \times 100}{\underset{1}{\cancel{12}}} = \frac{3 \times 100}{1} = \frac{300}{1} = 300$$

In Example D, does $12k$ mean the same as $12 \times k$? What must you do to find the numeral that replaces k? 36 and 12 are crossed out to show that each is divided by ▒. What is done next? What is the numeral that replaces k?

E $\frac{30}{d}=\frac{20}{16}$

$20d = 16 \times 30$

$$\frac{16 \times \overset{3}{\cancel{30}}}{\underset{2}{\cancel{20}}} = \frac{\overset{8}{\cancel{16}} \times 3}{\underset{1}{\cancel{2}}} = \frac{24}{1} = 24$$

$$\frac{\overset{8}{\cancel{16}} \times \overset{3}{\cancel{30}}}{\underset{2}{\cancel{20}}} = \frac{24}{1} = 24$$

Look at the work for Example E. To find the numeral that replaces d, you divide 16×30 by ▒. 30 and 20 are crossed out to show that each is divided by 10. Now you have 16×3 above the line and ▒ below the line.

Then both 16 and 2 are divided by ▒. How is this shown?

A shorter way to write the work is shown in red. Is 24 the numeral that replaces d?

F $\frac{85}{340}=\frac{t}{100}$

$340t = 85 \times 100$

$$\frac{85 \times \overset{5}{\cancel{100}}}{\underset{17}{\cancel{340}}} = \frac{85 \times 5}{17} = \frac{425}{17} = 25$$

Look at the work for Example F. Explain the division shown by the crossed-out numerals. How do you get 425 over 17? What do you get when you divide 425 by 17? What numeral replaces t?

Could 85 and 17 have been divided by the same number? How would you show this?

Use two or more ways to find the answer.

A $\frac{15 \times 45}{9} = \blacksquare$

B $\frac{30 \times 7}{3} = \blacksquare$

C $\frac{100 \times 60}{20} = \blacksquare$

D $\frac{28 \times 3}{21} = \blacksquare$

E $\frac{25 \times 44}{20} = \blacksquare$

F $\frac{78 \times 3}{18} = \blacksquare$

In each exercise below, find the numeral to replace the letter. Use the ratio test.

A $\frac{48}{15} = \frac{a}{10}$

B $\frac{c}{9} = \frac{40}{12}$

C $\frac{12}{m} = \frac{15}{40}$

D $\frac{15}{18} = \frac{50}{w}$

E $\frac{k}{63} = \frac{68}{17}$

F $\frac{9}{15} = \frac{n}{50}$

G $\frac{c}{16} = \frac{35}{20}$

H $\frac{b}{90} = \frac{21}{14}$

I $\frac{12}{z} = \frac{16}{40}$

Now you can solve equations of ratios by using the ratio test.

Using arithmetic

A Mr. Anderson and Tom put a picket fence around the yard. The perimeter of the yard is about 600 ft. They used 25 pickets to make 6 ft. of fence. About how many pickets did they use for the 600-ft. fence?

$\frac{25}{6} = \frac{t}{600}$

How do you know you can find the numeral to replace t by multiplying 25 by 100?

THINK You can also use the ratio test to find the numeral to replace t. You get the equation $6t = 25 \times 600$. How do you solve this equation?

B It took Mr. Anderson and Tom 2 hr. to put up 80 ft. of the picket fence. At this rate, how long did it take them to put up the entire fence?

$\frac{80}{2} = \frac{600}{b}$

THINK Can 80 be multiplied by a whole number to get 600? Use the ratio test to find the numeral to replace b.

C For every 10 ft. of fence that Mr. Anderson put up, Tom put up 5 ft. of fence. When Mr. Anderson had put up 30 ft. of fence, how many feet of fence had Tom put up?

$\frac{10}{5} = \frac{30}{y}$

THINK When you use the ratio test, what equation do you get? How do you solve this equation?

D Mr. Anderson bought the fence pickets on sale for $72. The price he paid was 80% of the usual price. How much would he have paid for the pickets at the usual price?

$$\frac{80}{100} = \frac{72}{d}$$ ← Use the ratio test to find the numeral to replace d.

E Mrs. Anderson cooked a 5-lb. roast for dinner. The roast had to be cooked 35 minutes per pound. How many minutes did it take to cook the roast?

$$\frac{35}{1} = \frac{f}{5}$$

THINK When you use the ratio test to solve Problem E, why do you not have to divide to solve the equation?

F Kathy Anderson and her mother made 56 raisin cookies and 112 spice cookies. They made how many times as many spice cookies as raisin cookies?

$$\frac{112}{56} = \frac{n}{1}$$

THINK Why do you use n over 1 as one of the equal ratios? When you use the ratio test, you get $56n = 112 \times 1$. What numeral replaces n? How do you read it as the answer to Problem F?

G Mr. Anderson paid $23 for a 100-ft. roll of screening. He paid how much per yard for the screening?

THINK Problem G will be easier if you think of $23 as 2300¢. You can use n over 3 for one of the ratios in your equation. What will you use for the other ratio? Will the numeral that replaces n represent dollars or cents?

H If the screening had been bought by the yard, it would have cost 81¢ per yard. How much would Mr. Anderson have had to pay for 100 ft. of the screening if he had bought it by the yard?

I Mr. Anderson used 5 ft. of screening for one screen. How much did he pay for the screening for one screen?

J When the Andersons decided to put wallpaper on two rooms, they found that they needed 14 rolls of paper for the walls and 7 rolls for the ceilings. They needed how many times as many rolls of paper for the walls as for the ceilings?

K Mr. Anderson paid $6 for 20 yards of border paper. What was the price of the border paper per yard?

THINK Problem K will be easier if you think of $6 as 600¢. Will your answer represent dollars or cents?

L All the paper for the two rooms cost $40. The amount paid for the border paper was what per cent of the total cost?

Checking up

Test 1

Use the ratio test when you solve these equations.

A $\frac{64}{a}=\frac{10}{5}$

B $\frac{100}{3}=\frac{w}{45}$

C $\frac{r}{91}=\frac{6}{21}$

D $\frac{3}{4}=\frac{51}{m}$

E $\frac{b}{40}=\frac{200}{500}$

F $\frac{35}{55}=\frac{x}{11}$

G $\frac{123}{t}=\frac{3}{2}$

H $\frac{16}{10}=\frac{24}{c}$

I $\frac{d}{85}=\frac{6}{5}$

Test 2

A 20% of 450 is ▒.
B ▒ is 25% of 500.
C 175% of 48 is ▒.
D 42% of 700 is ▒.
E ▒ is 225% of 160.
F 50% of 96 is ▒.
G ▒ is 125% of 64.
H 90% of 180 is ▒.
I ▒ is 124% of 175.

Test 3

A 40 is ▒% of 200.
B 130 is ▒% of 65.
C ▒% of 96 is 24.
D 234 is ▒% of 52.
E ▒% of 72 is 468.
F 180 is ▒% of 450.
G 405 is ▒% of 900.
H ▒% of 2200 is 792.
I 96 is ▒% of 480.

Test 4

A 160 is 4% of ▒.
B 36% of ▒ is 81.
C 80 is 5% of ▒.
D 75% of ▒ is 150.
E 110% of ▒ is 11.
F 6 is 1% of ▒.
G 16 is 80% of ▒.
H 104% of ▒ is 52.
I 96 is 24% of ▒.

Test 5

A 20% of 70 is ▒.
B 80 is ▒% of 160.
C 54 is 25% of ▒.
D 75 is 150% of ▒.
E 184% of 25 is ▒.
F 600 is ▒% of 40.
G 130% of ▒ is 65.
H 130 is ▒% of 650.
I 32% of 625 is ▒.

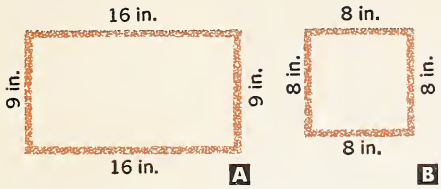
Keeping skilful

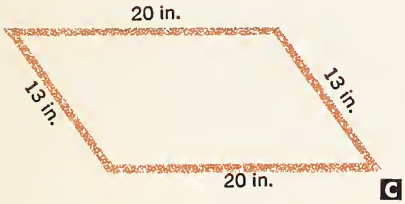
Find the sum.

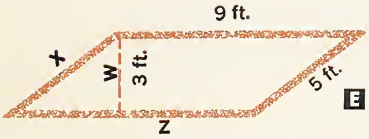
A 2074, 69, 39216
B 895, 636, 2152
C 113862, 94679
D 562, 36988, 1140
E 64862, 938, 4000
F 7213, 649, 866, 47

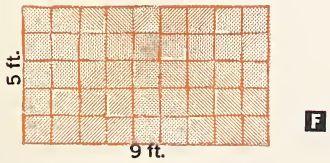
A $s+38276=70001$
B $114003-6295=a$
C $24836-t=158$
D $m-84531=3654$
E $198746+k=400623$
F $30705+96241=d$
G $n+81062=114050$
H $27383-r=9114$
I $9608+f=14327$
J $y-79836=8439$
K $g+16434=25321$
L $41628-z=17919$
M $30065+d=50096$
N $t-63824=8756$

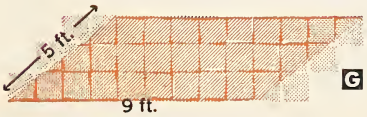
Thinking straight











A Look at Polygons A, B, and C. Each of these polygons has ▒ straight sides.

B Which of the three polygons have 4 right angles? Which polygon has no right angles?

C Are any of the sides of Polygon A perpendicular to each other? Are any of the sides of Polygon C perpendicular to each other? Do sides that are perpendicular also form right angles?

D In each of the three polygons, the opposite sides are parallel. How do you know that the opposite sides are also equal in length?

E Polygons A, B, and C are parallelograms because their opposite sides are parallel. The rectangle is a special kind of parallelogram. In what way is it special? Which of the three polygons are rectangles?

F The square is a special kind of rectangle. In what way is it special? Which polygon is a square? Is a square also a parallelogram?

G Polygons D and E are parallelograms. Have they equal perimeters?

THINK How do you find the perimeter of any polygon?

H Look at Polygon D. Line p represents the base. The length of the base is ▒ ft. Which lines represent the altitude? Is the altitude perpendicular to the base? The length of the altitude is ▒ ft.

I In Polygon E, Line z represents the base. The length of the base is ▒ ft. Is Line w perpendicular to the base? Is Line w the altitude? The length of the altitude is ▒ ft.

J In Polygon E, can Line x also represent the altitude? Why or why not?

K Picture F shows a rectangle the same size as Rectangle D. Picture G shows a parallelogram the same size as the one in Picture E. The small squares covering Parallelograms F and G represent square feet. Are the areas of Parallelograms F and G equal? Which parallelogram, F or G, has the greater area?

L Each parallelogram has a base ▒ ft. long. Which parallelogram has the longer altitude? Is the parallelogram with the longer altitude also the parallelogram with the greater area?

M Have Parallelograms F and G equal perimeters? Is it possible for two parallelograms to have equal perimeters and areas that are not equal? How do you know?

N If two parallelograms have bases that are of equal length and altitudes that are not equal, will they have areas that are not equal? How do you know?

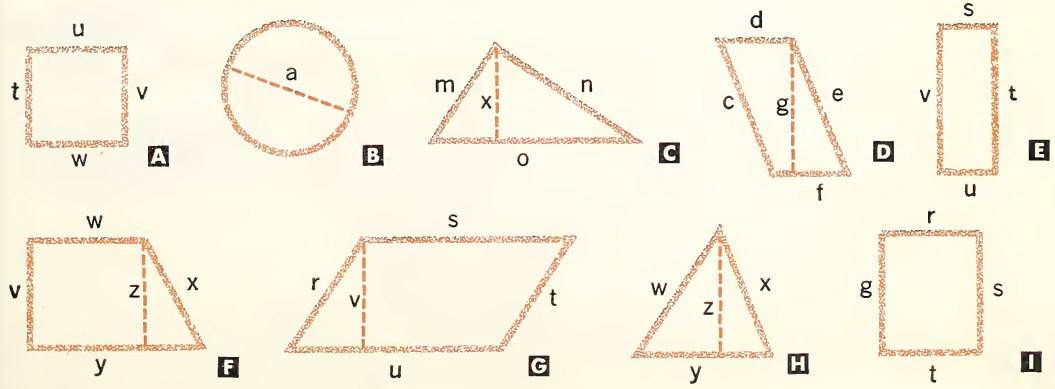
O Which pictures below show parallelograms? Which of these parallelograms are rectangles? Which rectangle is also a square?

P For each parallelogram, name a line that represents the altitude.

Q Which pictures show shapes that are not parallelograms?

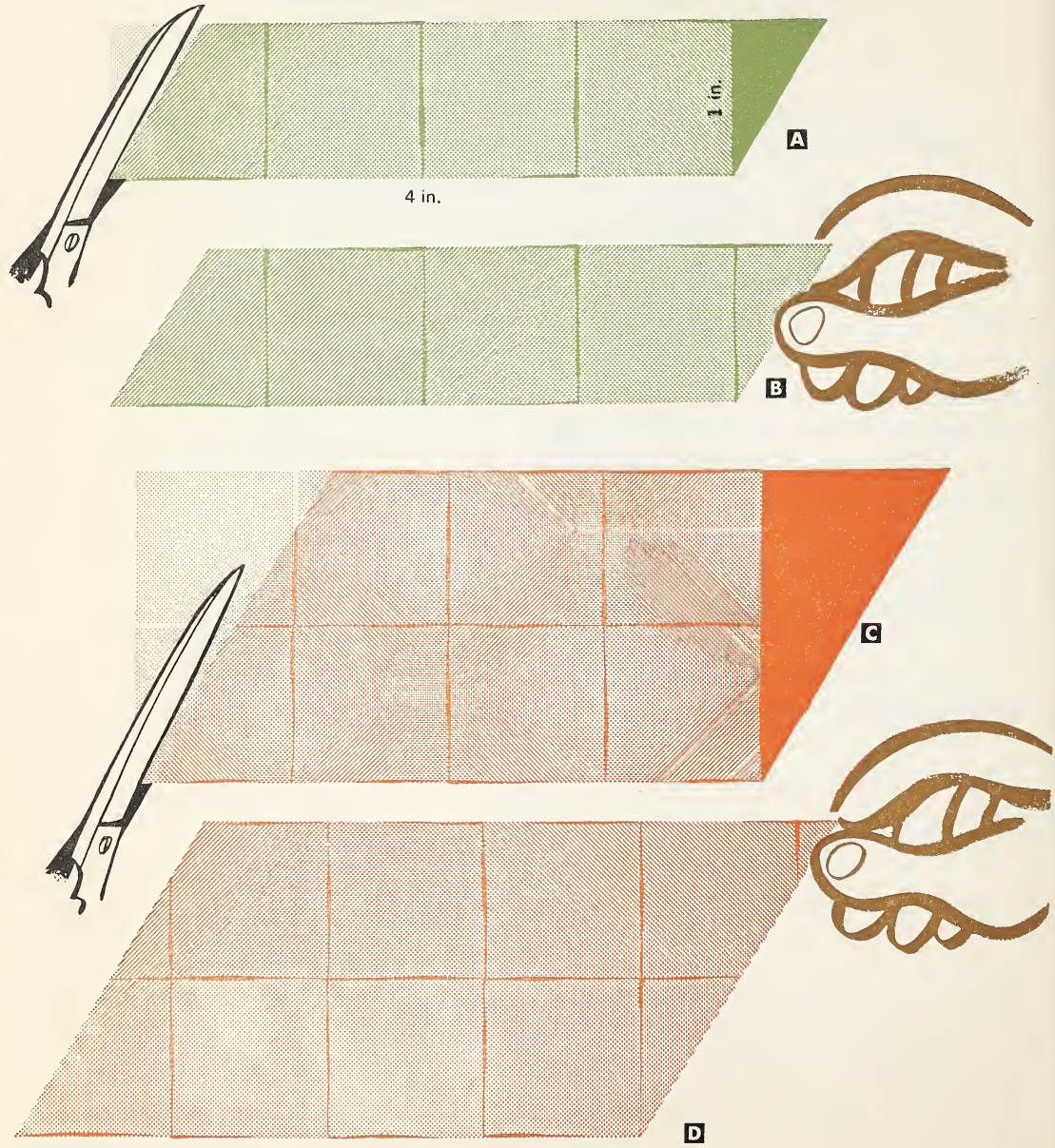
R For each triangle, name the line that represents the altitude.

S What do you call Line a on Circle B?



Moving forward

Now you will learn more about parallelograms.



A In Pictures A and B you can see a parallelogram that is not a rectangle. It has a base 4 in. long and an altitude 1 in. long. How do the 1-inch squares placed on this parallelogram show that its area is 4 square inches?

B How many rows of 1-inch squares are there in the parallelogram shown in Pictures A and B?

C Are there as many square inches in the row as there are inches in the length of the base?

D Now look at Pictures C and D. They show a parallelogram with a base 4 in. long and an altitude 2 in. long. How many 1-inch squares does it take to cover this parallelogram? The area of this parallelogram is ▒ sq. in.

E How many rows of square inches are there in the parallelogram in Pictures C and D? How do you know that it takes four 1-inch squares to cover each row? Are there as many square inches per row as there are inches in the length of the base?

F Are there as many rows as there are inches in the length of the altitude?

G Now look at Picture A again. You can see that the 1-inch squares form a rectangle with a base of 4 in. and an altitude of 1 in. What is its area? Is the area of this rectangle the same as the area of the parallelogram that is not a rectangle?

H In Picture C, the 1-inch squares also form a rectangle. Has this rectangle the same base and the same altitude as the parallelogram that is not a rectangle? What is the area of the rectangle?

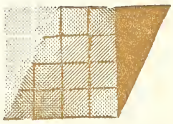
I Imagine that each small square placed on the parallelograms below is a square inch. How long are the base and altitude of each parallelogram?

J Use the small squares to find the area of each parallelogram.

E

F

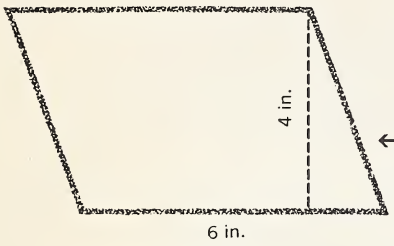
G



H

Now you know that if a rectangle and a parallelogram that is not a rectangle have equal bases and altitudes, they also have equal areas.

Exploring problems



A A piece of plywood is in the shape of a parallelogram with a base 6 in. long and an altitude 4 in. long. What is its area in square inches?

Look at the diagram.

Imagine that 1-inch squares are being laid in rows to cover the parallelogram. There will be 6 square inches in each row because the base of the parallelogram is 6 in.

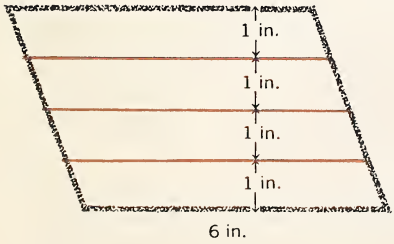
B This is a rate of 6 square inches per 1 row. Express this rate as a ratio.

$$\frac{6}{1}$$

6 ← Number of square inches in each row

1 ← Number of rows

Now you must find how many rows of square inches there will be. The altitude is 4 in., and each row will use 1 in. of the altitude. There will be 4 rows.



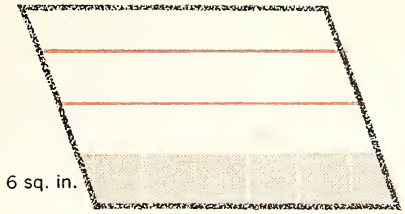
C You do not know how many square inches will be used for all 4 rows, but you can express the rate as x square inches per 4 rows.

$$\frac{x}{4}$$

x ← Total number of square inches

4 ← Total number of rows

D



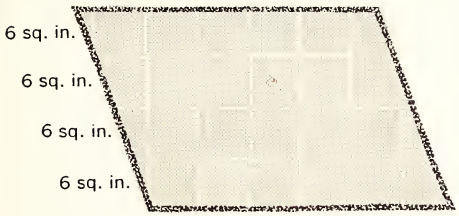
The rate at which square inches will be used will be the same for the 4 rows as for 1 row. So you can make an equation of the ratios.

$$\frac{6}{1} = \frac{x}{4}$$

Do these terms stand for number of square inches or number of rows? You must find the numeral that replaces x.

What do these terms stand for?

E

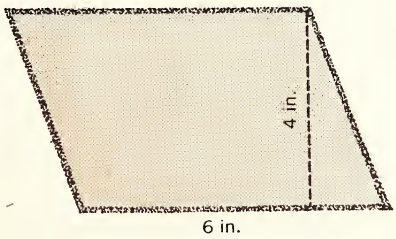


6 square inches are used for 1 row. You can see that 24 square inches are used for all 4 rows. You can also compute to find how many square inches there are in the parallelogram.

$$\frac{6}{1} = \frac{x}{4}$$

4 can be found by multiplying 1 by 4. So the numeral that replaces x can be found by multiplying 6 by 4.

F

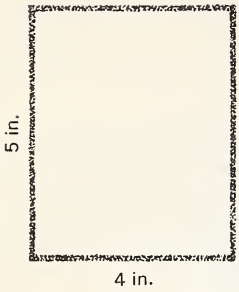


If you use the ratio test when you solve this equation, you get $1x = 6 \times 4$. What do you do next?

$$\frac{6}{1} = \frac{24}{4}$$

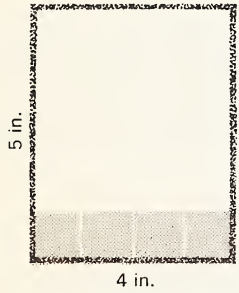
24 replaces x.

The area of the piece of plywood is 24 sq. in.



G A piece of plywood is in the shape of a parallelogram with a base 4 in. long and an altitude 5 in. long. What is its area in square inches?

Look at the diagram. This parallelogram has right angles. What special name do you know for this kind of parallelogram?

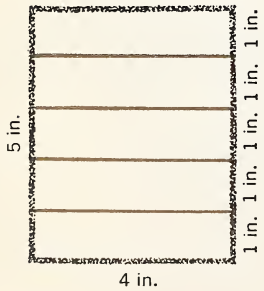


The base of this rectangle is 4 in. long.

H So in 1 row there will be ▒ sq. in. The rate is ▒ square inches per 1 row. Express this rate as a ratio.

$\frac{4}{1}$ ← What does this term stand for?

What does this term stand for? →

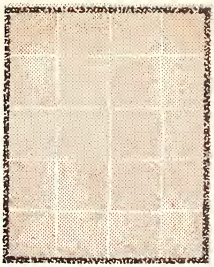


The altitude of the rectangle is 5 in. long.

I There will be ▒ rows of square inches. Why can the rate also be expressed as b square inches per 5 rows?

$\frac{b}{5}$ ← What does this term stand for?

What does this term stand for? →



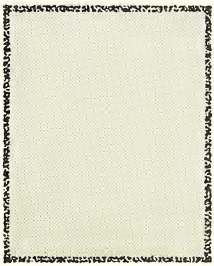
Square inches will be used at the same rate for 5 rows as for 1 row.

J

$$\frac{4}{1} = \frac{b}{5}$$

What do the 4 and the b stand for?

What do the 1 and the 5 stand for?



K

$$\frac{4}{1} = \frac{b}{5}$$

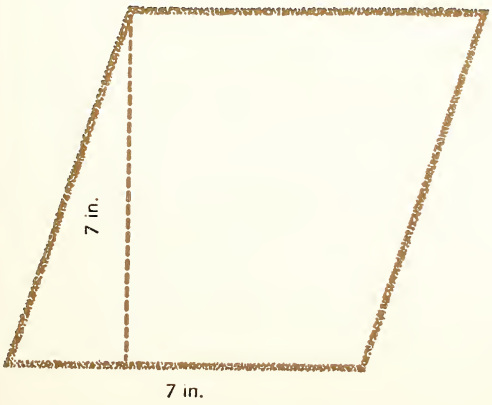
Why do you multiply 4 by 5 to find the numeral that replaces b? ▒ replaces b.

What equation do you get when you use the ratio test?

The area of this parallelogram is ▒ sq. in.

The base of Parallelogram L is 7 in. long. The altitude is 7 in. long. Find the area in square inches.

L



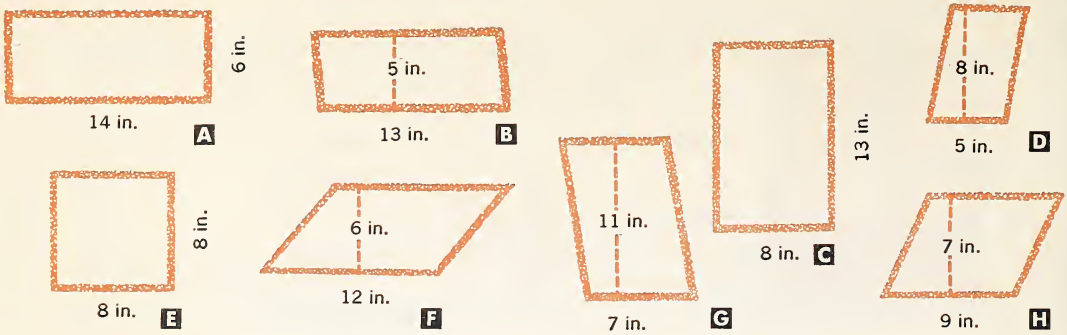
$$\frac{7}{1} = \frac{k}{7}$$

This ratio means ▒ sq. in. per 1 row.

When you have found the numeral that replaces k, you will know how many square inches there are in ▒ rows.

Why can you also use the equation $1k = 7 \times 7$?

The area of Parallelogram L is ▒ sq. in.



Each diagram above represents a piece of cardboard that is in the shape of a parallelogram. Find the area of each piece of cardboard in square inches.

A Henry is cutting a piece of glass in the shape of a rectangle 4 in. long and 3 in. wide. What is the area of the piece of glass in square inches?

THINK If you use 4 inches for the base, what do you use for the altitude?

B Don has a piece of plywood in the shape of a parallelogram with a base 15 in. long and an altitude 6 in. long. Find the area of the piece of plywood in square inches.

C Mr. Beach grows mushrooms in rectangular trays that are 17 in. long and 15 in. wide. What is the area in square inches of the bottom of one of these trays?

D Mr. May plans to put a tile floor in a hall. The hall floor has the shape of a rectangle. It is 60 in. long and 42 in. wide. What is the area of the floor in square inches?

E The tiles Mr. May will use are 6 in. square. Find the area of one tile in square inches.

F The tiles also come in 9-inch squares. A 9-inch tile will cover how many more square inches of floor than a 6-inch tile?

THINK When you have found the area of a 9-inch tile, what should you do next?

G Mr. Henry is setting tiles in a table top. The tiles are about 1 inch square. The space to be filled with tiles is a rectangle 22 inches by 17 inches. About how many 1-inch tiles will Mr. Henry need for the table top?

Now you should know how to solve problems about the areas of parallelograms.

Looking back

In this lesson you will learn more about the measurement of area.

A

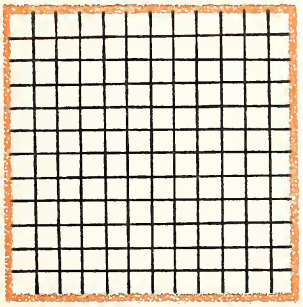


Diagram A represents a 1-foot square of paper marked off in square inches. The area of the paper square is 1 square foot. How can you find the number of square inches in the square foot?

You can use the equation below to find the area of 1 square foot in square inches.

$$\frac{12}{1} = \frac{b}{12}$$

This ratio expresses a rate of 12 sq. in. per 1 row.

What does this ratio express? How do you find the numeral that replaces b?

1 square foot = 144 square inches

B

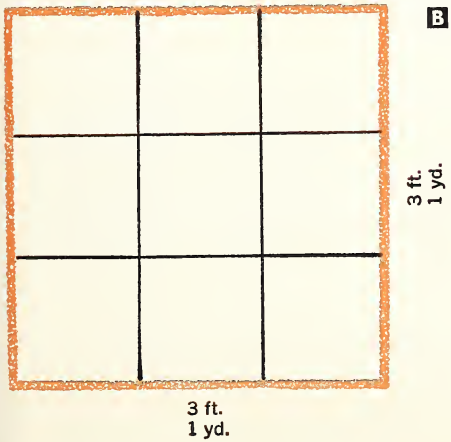


Diagram B represents a 1-yard square of paper marked off in square feet. The area of the paper square is 1 square yard. You can see that the area is also 9 square feet. But you can also use the equation below to find the number of square feet in 1 square yard.

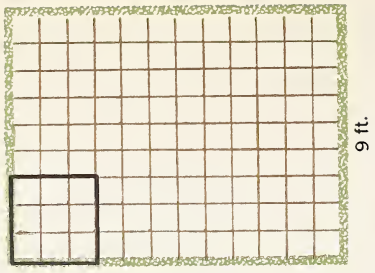
$$\frac{3}{1} = \frac{x}{3}$$

This ratio expresses a rate of 3 sq. ft. per 1 row.

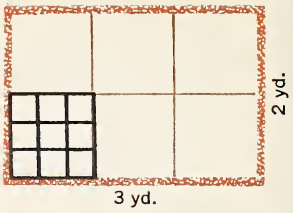
What does this ratio express? How do you find the numeral that replaces x?

1 square yard = 9 square feet

C



D



E

A Diagram C represents a rectangular rug 12 ft. long and 9 ft. wide. Find the area of the rug in square feet.

B You can also find the area in square yards. One way to do this is to change 108 sq. ft. to square yards.

The rate of square feet per square yards is always the same. Why does 9 per 1 express this rate? Look at Diagram D. In Problem B, the rate of square feet per square yards can also be expressed as 108 per t. What does t represent?

$$\frac{9}{1} = \frac{108}{t}$$

The numeral that replaces t will tell you how many square yards there are in 108 sq. ft.

If you use the ratio test to solve this equation, you get $9t = 1 \times 108$. How do you find the numeral that replaces t?

108 sq. ft. = ▩ sq. yd.

C Diagram E represents a rectangular tablecloth 3 yd. long and 2 yd. wide. There are how many square yards of cloth in the tablecloth?

D There are how many square feet of cloth in the tablecloth? You can change ▩ sq. yd. to square feet.

How does Diagram E show that 9 per 1 expresses the rate of square feet per square yards? In Problem D, does n per 6 also express the rate of square feet per square yards? What does n represent?

$$\frac{9}{1} = \frac{n}{6}$$

When you find the numeral that replaces n, you will know how many square feet there are in 6 sq. yd.

If you use the ratio test when you solve this equation, you get $1n = 9 \times 6$. What do you do now?

6 sq. yd. = ▩ sq. ft.

E A table top is in the shape of a rectangle 48 in. long and 33 in. wide. What is its area in square inches?

F What is the area of the table top in square feet?

The rate of square inches per square feet can be expressed as 144 per 1. Why?

$$\frac{144}{1}=\frac{1584}{b}$$

Why is 1584 in the first term of the second ratio? What does b represent?

G A piece of plywood has an area of 7 sq. ft. Find its area in square inches.

$$\frac{144}{1}=\frac{w}{7}$$

What does 144 represent? What does w represent?

H A rectangular floor is 20 ft. long and 18 ft. wide. What is its area in square yards?

The dimensions are given in feet. So first find the area in square feet. Then make an equation that you can use to change square feet to square yards.

$$\frac{9}{1}=\frac{360}{x}$$

Why must you use 9 over 1 as one of the ratios in the equation? Why is 360 in the first term of the second ratio? What does x represent?

I The dimensions of a rectangular mirror are 24 in. and 30 in. Find the area of the mirror in square feet.

First find the area in square inches. Then make an equation that you can use to change square inches to square feet.

$$\frac{144}{1}=\frac{720}{k}$$

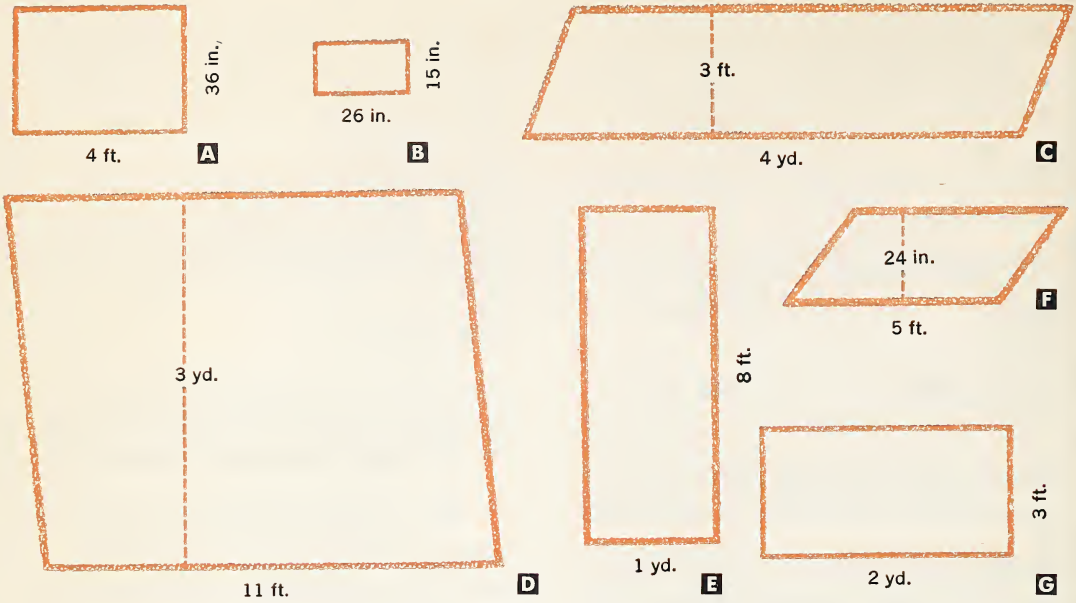
Why is 720 in the first term of the second ratio? What does k represent?

J Mrs. Jackson bought a rectangular piece of carpet 3 yd. wide and 5 yd. long. How many square feet of carpet did she buy?

$$\frac{9}{1}=\frac{m}{15}$$

What does m represent? Why is 15 in the second term of the second ratio?

A 17 sq. ft. = ▒ sq. in.
B 19 sq. yd. = ▒ sq. ft.
C 72 sq. ft. = ▒ sq. yd.
D 864 sq. in. = ▒ sq. ft.
E 117 sq. ft. = ▒ sq. yd.
F 15 sq. ft. = ▒ sq. in.
G 23 sq. yd. = ▒ sq. ft.
H 1440 sq. in. = ▒ sq. ft.
I 9 sq. ft. = ▒ sq. in.
J 630 sq. ft. = ▒ sq. yd.
K 3024 sq. in. = ▒ sq. ft.
L 63 sq. yd. = ▒ sq. ft.
M 26 sq. ft. = ▒ sq. in.



A In Diagram A, the length of the base is given in feet, and the length of the altitude is given in inches. To find the area in square inches, first change 4 ft. to inches. Then what do you do?

B Suppose you want to find the area of Rectangle A in square feet. You can change 1728 sq. in. to square feet. Or you can first change 36 in. to feet. Then what do you do?

C Find the area of each of the other parallelograms above. You may have to change one dimension before you make your equation.

Now you know how to find areas in square inches, square feet, and square yards. You can also change square measures when you need to.

Keeping skilful

A $3765 + d = 5002$

B $n - 8309 = 4667$

C $20745 - 9088 = v$

D $f + 87123 = 90551$

E $52430 - w = 38909$

F $a + 67312 = 100000$

G $m - 49325 = 77468$

H $200714 - 163552 = z$

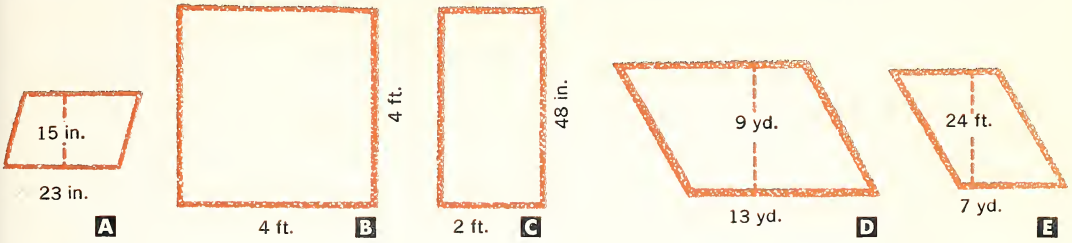
I $351426 + n = 508976$

J $723905 - c = 481078$

K $150013 - 94782 = g$

L $e - 63579 = 402180$

Checking up



Test 1

Find the area of each parallelogram shown above.

Test 2

Find the area of each parallelogram whose dimensions are given below.

- **A** 25 ft. square
- **B** base, 5 ft.; altitude, 32 in.
- **C** base, 17 ft.; altitude, 51 ft.
- **D** base, 30 yd.; altitude, 14 yd.
- **E** base, 36 in.; altitude, 7 ft.
- **F** base, 24 ft.; altitude, 11 yd.

Test 3

- **A** 6 sq. ft. = ▒ sq. in.
- **B** 45 sq. ft. = ▒ sq. yd.
- **C** 15 sq. yd. = ▒ sq. ft.
- **D** 432 sq. in. = ▒ sq. ft.
- **E** 36 sq. yd. = ▒ sq. ft.
- **F** 18 sq. ft. = ▒ sq. in.
- **G** 135 sq. ft. = ▒ sq. yd.
- **H** 1872 sq. in. = ▒ sq. ft.
- **I** 21 sq. ft. = ▒ sq. in.
- **J** 180 sq. ft. = ▒ sq. yd.
- **K** 2880 sq. in. = ▒ sq. ft.

Keeping skilful

- **A** $4\frac{5}{8}+2\frac{7}{8}=z$
- **B** $t+1\frac{5}{6}=8\frac{7}{12}$
- **C** $46\frac{3}{4}+n=49\frac{2}{3}$
- **D** $12-a=7\frac{1}{5}$
- **E** $\frac{1}{4}+3\frac{9}{10}+6\frac{3}{5}=f$
- **F** $p-11\frac{1}{3}=5\frac{1}{9}$
- **G** $10\frac{2}{5}-3\frac{1}{2}=b$
- **H** $w+24\frac{13}{16}=52$
- **I** $5\frac{1}{3}+7\frac{1}{8}+8\frac{5}{6}=c$
- **J** $11\frac{1}{6}-6\frac{4}{5}=m$
- **K** $13\frac{5}{8}+4\frac{1}{2}+9\frac{3}{16}=x$
- **L** $8\frac{2}{5}+d=10\frac{1}{3}$
- **M** $g-6\frac{5}{12}=13\frac{11}{12}$
- **N** $n+14\frac{5}{6}=21\frac{5}{6}$
- **O** $70\frac{3}{4}+23\frac{3}{10}=v$
- **P** $11\frac{5}{9}+e=15$

- **A** $3500\div700=n$
- **B** $64000\div8000=p$
- **C** $60000\div1200=r$
- **D** $16000\div3200=c$
- **E** $10080\div140=t$
- **F** $16200\div270=a$
- **G** $450000\div5000=h$
- **H** $720000\div9000=k$
- **I** $322000\div7000=b$

Side trip

Look at the division example below. Is the computation correct?

```
34)21098 | 600
  20400  |
    698  |  20
    680  |
     18  | 620
```

Here is another way to do division. In this way you work with parts of numbers in each step of the computation.

A

```
34)21098
```

Divide the 21 in 21098 by the 3 in 34. Try 7 in the quotient.

B

```
   7
34)21098
  238
```

Write 7 here. It stands for 700. Multiply the divisor 34 by 7.

Write the product of 34 and 7 here. 238 is really 23,800. 238 is greater than 210. So 7 is too large.

C

```
   6
34)21098
  204
```

Try 6 in the quotient. 6 stands for 600. Multiply the divisor 34 by 6. 204 is really 20,400.

D

```
   6
34)21098
  204
    6
```

204 is smaller than 210. So subtract.

This number is smaller than the divisor. So you know 6 is the correct number to use in the quotient.

E

```
   6
34)21098
  204
   69
```

Bring down the 9. 69 is really 698.

F

```
   6
34)21098
  204
   69
```

Divide the 6 in 69 by the 3 in 34. Try 2 in the quotient.

G

```
   62
34)21098
  204
   69
   68
```

Write 2 here. It stands for 20. Multiply the divisor 34 by 2.

Write the product here. 68 is really 680. 68 is smaller than 69. So 2 in the quotient is not too large.

H
```
    62
34)21098
   204
    69
    68
     1
```
Now subtract.

This number is smaller than the divisor. So you know that 2 is the correct number in the quotient.

I
```
    62
34)21098
   204
    69
    68
     18
```
Bring down the 8.

J
```
   620
34)21098
   204
    69
    68
    18
```
You do not divide because 18 is less than the divisor 34. But you must have a numeral in ones' place in the quotient. So write 0 in the quotient.

The quotient is 620 with 18 remainder, or $620\frac{18}{34}$, or $620\frac{9}{17}$.

Divide 29,568 by 96.

A
```
    3
96)29568
  288
    7
```
Why is 3 a good number to try in the quotient? Is this really 3 or 300?

How do you get this number?

How does this number show that 3 is the correct number to use in the quotient? What do you do next?

B
```
   30
96)29568
  288
   76
```
You do not divide 76 by 96. But you must have a numeral in tens' place in the quotient. What is this numeral? Now what do you do?

C
```
   308
96)29568
  288
  768
  768
```
Why is 8 the correct number to use in the quotient?

How do you get this number?

How do you get this number? Is there any more work to do?

The quotient is 308.

A $785623 \div 145 = n$

$$
\begin{array}{r}
5418\frac{13}{145} \\
145\overline{)785623} \\
725 \\
\hline
606 \\
580 \\
\hline
262 \\
145 \\
\hline
1173 \\
1160 \\
\hline
\cancel{13}
\end{array}
$$

B $139734 \div b = 18$

$$
\begin{array}{r}
7763 \\
18\overline{)139734} \\
126 \\
\hline
137 \\
126 \\
\hline
113 \\
108 \\
\hline
54 \\
54
\end{array}
$$

C $278901 \div 695 = w$

$$
\begin{array}{r}
401\frac{206}{695} \\
695\overline{)278901} \\
2780 \\
\hline
901 \\
695 \\
\hline
\cancel{206}
\end{array}
$$

For each exercise below, write your computation in the way shown in this lesson.

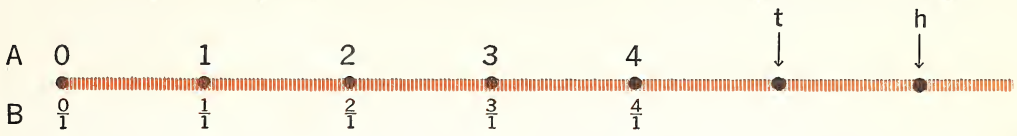
A $5297 \div 11 = t$
B $84325 \div 62 = n$
C $217 \div 7 = x$
D $438 \div r = 5$
E $17300 \div m = 25$
F $1747 \div 8 = q$
G $338865 \div 89 = a$
H $965413 \div 583 = b$
I $76834 \div 412 = c$
J $934462 \div 38 = p$
K $4967 \div h = 112$
L $10000 \div 333 = y$
M $47631 \div 432 = z$
N $191057 \div d = 651$
O $26513 \div 99 = g$
P $5959 \div w = 28$
Q $8086 \div 192 = s$
R $132399 \div 575 = v$

Keeping skilful

A $\frac{3}{8} + \frac{3}{4} + \frac{5}{6} = m$
B $16\frac{7}{10} - 3\frac{1}{2} = x$
C $a + 17\frac{1}{3} = 25$
D $32\frac{3}{5} + 6\frac{1}{2} + 5\frac{2}{5} = n$
E $11\frac{3}{4} - t = 10\frac{5}{6}$
F $7 + q = 21\frac{1}{8}$
G $f - 15\frac{2}{3} = 3\frac{4}{5}$
H $18\frac{2}{3} + 5\frac{3}{8} + \frac{5}{6} = b$
I $29\frac{5}{12} - 3\frac{5}{9} = e$
J $18\frac{1}{2} + v = 32\frac{1}{3}$
K $47\frac{7}{16} - s = 4\frac{1}{4}$
L $p - 13\frac{5}{8} = 2\frac{1}{2}$
M $c + 6\frac{1}{12} = 14\frac{1}{3}$
N $\frac{3}{5} + 8\frac{4}{5} + 18\frac{7}{10} = r$
O $79\frac{4}{7} - 12\frac{2}{5} = z$
P $1\frac{1}{9} - h = \frac{1}{6}$
Q $y + 11 = 33\frac{1}{3}$
R $27 + 6\frac{1}{2} + 7\frac{2}{3} = d$
S $83\frac{1}{10} + n = 96\frac{5}{8}$
T $46\frac{3}{5} + 16\frac{3}{4} = t$
U $38 - 14\frac{3}{10} = q$

Moving forward

In this lesson you will learn that a number may have many different names.

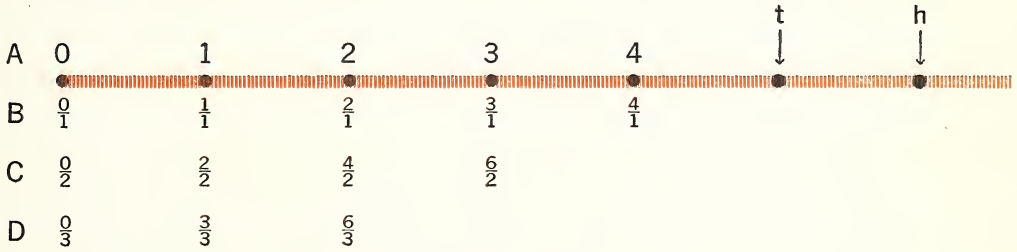


A The numerals in Row A above are a set of names for some of the points on the line. What numerals can you use to name Points t and h? Can the line be extended so there is a point for 100? For 1,000,000? Give six other numerals that belong in Set A.

B Look at the pairs of numerals in Row B below the line. They belong in another set of names for the points on the line. Each pair has ▒ in the denominator.

C What pair of numerals from this set can you use to name Point t? Point h?

D If you extend the line, will all the other pairs of numerals in Set B have the same denominator? Give six other pairs of numerals that belong in Set B.



E The pairs of numerals in Row C belong in another set of names for the points on the line. Each pair has ▒ in the denominator. What pair of numerals from this set can you use to name Point 4? Point t? Point h?

F Suppose you extend the line farther to the right. Give six other pairs of numerals that belong in Set C.

G The pairs of numerals in Set D have what denominator? Name the pairs of numerals that belong in this set for the other points shown.

H Give six other pairs of numerals that belong in Set D.

I How many names are shown for Point 2? Give four other names for Point 2.

s
t
A 0 $0\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 $2\frac{1}{2}$ 3 $3\frac{1}{2}$ 4
B $\frac{0}{1}$ $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$
C $\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$ $\frac{7}{2}$

J More points are now named on the line. What numerals are used in Set A to name these new points? What numerals can you use to name Points s and t? Give six other numerals that belong in Set A.

K Are there pairs of numerals from Set B that can be used to name the new points on the line? Is there a pair of numerals from Set B for Point s? For Point t?

L Is there any end to the pairs of numerals that belong in Set B?

M Now look at Set C. What pair of numerals that belongs in this set can you use to name Point 4? Point s? Point t? Give six other pairs of numerals that belong in Set C.

---

m n p q r
u v w x y z
A 0 1 $1\frac{1}{4}$ $1\frac{1}{3}$ $1\frac{1}{2}$ $1\frac{2}{3}$ $1\frac{3}{4}$ 2
B $\frac{0}{1}$ $\frac{1}{1}$ $\frac{2}{1}$
C $\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$
D $\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$
E $\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$

N Notice that other points have been named on the line above. What numerals that belong in Set A can you use to name Points m, n, p, q, and r? Points u, v, w, x, y, and z?

O Give six other numerals that belong in Set A.

P What pairs of numerals that belong in Sets B, C, D, and E can you use to name Points u, v, w, x, y, and z? Is there a pair of numerals in each set for each point?

Q Give six other pairs of numerals that belong in Sets D and E.

R How many names have been given to Point p? Give four other names for Point p. Give three other names for Point $1\frac{2}{3}$.

S Suppose $8\frac{2}{3}$ were shown in Set A. Will Set C or Set D have a pair of numerals to name the same point?

T Suppose $2\frac{2}{5}$ were shown in Set A. Could you use a pair of numerals from Sets B, C, D, or E to name the same point? What pair of numerals with denominator 5 can you use to name Point 0? To name Point 1? To name Point $2\frac{2}{5}$?

U If 25 were shown in Set A, what pairs of numerals could you use from Sets B, C, D, and E to name the same point?

V Which set has a pair of numerals to name Point w? Write five other names for Point w.

W When you add $\frac{1}{2}$ and $\frac{3}{4}$, what name do you use for $\frac{1}{2}$? Has Set C a pair of numerals to name $\frac{1}{2}$? To name $\frac{3}{4}$? Has Set D pairs of numerals to name either $\frac{1}{2}$ or $\frac{3}{4}$? Has Set E pairs of numerals to name both $\frac{1}{2}$ and $\frac{3}{4}$? When you add $\frac{1}{2}$ and $\frac{3}{4}$, you think of $\frac{1}{2}$ as $\frac{2}{4}$, so that both $\frac{2}{4}$ and $\frac{3}{4}$ will be in the same set.

X When you subtract $\frac{1}{2}$ from 1, what name do you use for 1? Has Set B a pair of numerals to name 1? To name $\frac{1}{2}$? Which set has pairs of numerals to name both $\frac{1}{2}$ and 1? When you subtract $\frac{1}{2}$ from 1, why do you think of 1 as $\frac{2}{2}$?

Y When you subtract $\frac{1}{3}$ from $\frac{1}{3}$, you do not need to change the name. How do you get the answer $\frac{0}{3}$? Is $\frac{0}{3}$ another name for 0? Can you use $\frac{0}{70}$ to name 0? Can you use $\frac{0}{800}$? Can you use $\frac{0}{10,000}$?

Z When you subtract $\frac{1}{12}$ from $\frac{5}{12}$, do you need to use different names for the numbers? Is the answer $\frac{4}{12}$? Is $\frac{4}{12}$ another name for $\frac{1}{3}$?

You may think of all the pairs of numerals in Sets B, C, D, and E on page 166 as one set of names for the points on the line. Use this set of names to decide which sign, $>$, $<$, or $=$, makes a true statement for each exercise below.

A $\frac{4}{3} \sim\sim \frac{5}{4}$

B $\frac{5}{3} \sim\sim \frac{7}{4}$

C $\frac{4}{4} \sim\sim \frac{1}{1}$

D $1\frac{1}{2} \sim\sim 1\frac{2}{3}$

E $\frac{4}{2} \sim\sim \frac{0}{3}$

F $\frac{3}{2} \sim\sim \frac{5}{3}$

G $\frac{1}{2} \sim\sim \frac{2}{4}$

H $1\frac{3}{4} \sim\sim 1\frac{2}{3}$

I $\frac{0}{4} \sim\sim \frac{1}{3}$

J $\frac{4}{3} \sim\sim \frac{3}{2}$

K $\frac{1}{2} \sim\sim \frac{1}{3}$

L $\frac{3}{3} \sim\sim \frac{0}{1}$

M $1\frac{1}{4} \sim\sim 1\frac{1}{3}$

N $\frac{2}{1} \sim\sim \frac{5}{3}$

O $\frac{6}{4} \sim\sim \frac{3}{2}$

P $\frac{2}{1} \sim\sim \frac{4}{3}$

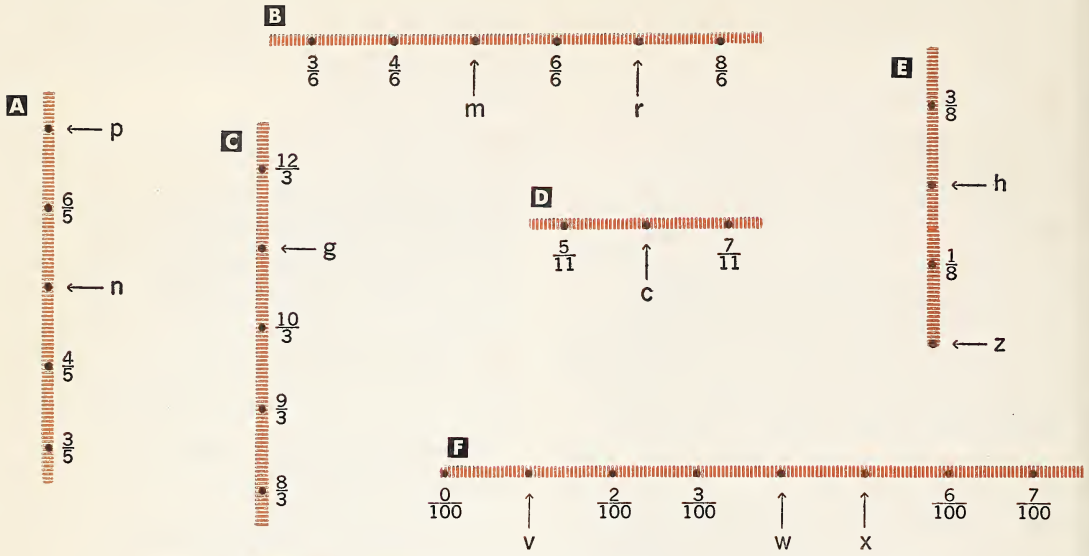
Q $\frac{1}{1} \sim\sim \frac{1}{3}$

R $\frac{4}{2} \sim\sim \frac{2}{1}$

S $\frac{0}{1} \sim\sim \frac{0}{3}$

T $1\frac{1}{3} \sim\sim 1\frac{1}{2}$

Study each line below. Then replace each letter with the pair of numerals that belongs in the set.



Now you should be able to give different names to fractions, whole numbers, and mixed numbers.

Keeping skilful

A 78 is 39% of ▭.
B 16% of 25 is ▭.
C 25 is ▭% of 20.
D 50% of ▭ is 5.
E 230% of 50 is ▭.
F 26% of ▭ is 130.
G 188 is ▭% of 400.
H 140% of ▭ is 28.
I 82% of 250 is ▭.
J 24 is ▭% of 80.
K 175% of ▭ is 7.
L 40% of 70 is ▭.

M 92 is ▭% of 25.
N 88% of ▭ is 66.
O 135% of 20 is ▭.
P 92 is ▭% of 200.
Q 72% of 275 is ▭.
R 45% of ▭ is 144.
S 77 is ▭% of 25.
T ▭% of 150 is 3.
U 30 is 40% of ▭.
V 160% of 5 is ▭.
W 39% of ▭ is 39.
X 15% of 80 is ▭.

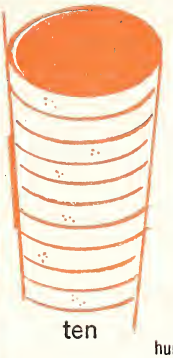
A 5 bu. = ▭ qt.
B 36,000 sec. = ▭ hr.
C 11 sq. yd. = ▭ sq. ft.
D 130 lb. = ▭ oz.
E 23 T. = ▭ lb.
F 192 tbs. = ▭ c.
G 4 mi. = ▭ yd.
H 12 gal. = ▭ pt.
I 1152 sq. in. = ▭ sq. ft.
J 18 wk. = ▭ da.
K 52 pk. = ▭ bu.
L 288 in. = ▭ yd.

Moving forward

In this lesson you will learn how to use decimal fraction numerals as names for numbers.

ten

ten



ten

one

two tenths

hundreds	tens	ones	tenths
	3	1	2

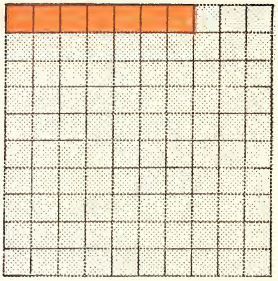
$31\frac{2}{10}$ 31.2

thirty-one and two tenths

A

one

one



seven hundredths

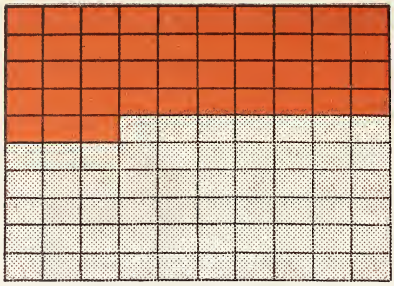
hundreds	tens	ones	tenths	hundredths
		2		7

$2\frac{7}{100}$ 2.07

two and seven hundredths

B

one



forty-three hundredths

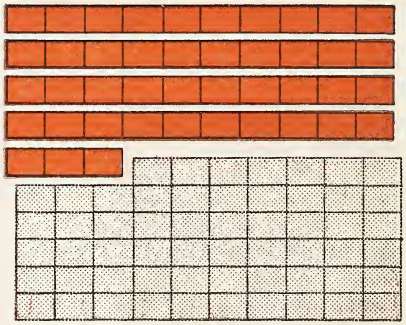
hundreds	tens	ones	tenths	hundredths
		1		43

$1\frac{43}{100}$ 1.43

one and forty-three hundredths

C

one



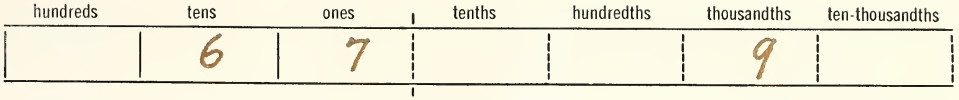
forty-three hundredths or
four tenths and three hundredths

hundreds	tens	ones	tenths	hundredths
		1	4	3

$1\frac{43}{100}$ 1.43

one and forty-three hundredths

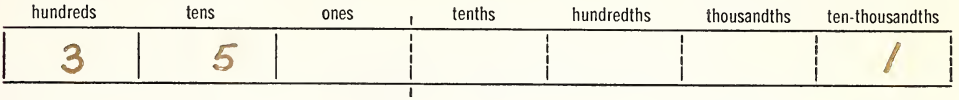
A



$67\frac{9}{1000}$ 67.009

sixty-seven and nine thousandths

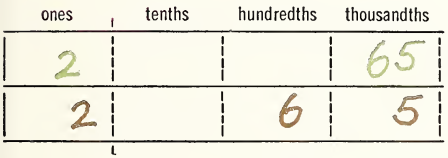
B



$350\frac{1}{10{,}000}$ 350.0001

three hundred fifty and one ten-thousandth

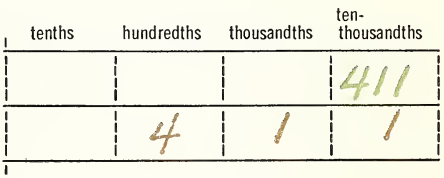
C



$2\frac{65}{1000}$ 2.065

two and sixty-five thousandths

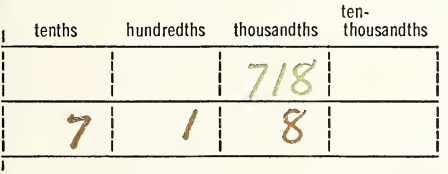
D



$\frac{411}{10{,}000}$ .0411

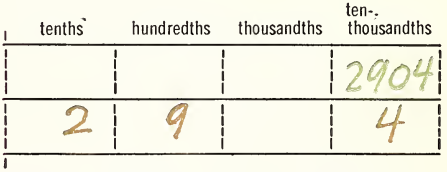
four hundred eleven ten-thousandths

E



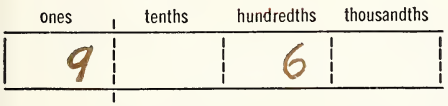
$\frac{718}{\equiv}$ .718 718 ~~~

F



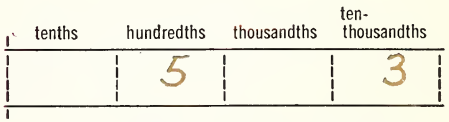
$\frac{2904}{\equiv}$ .2904 2904 ~~~

G



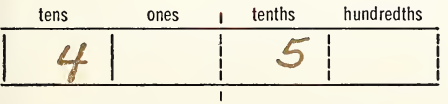
$9\frac{6}{\equiv}$ 9.06 9 ~~~ 6 ~~~

H



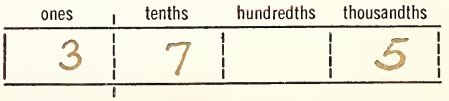
$\frac{503}{\equiv}$ .0503 503 ~~~

I



$40\frac{5}{\equiv}$ 40.5 40 ~~~ 5 ~~~

J



$3\frac{705}{\equiv}$ 3.705 3 ~~~ 705 ~~~

Say "and."

Read as a whole number. → 4326.1642 ← Read as you would a whole number. Then say "ten-thousandths," because the last numeral is in ten-thousandths' place.

Say "and."

Say "fifty-seven." → 57.005 ← Read as "five." Then say "thousandths," because the last numeral is in thousandths' place.

There is no whole number. So you do not say "and" for the point. → .14 ← Read this as "fourteen." Then say "hundredths."

Read each numeral below.

					tenths	hundredths	thousandths	ten-thousandths
A		8	3	.	4	9		
B	5	0	6	.	0	1	4	
C	3	7	8	.	5			
D			9	.	0	6	4	3
E		2	8	.	0	0	7	

F 7208.64
G 5.031
H 227.516
I 39.3028
J 7700.2
K 11.0006
L 480.09
M 89543.68
N 627.003
O 2324.0075

Write as decimal fraction numerals.

A 36 and 9 tenths
B 518 and 213 thousandths
C 69 and 25 hundredths
D 7 and 2 thousandths
E 88 and 4 tenths
F 375 and 53 hundredths
G 1634 and 1256 ten-thousandths
H 930 and 784 ten-thousandths
I 2060 and 5 thousandths
J 429 and 1 hundredth
K 70 and 6 ten-thousandths
L 539 and 41 thousandths
M 2103 and 1 ten-thousandth
N 6254 and 7 hundredths

Now you should be able to read and write decimal fraction numerals.

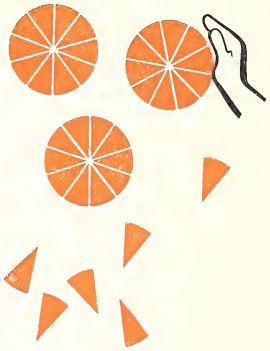
Moving forward

In this lesson you will learn how to change decimal fraction numerals as you will when you compute.

tens	ones	tenths
	2	16

$2\frac{16}{10}$

two and sixteen tenths **A**



tens	ones	tenths
	3	6

$3\frac{6}{10}$ 3.6

three and six tenths

tens	ones	tenths
1		3

$10\frac{3}{10}$ 10.3

ten and three tenths **B**

tens	ones	tenths
	9	13

$9\frac{13}{10}$

nine and thirteen tenths

A

tens	ones	tenths	hundredths
4	3	9	16
4	?	?	6

B

ones	tenths	hundredths	thousandths
	19	28	3
?	1	?	3

C

ones	tenths	hundredths	thousandths
6	5		2
6	?	?	12

D

tens	ones	tenths	hundredths
5	2	3	8
?	11	12	?

E

tenths	hundredths	thousandths	ten-thousandths
3	17	9	11
?	8	?	1

F

tenths	hundredths	thousandths	ten-thousandths
5			6
?	?	?	16

Write in the usual way each number represented below.

	ones	tenths	hundredths	thousandths
A		9	18	
B	2	1	9	28
C		22	3	6
D		2	0	14
E	9	9	20	
F	14	5	0	27
G			18	7
H	24		19	
I		2	3	20
J	9	9	9	10
K			9	11
L	4	6	9	45

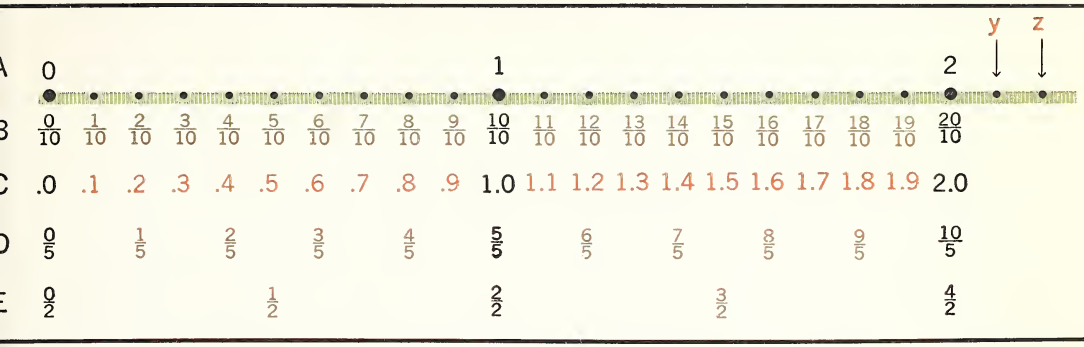
What numeral belongs where each question mark is?

	ones	tenths	hundredths	thousandths
A	1	0	3	
	?	9	?	
B	7	6	0	9
	?	?	?	19
C	9	1	4	2
	?	11	?	12
D	4	9	0	0
	4	?	?	10
E		4	3	0
		?	12	?
F	1	0	0	3
	?	?	?	13

Now you should know how to change decimal fraction numerals when you need to.

Moving forward

In this lesson you will learn more about decimal fraction numerals and the number system.


y
z
A 0 1 2
B $\frac{0}{10}$ $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{10}$ $\frac{11}{10}$ $\frac{12}{10}$ $\frac{13}{10}$ $\frac{14}{10}$ $\frac{15}{10}$ $\frac{16}{10}$ $\frac{17}{10}$ $\frac{18}{10}$ $\frac{19}{10}$ $\frac{20}{10}$
C .0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
D $\frac{0}{5}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$ $\frac{6}{5}$ $\frac{7}{5}$ $\frac{8}{5}$ $\frac{9}{5}$ $\frac{10}{5}$
E $\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$

A Five sets of names are shown for points on the line above. For how many points are there names in Set A?

B Look at Set B and Set C. Is there a name in each set for each point shown on the line? Do you read the numerals in the two sets in the same way? Are the denominators the same?

C Notice how the name for Point 1 is written in Set C. You can read it as "one and zero tenths" or as "one."

D How do you read the name in Set C for Point 2?

E Why are names not given in Set D and Set E for all the points?

F What name from Set B would you use for Point y? For Point z?

G What name from Set C would you use for Point y? For Point z?

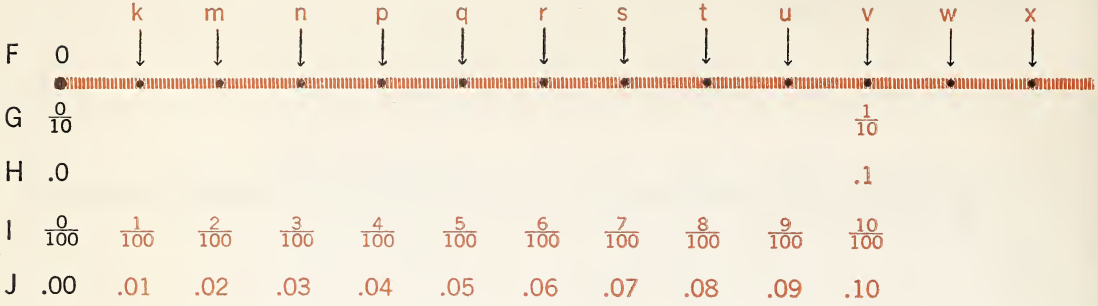
H Is there a name in Set D for Point y? For Point z? How do you know that there is no name in Set E for either Point y or Point z?

I If Point $8\frac{1}{2}$ were shown on the line, what name could you give for this point from Set B? From Set C? Would there be a name for Point $8\frac{1}{2}$ from Set D? What name could you use from Set E for this point?

J Suppose Point 15 were shown on the line. Give a name from each set for this point.

K Can you use $\frac{0}{4}$ to name Point 0? Which of these pairs of numerals can you use to name points on the line: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{4}{4}$, $\frac{7}{4}$, $\frac{8}{4}$?

L Can you use $\frac{0}{6}$ to name Point 0? Which of these pairs of numerals can you use to name points on the line: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{6}{6}$, $\frac{8}{6}$, $\frac{9}{6}$?



M The line above shows only part of the line on page 175. What part? Are the points named in Sets G and H on the line above also named on the line on page 175?

N Look at Sets I and J. Is there a name from each set for each point shown on the line? Do you read the numerals in the two sets in the same way?

O What name can you use in Set I for Point w? For Point x? What names can you use from Set J for these two points?

P Can you use $\frac{0}{50}$ to name Point 0? For which point on the line can you use the name $\frac{1}{50}$? For which point can you use $\frac{2}{50}$? $\frac{3}{50}$? $\frac{4}{50}$? $\frac{5}{50}$? $\frac{6}{50}$?

Q Can you use $\frac{0}{25}$ to name Point 0? For which point on the line can you use $\frac{1}{25}$? $\frac{2}{25}$? $\frac{3}{25}$?

R Look at the line on page 175 again. Can you use $\frac{100}{100}$ to name Point 1? Can you also use 1.00 to name Point 1?

S Make a line like the one above to show the thousandths between 0 and $\frac{1}{100}$. Can you use $\frac{0}{1000}$ to name Point 0? Can you also use .000? Can you use $\frac{10}{1000}$ to name $\frac{1}{100}$? What decimal fraction numeral equal to $\frac{10}{1000}$ can you use to name $\frac{1}{100}$?

Rewrite each exercise below, using the sign $>$, $=$, or $<$, to make a true statement.

A .01 ~~~ .1
B 1.0 ~~~ 1.000
C .10 ~~~ .04
D 1.3 ~~~ .13
E .00 ~~~ .000
F 2.8 ~~~ 2.0
G .09 ~~~ .1
H .005 ~~~ .03
I 1.6 ~~~ 1.08
J .12 ~~~ .012
K .01 ~~~ .010
L .009 ~~~ .07
M 3.60 ~~~ .361
N .2 ~~~ .20

Now you should understand how decimal fraction numerals can be used to name points of a line.

Learning how

In this lesson you will learn how to add and subtract numbers expressed with decimal fraction numerals.

see

A Bill read that Edmonton had 4.34 in. of rainfall in June and 3.67 in. of rainfall in August of that year. Edmonton had a total of how many inches of rain in these two months?

$4.34 + 3.67 = p$

You must find the numeral to replace p.

First add in the hundredths' column. The sum is 11 hundredths. Think of 11 hundredths as 1 tenth and 1 hundredth.

$$\begin{array}{r} 4.34 \\ 3.67 \\ \hline 1 \end{array}$$

Remember the 1 tenth. Write 1 hundredth in hundredths' place.

Write 1 in tenths' column to show there is 1 more tenth. Add in the tenths' column. The sum is 10 tenths. Think of 10 tenths as 1.

$$\begin{array}{r} {\scriptstyle 1} \\ 4.34 \\ 3.67 \\ \hline 01 \end{array}$$

Remember the 1. Write 0 in tenths' place.

Write 1 in ones' column to show there is 1 more one.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 4.34 \\ 3.67 \\ \hline .01 \end{array}$$

Put the point here. You have really added $\frac{34}{100}$ and $\frac{67}{100}$. So the fraction part of your sum must be hundredths.

Add the ones. The sum is 8.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 4.34 \\ 3.67 \\ \hline 8.01 \end{array}$$

Write 8 in ones' place.

Edmonton had a total of 8.01 in. of rain in August and September.

$4.34 + 3.67 = 8.01$

B Mr. Miller has a steel rod that is 3.028 in. long. He needs to cut .175 in. from the rod. How long will the rod be when he has cut it down?

$3.028 - .175 = q$

You must find the numeral to replace q.

Now turn the page.

$$\begin{array}{r} 3.028 \\ .175 \\ \hline 3 \end{array}$$

First subtract in the thousandths' column. The answer is 3 thousandths.

Write 3 in thousandths' place.

$$\begin{array}{r} 3.028 \\ .175 \\ \hline 3 \end{array}$$

You cannot subtract 7 hundredths from 2 hundredths. You need more hundredths. Get them from the tenths.

$$\begin{array}{r} {\scriptstyle 2\ 10} \\ 3.028 \\ .175 \\ \hline 3 \end{array}$$

But there are no tenths. So to get the tenths you need, think of the 3 ones as 2 ones and 10 tenths.

$$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 2\ 10\ 12} \\ 3.028 \\ .175 \\ \hline 53 \end{array}$$

Now think of the 10 tenths as 9 tenths and 10 hundredths. You have 12 hundredths. Now subtract.

Write 5 in hundredths' place.

$$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 2\ 10\ 12} \\ 3.028 \\ .175 \\ \hline 853 \end{array}$$

Now subtract in the tenths' column. 9 tenths minus 1 tenth equals 8 tenths.

Write 8 in tenths' place.

$$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 2\ 10\ 12} \\ 3.028 \\ .175 \\ \hline .853 \end{array}$$

Put a point here. You have subtracted thousandths. So this part of your answer must be thousandths.

$$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 2\ 10\ 12} \\ 3.028 \\ .175 \\ \hline 2\,853 \end{array}$$

There are no ones to subtract. So write 2 in ones' place.

The steel rod will be 2.853 in. long.
$3.028 - .175 = 2.853$

---

think A Mr. Jackson noticed that the odometer in his car showed 9623.4 mi. Then he drove 29.3 mi. to High Falls. What did his odometer show when he reached High Falls?

$9623.4 + 29.3 = t$

You must find the numeral to replace t.

$$\begin{array}{r} 9623.4 \\ 29.3 \\ \hline 7 \end{array}$$

First add in the tenths' column.

Why do you write 7 here?

9623.4
29.3
9652.7

Now add the whole numbers.

Why do you put the point here?

Mr. Jackson's odometer showed ▒ mi. when he reached High Falls.
9623.4 + 29.3 = t

B The temperature at noon one day was 92.6°. By 1 P.M. the temperature had dropped 8.8°. What was the temperature at 1 P.M.?
92.6 − 8.8 = d

You must find the numeral to replace *d*.

92.6
8.8

What must you do before you can subtract the tenths?

1 16
92.6
8.8
8

Now you have 1 one and 16 ~~~. What do you do next?

Why do you write 8 in tenths' place?

8 11 16
92.6
8.8
83.8

Now subtract the ~~~. Why do you not need to subtract in tens' column?

Why do you put the point here?

The temperature at 1 P.M. was ▒°.
92.6 − 8.8 = d

try

A 27.176 + 18.932 = v

27.176
18.932
46.108

Does the decimal fraction numeral in the answer express hundredths or thousandths?

C 2.1 + 16.8 + 12.1 = n

2.1
16.8
12.1
31.0

Why is there a zero in tenths' place in the answer? Can you think of the answer as 31?

B 1.68 − .75 = f

1.68
.75
.93

Could you put a zero in ones' place in the answer?

D 65.085 − 18.716 = a

65.085
18.716
46.369

Since all the decimal fraction numerals in an addition or subtraction example must have the same denominator, the points will be in a straight line.

do

A $.96 - .95 = a$

B $12.29 + 6.38 = m$

C $6.02 - 1.70 = r$

D $1.2934 - .0062 = b$

E $85.176 + .824 = y$

F $163.3 + 1.8 = f$

G $.263 - .205 = p$

H $1.5 + 3.2 + 6.3 = x$

I $.0071 + .3602 = c$

J $25.5 - 5.5 = h$

K $3.196 + 12.275 = z$

L $11.116 - 9.783 = n$

M $14.005 + .137 = v$

N $.42 - .40 = t$

O $54.6 - 13.9 = g$

P $.11 + .76 + .85 = m$

Q $3.4975 + .8025 = d$

R $6.5 - 3.5 = y$

S $.087 + .334 = k$

T $.13 - .08 = p$

U $1.55 + 3.45 = n$

V $1.942 + 1.066 = q$

W $2.3285 - .6055 = x$

X $.28 - .11 = f$

A $5.6 - .6 = n$

B $.32 - .05 = h$

C $.4 + .6 + .8 = s$

D $345.17 + 2.75 = n$

E $2.95 - .08 = v$

F $6.8921 - 6.8304 = k$

G $.13 + .82 + .19 = e$

H $52.095 + .175 = r$

I $6.112 - 5.006 = m$

J $12.2 + 3.8 = z$

K $1.938 + 2.035 = w$

L $52.08 - 14.49 = d$

Now you should be able to add and subtract numbers expressed with decimal fraction numerals.

Keeping skilful

A $\frac{8}{16} = \frac{38}{t}$

B $\frac{45}{10} = \frac{f}{4}$

C $\frac{96}{72} = \frac{b}{48}$

D $\frac{25}{35} = \frac{15}{p}$

E $\frac{14}{18} = \frac{21}{n}$

F $\frac{130}{20} = \frac{y}{34}$

G $\frac{9}{6} = \frac{30}{g}$

H $\frac{70}{15} = \frac{k}{33}$

I $\frac{16}{22} = \frac{24}{d}$

J $\frac{54}{12} = \frac{36}{z}$

K $\frac{175}{95} = \frac{n}{38}$

L $\frac{8}{6} = \frac{s}{33}$

M $\frac{30}{22} = \frac{45}{n}$

N $\frac{100}{40} = \frac{m}{28}$

A $11665 \div 32 = r$

B $754 \times 186 = a$

C $83872 \div n = 120$

D $50000 \div 40 = c$

E $361 \times 79 = v$

F $230 \times 150 = t$

G $84362 \div q = 36$

H $79 \times 5932 = e$

I $63977 \div 110 = y$

J $38485 \div k = 75$

K $128 \times 128 = b$

L $940 \times 670 = z$

M $140625 \div 375 = s$

N $387 \times 622 = m$

O $13642 \div d = 16$

P $293624 \div 725 = n$

A $5\frac{2}{3} + 4\frac{1}{5} = n$

B $17\frac{1}{2} - 3\frac{3}{4} = b$

C $\frac{3}{8} + \frac{5}{6} + \frac{1}{4} + \frac{1}{2} = z$

D $3\frac{1}{7} + 8\frac{1}{2} = m$

E $11\frac{1}{3} - d = 2\frac{1}{3}$

F $q + 7\frac{1}{8} = 10$

G $4\frac{4}{5} + 2\frac{7}{10} = x$

H $13\frac{5}{12} - a = 2\frac{5}{6}$

I $2\frac{3}{4} + t = 7\frac{2}{3}$

J $16\frac{3}{10} + 1\frac{1}{10} = z$

K $\frac{11}{12} - \frac{1}{6} = n$

L $s - 2 = 4\frac{4}{5}$

M $7\frac{3}{8} + c = 10\frac{1}{2}$

N $w + 8\frac{3}{4} = 9\frac{1}{10}$

Using arithmetic

A Mr. Thomas kept the temperature for hatching chicks at 102.8°. When the chicks hatched, he moved them to a brooder, where he kept the temperature at 94.6°. The brooder temperature was how many degrees lower than the hatching temperature?

B When the chicks were one week old, Mr. Thomas lowered the brooder temperature to 89.6°. He lowered the brooder temperature how many degrees at the end of the first week?

C When one chick was 12 weeks old, its weight was 2.10 lb. At 24 weeks, its weight was 4.25 lb. What was the gain in weight during that time?

D When the chickens were 12 weeks old, Mr. Thomas increased the amount of feed for each chicken by .5 lb. per week. Then each chicken was eating 1.1 lb. of feed per week. About how much feed per week was a chicken eating before the feed was increased?

E The feed for a 12-week-old chicken for one week contained .4 lb. of wheat and .5 lb. of corn. How much did the other material in the feed weigh?

F At the fair, one of Mr. Allen's cows won first prize for milk production. This cow had produced 23,686.80 lb. of milk in one year. The cow that won second prize produced 22,945.35 lb. of milk that year. Mr. Allen's cow had produced how many more pounds of milk than the cow that won second prize?

G Mr. Allen's prize cow produced 64.88 lb. of milk one day. The amount produced in the morning milking was about 38.32 lb. About how much milk was produced in the evening milking?

H The 23,686.80 lb. of milk that Mr. Allen's prize cow produced in one year contained 919.47 lb. of butterfat. What would be the weight of the remaining milk if all butterfat were removed?

I The Allens use the milk produced by their cows in three ways. About .8 gal. is used each day by the family. About 3.4 gal. is used each day for feeding animals. About 52.6 gal. is sold per day. About how many gallons of milk do the cows produce per day?

J Mr. Allen's milk storage tank will hold 200.0 gal. of milk. One day the gauge showed that the tank contained 172.6 gal. How many more gallons of milk could have been put in the tank?

K Mr. Allen built a feed box for his cows. The bottom of the box is 45.8 ft. long and 3.6 ft. wide. What is the perimeter of the bottom of the box?

L Mr. Allen put a steel binding around the edge of the feed box. He had a piece of steel binding 72.5 ft. long. How many more feet of steel binding did he need for the feed box?

M Mr. Townsend has two pastures for his cattle. The area of one pasture is 12.9 acres, and the area of the other is 8.3 acres. How many acres are there in both pastures?

N If grass seed were sown in the pastures, 18.50 lb. of seed per acre would be needed. If clover were sown, 9.75 lb. per acre would be needed. How much more grass seed than clover seed would be needed per acre?

O Mr. Townsend sowed 5.5 acres of pasture in the morning and 5.8 acres in the afternoon. How many acres of pasture did he have left to sow?

Checking up

Test 1

A	2.014	two and fourteen
B	.23	twenty-three
C	6.2	six and two
D	5.0025	five and twenty-five
E	.380	three hundred eighty
F	100.6	one hundred and six
G	3.70	three and seventy
H	.5000	five thousand

Test 2

Write as decimal fraction numerals.

A seventeen and three hundredths
B six hundred ten-thousandths
C eight hundred seven thousandths
D five and two tenths
E nine and eleven thousandths
F two and sixteen hundredths
G eight and seventeen thousandths

Test 3

Find the sum.

A 3.178, 6.204
B 12.92, 11.18, 2.79
C .6007, .1752, .6324
D .9, .8, .5, .6, .2
E 5.48, 6.52
F 12.735, 6.384
G 2.3975, 5.7525

Test 4

Find the difference.

A .724, .308
B 1.7, .9
C 16.3025, 13.4575
D 29.38, 18.30
E 4.2, 3.2
F 1.293, .875
G 3.75, .83

Test 5

A $1.79 + .68 = n$
B $.6028 - .0342 = b$
C $18.9 + y = 27.3$
D $p - 2.111 = 4.728$
E $14.92 - c = 10.66$
F $s + 5.116 = 5.728$
G $2.8 + 6.7 = z$
H $h - 4.637 = 14.892$

Thinking straight

A Mrs. Parker bought 2 lb. of bananas for 29¢. You can use the ratio at the right to express this rate. In the ratio, 2 stands for the number of pounds. What does 29 stand for?

$$\frac{2}{29}$$

B Mrs. Page bought $1\frac{3}{4}$ lb. of bananas for 29¢. You can use the ratio at the right to express this rate. In the ratio, what does $1\frac{3}{4}$ stand for? What does 29 stand for?

$$\frac{1\frac{3}{4}}{29}$$

C Ann used 1 yd. of material for a blouse and 2 yd. for a skirt. You can use the ratio at the right to express this comparison. In the ratio, what does 1 stand for? What does 2 stand for?

$$\frac{1}{2}$$

D Sue used 1 yd. of material for a blouse and $2\frac{1}{4}$ yd. for a skirt. You can use the ratio at the right to express this comparison. What does each term in the ratio stand for?

$$\frac{1}{2\frac{1}{4}}$$

E Mary lives twice as far from school as Alice. This means that Mary lives 2 blocks from school for each block Alice lives from school. You can use the ratio above to express this comparison. What does each term stand for?

$$\frac{2}{1}$$

F Peggy lives $2\frac{1}{3}$ times as far from school as Sue. This means that Peggy lives $2\frac{1}{3}$ blocks from school for each block Sue lives from school. What does $2\frac{1}{3}$ in the ratio stand for? What does 1 stand for?

$$\frac{2\frac{1}{3}}{1}$$

G June lives $\frac{3}{4}$ as far from school as Tony. This means that June lives $\frac{3}{4}$ of a block from school for each block Tony lives from school. What does each of the terms of the ratio stand for?

$$\frac{\frac{3}{4}}{1}$$

Write a ratio for each exercise below.

A 100 mi. in $2\frac{2}{3}$ hr.

B $8\frac{1}{2}$ pages to 7 pages

C $3\frac{7}{10}$ times as many buttons

D 75 lb. to $70\frac{5}{8}$ lb.

E \$2.60 per $3\frac{1}{2}$ lb.

F $4\frac{1}{2}$ mi. per 1 in.

G $\frac{4}{5}$ as many records

H $33\frac{1}{3}$ revolutions per minute

I $1\frac{3}{4}$ cups to 2 cups

J $16\frac{1}{2}$¢ for 1 can

K $1\frac{2}{3}$ times as many hours

L $2\frac{5}{8}$ in. per 100 mi.

M $5\frac{1}{3}$ yd. per window

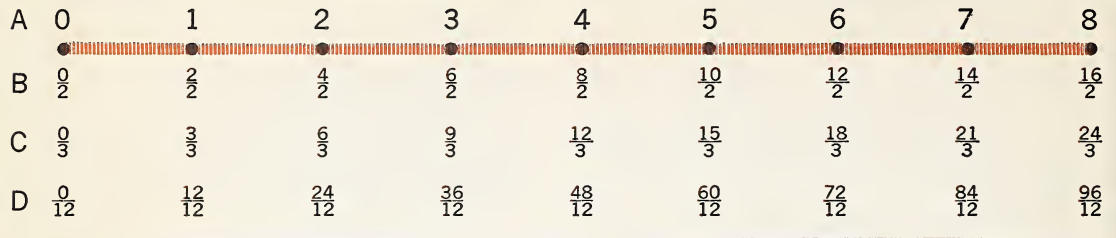
N 1 in. per $5\frac{5}{12}$ ft.

O $\frac{1}{2}$ as many cookies

P $5\frac{1}{2}$ times as much money

Q 10 mi. per $\frac{3}{8}$ in.

Thinking straight


A 0 1 2 3 4 5 6 7 8
B $\frac{0}{2}$ $\frac{2}{2}$ $\frac{4}{2}$ $\frac{6}{2}$ $\frac{8}{2}$ $\frac{10}{2}$ $\frac{12}{2}$ $\frac{14}{2}$ $\frac{16}{2}$
C $\frac{0}{3}$ $\frac{3}{3}$ $\frac{6}{3}$ $\frac{9}{3}$ $\frac{12}{3}$ $\frac{15}{3}$ $\frac{18}{3}$ $\frac{21}{3}$ $\frac{24}{3}$
D $\frac{0}{12}$ $\frac{12}{12}$ $\frac{24}{12}$ $\frac{36}{12}$ $\frac{48}{12}$ $\frac{60}{12}$ $\frac{72}{12}$ $\frac{84}{12}$ $\frac{96}{12}$

$3 \times 2 = n$

What numeral replaces n?

A In the work labelled A, what numeral replaces n?

$\frac{12}{4} \times \frac{6}{3} = n$

What new names are used for 3 and 2?

What numeral replaces n?

B

B Now look at the number line above. What numerals have been used to name Point 2? What numerals have been used to name Point 3?

C Now look at the work labelled B. Is $\frac{12}{4}$ another name for 3? Is $\frac{6}{3}$ another name for 2?

$\frac{12}{4} \times \frac{6}{3} = 72$ ← Write 72 here.

The product of 12 and 6 is 72.

C

D To find the product of 3 and 2 by using $\frac{12}{4}$ and $\frac{6}{3}$, look at the work labelled C. What numbers are multiplied? Where is their product written?

$\frac{12}{4} \times \frac{6}{3} = \frac{72}{12}$

The product of 4 and 3 is 12.

Write 12 here.

D

E Now look at the work labelled D. What numbers are multiplied now? Where is their product written?

$\frac{12}{4} \times \frac{6}{3} = \frac{72}{12} = \frac{6}{1} = 6$

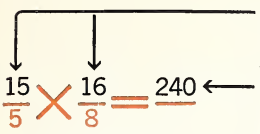
E

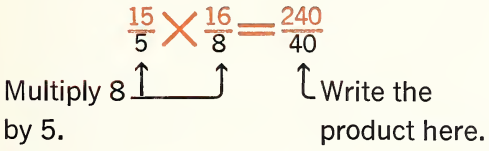
F Examine the work labelled E. What is the product when $\frac{6}{3}$ is multiplied by $\frac{12}{4}$? Is the product $\frac{72}{12}$ shown on the number line as another name for 6?

G Is the product of $\frac{12}{4}$ and $\frac{6}{3}$ equal to the product of 3 and 2?

H Can $\frac{15}{5}$ and $\frac{16}{8}$ be used as other names for 3 and 2?

$\frac{15}{5} \times \frac{16}{8} = d$





I Can $\frac{240}{40}$ be used as another name for 6?

J Is the product of $\frac{15}{5}$ and $\frac{16}{8}$ equal to the product of 3 and 2?

K What is the product of 10 and 7?

L Can $\frac{90}{9}$ and $\frac{35}{5}$ be used as other names for 10 and 7? The product of $\frac{90}{9}$ and $\frac{35}{5}$ is $\frac{\square}{\square}$.

M Can the name of the product in Exercise L be used as another name for 70? Is the product of $\frac{90}{9}$ and $\frac{35}{5}$ equal to the product of 10 and 7?

N Are $\frac{15}{1}$ and $\frac{27}{3}$ other names that can be used for 15 and 9? Is the product of $\frac{15}{1}$ and $\frac{27}{3}$ equal to the product of 15 and 9?

O Find the product of 7 and 8. Now look at the number line on page 184. For each of the points 7 and 8, select another name and find the product by using these names. Is the product equal to the product of 7 and 8?

In each exercise below, first select new names that can be used for each number. Then find the product.

A $4 \times 8 = t$
B $12 \times 5 = a$
C $3 \times 7 = r$
D $15 \times 2 = s$
E $6 \times 4 = d$
F $2 \times 9 = c$
G $6 \times 11 = w$
H $9 \times 5 = g$
I $13 \times 7 = v$
J $10 \times 8 = b$
K $12 \times 3 = m$
L $6 \times 14 = e$
M $5 \times 15 = z$
N $9 \times 11 = k$

Keeping skilful

Express each improper fraction below as a mixed number or a whole number.

A	B	C	D	E	F
$\frac{40}{10}$	$\frac{32}{5}$	$\frac{18}{3}$	$\frac{21}{8}$	$\frac{16}{2}$	$\frac{64}{8}$
G	**H**	**I**	**J**	**K**	**L**
$\frac{70}{12}$	$\frac{94}{20}$	$\frac{100}{50}$	$\frac{86}{12}$	$\frac{116}{20}$	$\frac{99}{9}$

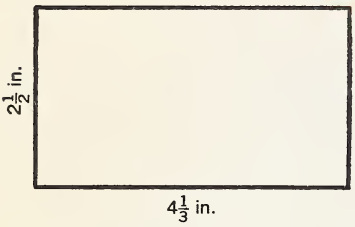
Express each mixed number below as an improper fraction.

A $6\frac{7}{12}$
B $3\frac{4}{5}$
C $9\frac{1}{32}$
D $20\frac{5}{6}$
E $16\frac{9}{10}$
F $44\frac{7}{8}$
G $4\frac{9}{16}$
H $30\frac{14}{15}$
I $18\frac{2}{3}$
J $51\frac{1}{2}$
K $100\frac{11}{12}$
L $64\frac{3}{10}$
M $8\frac{3}{4}$
N $25\frac{1}{5}$
O $9\frac{3}{8}$
P $11\frac{1}{6}$

Learning how

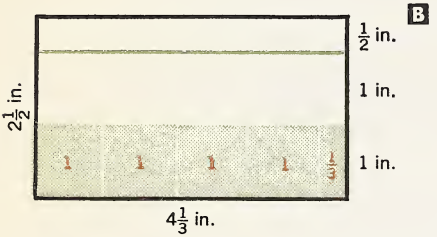
Now you will learn how to find the product when you use mixed numbers and proper fractions.

see



What are the dimensions of the piece of paper in Diagram A? Find its area in square inches.

Diagram B shows why you can use the equation below to find the area.



Number of square inches in 1 row → $\frac{4\frac{1}{3}}{1} = \frac{w}{2\frac{1}{2}}$ ← Number of square inches in $2\frac{1}{2}$ rows

When you find the numeral that replaces w, you will know the area of the piece of paper.

When you use the ratio test, you get this equation.

$$w = 4\frac{1}{3} \times 2\frac{1}{2}$$

To solve this equation, you must find the product of $4\frac{1}{3}$ and $2\frac{1}{2}$. To multiply these mixed numbers, first find other names for them. Express them with fraction numerals.

$\frac{13}{3}$ is another name for $4\frac{1}{3}$.

$$w = \frac{13}{3} \times \frac{5}{2}$$

$\frac{5}{2}$ is another name for $2\frac{1}{2}$.

Now multiply the numerators.

Multiply 5 by 13.

$\frac{13}{3} \times \frac{5}{2} = 65$ ← Write the product here.

Next multiply the denominators.

$\frac{13}{3} \times \frac{5}{2} = \frac{65}{6}$

Multiply 2 by 3.

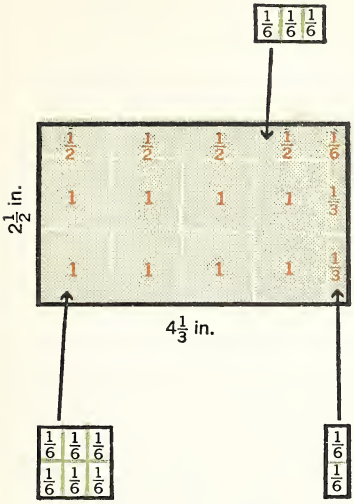
Write the product here.

Express $\frac{65}{6}$ as a mixed number.

$\frac{65}{6} = 10\frac{5}{6}$ ← $10\frac{5}{6}$ is another name for $\frac{65}{6}$.

C

The work you have just done shows that the area is $10\frac{5}{6}$ sq. in. Diagram C also shows that the area is $10\frac{5}{6}$ sq. in.



Each square inch is how many one sixths of a square inch?

Each $\frac{1}{2}$ square inch is how many one sixths of a square inch?

Each $\frac{1}{3}$ square inch is how many one sixths of a square inch?

Now use Diagram C to find how many one sixths of a square inch there are in the area. You can either count or add. Do not forget the $\frac{1}{6}$ sq. in. in the upper right corner.

Is the total area $\frac{65}{6}$ sq. in.?

How does the diagram show that the area can also be expressed as $10\frac{5}{6}$ sq. in.? The area of the paper is $10\frac{5}{6}$ sq. in.

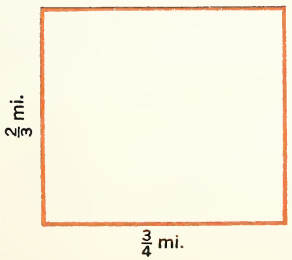
$4\frac{1}{3} \times 2\frac{1}{2} = 10\frac{5}{6}$

---

see

A

Mr. Jackson's farm is a rectangle $\frac{3}{4}$ mi. long and $\frac{2}{3}$ mi. wide. Find its area in square miles.



You can use the equation below to find the area.

$$\frac{\frac{3}{4}}{1} = \frac{t}{\frac{2}{3}}$$

When you find the numeral that replaces t, you will know the area.

Now turn the page.

When you use the ratio test, you get this equation.

$$t = \frac{3}{4} \times \frac{2}{3}$$

To solve the equation, you must find the product of $\frac{3}{4}$ and $\frac{2}{3}$.

Look at the work in the next column. Since the numbers are already expressed with fraction numerals, you do not need to use new names for them.

Multiply 2 by 3. The product is ▤.

$\frac{3}{4} \times \frac{2}{3} = \underline{6}$ ← Write 6 here.

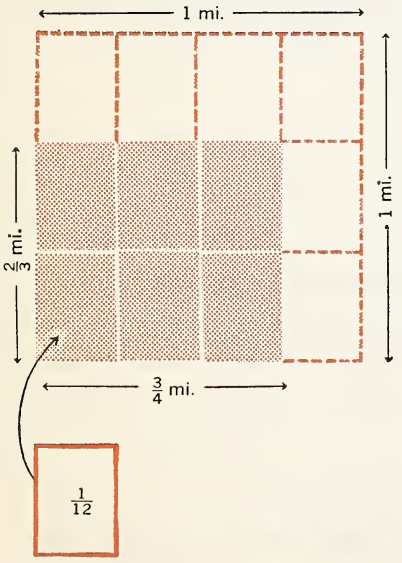
Multiply 3 by 4. The product is ▤.

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Write 12 here.

$\frac{6}{12} = \frac{1}{2}$

$\frac{1}{2}$ is another name for $\frac{6}{12}$.



B

Now look at Diagram B. The dotted lines help show the whole square mile of which Mr. Jackson's farm is a part.

One side of the square is marked off in fourths because the farm is $\frac{3}{4}$ mi. long. The other side is marked off in thirds because the farm is $\frac{2}{3}$ mi. wide. Now the whole square is marked off in twelfths.

You can see from the diagram that the area of Mr. Jackson's farm is $\frac{6}{12}$ sq. mi.

How does Diagram B show that you can also think of the area of the farm as $\frac{1}{2}$ sq. mi.?

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$, or $\frac{1}{2}$

The area of Mr. Jackson's farm is $\frac{1}{2}$ sq. mi.

think Jane said, "My story is $1\frac{1}{4}$ pages long." Ann said, "My story is twice as long as yours, Jane." Ann's story was how many pages long?

Is the equation below correct?

$$\frac{2}{1}=\frac{d}{1\frac{1}{4}}$$

How do you get the equation below?

$$d=2\times 1\frac{1}{4}$$

Before you multiply $1\frac{1}{4}$ by 2, express $1\frac{1}{4}$ and 2 with fraction numerals.

Are $\frac{2}{1}$ and $\frac{5}{4}$ correct names for 2 and $1\frac{1}{4}$?

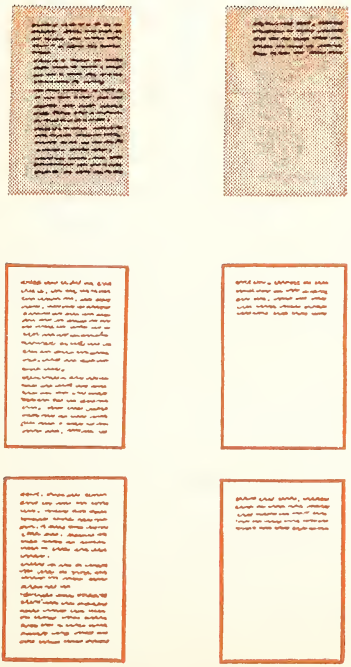
$$\frac{2}{1}\times\frac{5}{4}=\frac{10}{4}$$

How do you find the 10? The 4?

Now express $\frac{10}{4}$ as a mixed number.

$\frac{10}{4}=2\frac{\square}{2}$

Ann's story was how many pages long?



You can also use the diagram to find how many pages Ann used for her story.

For the 1 page that Jane used, Ann used ▩ pages.

For the $\frac{1}{4}$ page that Jane used, Ann used ▩ fourths of a page.

Count or add Ann's pages by fourths. Do you get $\frac{10}{4}$?

Can you use the diagram to show that Ann used $2\frac{1}{2}$ pages?

$2\times 1\frac{1}{4}=\square\frac{1}{2}$

think

A pipe that is $\frac{7}{8}$ ft. long weighs 1 lb. A longer pipe of the same kind weighs $2\frac{1}{2}$ lb. How long is this pipe?

Is the equation below correct?

$$\frac{\frac{7}{8}}{1} = \frac{n}{2\frac{1}{2}}$$

How is the equation below found?

$$n = \frac{7}{8} \times 2\frac{1}{2}$$

Before you multiply, for which number must you have a new name?

$$\frac{7}{8} \times \frac{5}{2} = \frac{35}{16}$$

← How is 35 found?

← How is 16 found?

Can $\frac{35}{16}$ be expressed as $2\frac{3}{16}$?

The pipe is how many feet long?

try

A Sue paid 28¢ for 1 lb. of plums. Jane bought $1\frac{1}{4}$ lb. at the same rate. How much did her plums cost?

$$\frac{28}{1} = \frac{a}{1\frac{1}{4}}$$

$$\frac{28}{1} = \frac{a}{1\frac{1}{4}}$$

$$a = 28 \times 1\frac{1}{4}$$

$$\frac{28}{1} \times \frac{5}{4} = \frac{140}{4} = 35$$

B Mr. Stone has a market garden. One day he set out $\frac{1}{4}$ row of tomato plants in 1 hr. At this rate, what part of a row had he planted in $\frac{1}{2}$ hr.?

$$\frac{\frac{1}{4}}{1} = \frac{s}{\frac{1}{2}}$$

$$\frac{\frac{1}{4}}{1} = \frac{s}{\frac{1}{2}}$$

$$\frac{1}{4} \times \frac{1}{2} = s$$

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

← What do you multiply to get 1?

C $8\frac{1}{10} \times 1\frac{2}{3} = d$

$$\frac{81}{10} \times \frac{5}{3} = \frac{405}{30} = 13\frac{15}{30} \text{ or } 13\frac{1}{2}$$

D $w = \frac{5}{16} \times 14$

$$\frac{5}{16} \times \frac{14}{1} = \frac{70}{16} = 4\frac{6}{16} \text{ or } 4\frac{3}{8}$$

← Why do you use $\frac{14}{1}$?

E $\frac{1}{5} \times 11\frac{1}{3} = r$

$$\frac{1}{5} \times \frac{34}{3} = \frac{34}{15} = 2\frac{4}{15}$$

F $b = 3\frac{3}{8} \times 4\frac{1}{6}$

$$\frac{27}{8} \times \frac{25}{6} = \frac{675}{48} = 14\frac{3}{48} \text{ or } 14\frac{1}{16}$$

do

A Mrs. Johnson bought a $2\frac{3}{4}$-lb. roast. Mrs. Price bought a roast that weighed $1\frac{1}{4}$ times as much as Mrs. Johnson's. Mrs. Price's roast weighed how many pounds?

$\frac{1\frac{1}{4}}{1} = \frac{k}{2\frac{3}{4}}$

$1\frac{1}{4} \times 2\frac{3}{4} = k$

B Kathy wrapped and tied 16 small packages that were all the same size. She used $\frac{3}{4}$ yd. of ribbon for each package. How many yards of ribbon did she use in all?

$\frac{\frac{3}{4}}{1} = \frac{v}{16}$

$v = \frac{3}{4} \times 16$

C $9\frac{1}{8} \times \frac{5}{6} = d$

D $m = \frac{14}{15} \times \frac{2}{3}$

E $7 \times 3\frac{1}{3} = a$

F $k = 9 \times \frac{1}{8}$

G $\frac{11}{24} \times \frac{3}{5} = w$

H $8\frac{1}{2} \times 1\frac{7}{10} = t$

I $s = 5\frac{1}{5} \times \frac{2}{3}$

J $n = \frac{15}{16} \times 4\frac{1}{5}$

K $2\frac{7}{10} \times 18 = b$

L $c = 32 \times \frac{1}{2}$

M $r = 6\frac{2}{3} \times 5\frac{9}{10}$

N $40 \times 1\frac{1}{8} = d$

O $h = 13\frac{1}{4} \times \frac{11}{12}$

P $\frac{5}{6} \times 12\frac{1}{2} = t$

Q $\frac{15}{32} \times \frac{4}{5} = a$

R $w = 8\frac{5}{9} \times 6\frac{2}{3}$

S $10 \times 8\frac{2}{5} = m$

T $b = \frac{13}{16} \times \frac{3}{4}$

U $1\frac{13}{15} \times 30 = r$

V $n = 7\frac{5}{6} \times 2\frac{3}{4}$

W $k = 7\frac{1}{4} \times \frac{9}{10}$

A $d = 5 \times \frac{1}{4}$

B $\frac{11}{12} \times 3\frac{7}{8} = t$

C $v = \frac{15}{16} \times \frac{2}{3}$

D $10 \times \frac{7}{12} = r$

E $m = 8\frac{1}{2} \times 1\frac{11}{15}$

F $14\frac{1}{6} \times 3\frac{3}{4} = s$

G $c = 2\frac{1}{3} \times \frac{7}{15}$

Now you know how to find the products of mixed numbers and of proper fractions.

Keeping skilful

A $11.003 + b = 18.074$

B $.67314 - .12869 = d$

C $22.7 + 30.5 + 11.9 = r$

D $5.06 + .84 + 1.22 + .49 = w$

E $92.001 - c = 91.745$

F $1387.5 - 879.6 = a$

G $.4 + .9 + .3 + .8 + .2 = n$

H $1.3725 - .8939 = b$

I $m + .7498 = 1.6302$

J $14.0002 - 6.9562 = h$

K $z - 52.618 = 76.992$

A 16 is ▒% of 64.

B 25% of 60 is ▒.

C 44 is 4% of ▒.

D 75% of 300 is ▒.

E 120 is ▒% of 40.

F 650 is 200% of ▒.

G 144 is ▒% of 100.

H 80% of 50 is ▒.

I 85 is 10% of ▒.

J 112 is ▒% of 140.

K 37% of 400 is ▒.

A $3879 \div d = 20$

B $65 \times 189 = c$

C $1740 \div m = 435$

D $319 \times r = 6699$

E $4867 \div 27 = s$

F $k \times 945 = 81270$

G $648 \times 327 = t$

H $11324 \div 165 = w$

I $d \times 472 = 148208$

J $7934 \div 68 = y$

K $230 \times a = 3680$

Using arithmetic

A Pete rides his bicycle to school. He rides 3 mi. a day on each round trip to and from school. How many miles does he ride in 5 days?

You can use an equation of ratios to solve Problem A.

$$\frac{3}{1}=\frac{k}{5}$$

Does the equation below also show how to solve Problem A?

$5 \times 3 = k$

When you use the ratio test, can you also write your equation as $k = 3 \times 5$ or as $3 \times 5 = k$? Does the same numeral replace k in all these equations? What numeral replaces k?

B Jim also rides his bicycle to school. He rides $2\frac{3}{4}$ mi. a day on each round trip to and from school. How many miles does he ride in 5 days?

Does the equation below show how to solve Problem B?

$$\frac{2\frac{3}{4}}{1}=\frac{a}{5}$$

Does the equation below show the result when you use the ratio test?

$a = 2\frac{3}{4} \times 5$

What numeral replaces a?

C Kathy bought a watermelon that weighed $16\frac{1}{4}$ lb. Ann bought one that weighed $\frac{1}{2}$ as much as Kathy's. How many pounds did Ann's watermelon weigh?

You know that Ann bought $\frac{1}{2}$ lb. of watermelon for each 1 lb. that Kathy bought. Does the equation below show how to solve Problem C?

$$\frac{\frac{1}{2}}{1}=\frac{W}{16\frac{1}{4}}$$

Does the equation below show the result when you use the ratio test?

$W = \frac{1}{2} \times 16\frac{1}{4}$

What numeral replaces w?

D Mrs. Brown has a flower bed that is $11\frac{1}{2}$ yd. long and $6\frac{1}{4}$ yd. wide. What is the area of the flower bed in square yards?

Write an equation of ratios for Problem D. Do you get the equation below by using the ratio test?

$11\frac{1}{2} \times 6\frac{1}{4} = d$

What numeral replaces d?

E May made 2 lb. of candy. Patty made $1\frac{1}{2}$ times as much candy as May. How much candy did Patty make?

If you are sure that you understand Problem E, write just the equation you would get from using the ratio test. If you are not sure, write an equation of ratios and use the ratio test.

F Ellen weighs $2\frac{1}{2}$ times as much as her little sister Beth. Beth weighs 20 lb. How much does Ellen weigh?
$2\frac{1}{2} \times 20 = n$

G Mr. Cook uses about $\frac{3}{4}$ T. of coal a week during the winter. He burns coal for about 18 weeks. About how much coal does he need for the winter?
$b = 18 \times \frac{3}{4}$

H Mr. Cook drove 117 mi. one day. Mr. Peters drove $\frac{2}{3}$ as far as Mr. Cook that day. Mr. Peters drove how many miles that day?

I Patty practised the piano for $\frac{3}{4}$ hr. each day for 4 days. How many hours had she practised at the end of the fourth day?

J A field is $\frac{1}{2}$ mi. long and $\frac{1}{4}$ mi. wide. The area of the field is what fraction of a square mile?

K There were 34 children at a party. For a game, each child was given $\frac{1}{2}$ of a sheet of paper. How many whole sheets of paper must have been cut in half?

L Bob's home is $3\frac{1}{2}$ blocks from the school. Andy's home is $2\frac{1}{2}$ times as far from school as Bob's. Andy's home is how many blocks from the school?

M Jack has two dogs. The larger one is fed a total of $\frac{3}{4}$ lb. of fresh meat each day. The smaller one is fed $\frac{1}{2}$ as much fresh meat as the larger one. How much fresh meat is the smaller dog fed each day?

N Mrs. Jackson used $10\frac{2}{3}$ yd. of cloth for bedroom curtains. She used about $\frac{5}{8}$ as much cloth for a chair cover. About how much cloth did she use for the chair cover?

Keeping skilful

A $6 \times \frac{3}{8} = m$
B $2\frac{1}{4} \times 8\frac{2}{3} = f$
C $\frac{1}{3} \times \frac{3}{5} = a$
D $4 \times 5\frac{1}{2} = w$
E $3\frac{5}{6} \times 3 = b$
F $\frac{7}{8} \times \frac{5}{12} = c$
G $5\frac{1}{2} \times 6\frac{2}{3} = n$
H $\frac{4}{9} \times 1\frac{1}{3} = z$

I $3\frac{7}{12} \times 1\frac{1}{8} = v$
J $\frac{1}{5} \times \frac{9}{16} = c$
K $6 \times \frac{1}{6} = d$
L $7\frac{9}{10} \times \frac{3}{4} = x$
M $5 \times 2\frac{4}{15} = g$
N $20 \times 3\frac{1}{2} = h$
O $\frac{3}{5} \times \frac{2}{5} = t$
P $2\frac{5}{6} \times \frac{1}{2} = k$

A $n + 3\frac{1}{2} = 6\frac{2}{3}$
B $\frac{15}{16} + \frac{7}{8} = d$
C $4\frac{3}{10} + s = 5\frac{1}{4}$
D $11 - y = 8\frac{5}{6}$
E $a - 13\frac{1}{3} = 24$
F $z + 6\frac{7}{12} = 15$
G $w - 4\frac{3}{10} = 1\frac{1}{3}$
H $6\frac{2}{3} - 1\frac{3}{5} = b$

I $14\frac{1}{5} - k = 11$
J $3\frac{5}{6} + \frac{7}{10} = m$
K $r - \frac{31}{32} = 1$
L $h + \frac{11}{16} = 4$
M $7\frac{5}{12} - r = 7\frac{1}{4}$
N $n - 1\frac{9}{10} = 3\frac{1}{4}$
O $8\frac{7}{15} + k = 19$
P $14 - 5\frac{3}{8} = c$

Side trip

A How do you read the decimal fraction numeral at the right? How do you read the sign that follows the numeral?

98.6°

B Another way to read the decimal fraction numeral 98.6 is to say, "Ninety-eight point six." Or you also may say, "Nine eight point six." In what three ways might you read 54.05?

C Read the decimal fraction numeral 8.183 in three ways.

D The numeral at the right represents a telephone number. When giving this number, would you say, "Thirteen thousand fifty-two," or would you say, "One three oh five two"? Why?

13052

E The numeral below represents a house number. Would you read this number as "Three thousand two hundred forty" or as "Three two four oh"? Why? Might you also read it as "Thirty-two forty"?

3240

F Why are house and telephone numbers often read differently from other numbers?

G The numeral below represents a year. You can read it as "Sixteen seven." How would you read the date 1867?

1607

H Three ways to write five tenths are shown here. The first two are common fraction numerals. The other is a decimal fraction numeral. You can read all three in the same way. Now write the numeral for each of five fractions in these three ways.

$\frac{5}{10}$ 5/10 .5

I You can also use the slant line to write a ratio, as shown at the right. Write five ratios, using the slant line.

25/60

J At the right are the abbreviations for "inch" or "inches" and "foot" or "feet."

3 in. 3″

$7\frac{1}{2}$ ft. $7\frac{1}{2}'$

Now write each dimension below, using these two abbreviations.

4 feet	$6\frac{1}{2}$ inches	1.1 inches
.7 foot	$8\frac{3}{16}$ inches	$10\frac{1}{2}$ feet
.03 inch	15 feet	$\frac{9}{10}$ inch

K The dimensions of a box are shown below in two ways. You read both of these in the same way.

18 ft. by 3 ft. by $1\frac{1}{2}$ ft.

$18' \times 3' \times 1\frac{1}{2}'$

Now write the dimensions of a board that is 6 ft. long and 12 in. wide. Use the sign that is read "by."

Thinking straight

A $\frac{2}{3} \times \frac{3}{2}$

B $\frac{15}{12} \times \frac{12}{15}$

C $\frac{6}{5} \times \frac{5}{6}$

D $\frac{100}{59} \times \frac{59}{100}$

E $8 \times \frac{1}{8}$

F $\frac{1}{6} \times 6$

A Look at Exercise A. Is the denominator of $\frac{2}{3}$ the same as the numerator of $\frac{3}{2}$? Is the numerator of $\frac{2}{3}$ the same as the denominator of $\frac{3}{2}$?

B Examine Exercises A, B, C, and D. In each exercise, is the numerator of one fraction numeral the same as the denominator of the other? Is the denominator of one fraction numeral the same as the numerator of the other?

C Look at Exercises E and F. Can you use $\frac{8}{1}$ as another name for 8? Can you use $\frac{6}{1}$ as another name for 6?

D Find the product in each exercise.

E When the product of two numbers is 1, one number is the reciprocal of the other. $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$. What is the reciprocal of $\frac{2}{3}$? What is the reciprocal of 8? Of $\frac{1}{8}$?

Write the reciprocal of each number below. Multiply each number by its reciprocal. Is each product 1?

A $\frac{15}{16}$ **B** 7 **C** $\frac{21}{11}$ **D** $\frac{5}{16}$ **E** $\frac{9}{10}$ **F** 24

Checking up

Test 1

A $\frac{5}{8} \times \frac{2}{3} = r$

B $\frac{1}{6} \times \frac{7}{12} = t$

C $\frac{3}{5} \times \frac{1}{2} = a$

D $\frac{6}{7} \times \frac{5}{9} = b$

E $\frac{15}{16} \times \frac{4}{5} = w$

F $\frac{31}{32} \times \frac{4}{5} = s$

G $\frac{21}{40} \times \frac{5}{7} = f$

Test 2

A $4 \times \frac{3}{4} = n$

B $\frac{1}{2} \times 10 = g$

C $\frac{7}{16} \times 12 = a$

D $8 \times \frac{9}{10} = r$

E $\frac{11}{12} \times 5 = k$

F $\frac{1}{10} \times 35 = w$

G $\frac{19}{32} \times 40 = d$

Test 3

A $10\frac{1}{2} \times 2 = b$

B $5\frac{2}{3} \times 9 = w$

C $2\frac{1}{9} \times 1\frac{1}{4} = d$

D $4\frac{1}{8} \times 3\frac{3}{5} = a$

E $12\frac{1}{3} \times 1\frac{1}{8} = v$

F $1\frac{1}{4} \times 3\frac{5}{6} = m$

G $18 \times 7\frac{5}{6} = t$

Test 4

A $3\frac{1}{3} \times \frac{3}{10} = f$

B $\frac{7}{10} \times \frac{3}{5} = t$

C $6\frac{4}{5} \times 1\frac{5}{12} = a$

D $\frac{11}{12} \times 36 = e$

E $\frac{9}{16} \times \frac{4}{5} = k$

F $21 \times 1\frac{1}{7} = z$

G $10\frac{5}{6} \times 1\frac{5}{7} = h$

Thinking straight

A Mrs. Long used 24 cups of grapes to make grape juice. She used 2 cups of grapes for each quart of juice. How many quarts of juice did she make?

Make an equation of ratios for Problem A.

When you use the ratio test, do you get the equation shown below?

$2d = 24$

To find the numeral that replaces d, you divide 24 by ▭. The expressions at the right show two ways to write the division.

$24 \div 2$

$\frac{24}{2}$

B Mrs. Parks used 21 cups of grapes to make grape juice. She used $1\frac{3}{4}$ cups of grapes for each quart of juice. How many quarts of juice did she make?

Make an equation of ratios for Problem B.

How do you get the equation below?

$1\frac{3}{4}w = 21$

To find the numeral that replaces w, you divide ▭ by $1\frac{3}{4}$. The expression at the right shows one way to write the division.

$21 \div 1\frac{3}{4}$

Another way to write the division is also shown at the right.

$\frac{21}{1\frac{3}{4}}$

C Mr. Cook had $1\frac{1}{2}$ lb. of sweet corn seed. He planted $\frac{1}{2}$ lb. of the seed every other week until he had used all of it. How many plantings had Mr. Cook made?

Make an equation of ratios for Problem C.

How is the equation below found?

$\frac{1}{2}x = 1\frac{1}{2}$

To find the numeral that replaces x, divide $1\frac{1}{2}$ by ▭. You may write the division in either of the two ways shown below.

$1\frac{1}{2} \div \frac{1}{2}$ $\qquad$ $\frac{1\frac{1}{2}}{\frac{1}{2}}$

D Mr. Henry sawed a 14-foot board into 3.5-foot lengths. How many of the shorter boards did he have?

Make an equation of ratios for Problem D.

How is the equation below found?

$3.5a = 14$

To find the numeral that replaces a, you divide ▭ by 3.5. You may write the division in either of these two ways.

$14 \div 3.5$ $\qquad$ $\frac{14}{3.5}$

For each problem that follows, write two expressions to show that you

divide to solve the problem. You do not need to compute. You will learn how to do the computation later.

E A gardener had $\frac{7}{8}$ lb. of prize seed. He put this seed into packages with $\frac{1}{8}$ lb. of seed in each package. How many packages did he make?

F 14 pt. of cream was put into $\frac{1}{2}$-pt. bottles. How many bottles were used?

G When Patty knitted a scarf, she found that she could knit about $\frac{3}{4}$ inch in 1 hr. About how many hours did it take her to knit a scarf 30 in. long?

H Mr. Peterson spent \$3.87 for wallpaper that was priced at \$1.29 per roll. How many rolls of this wallpaper did Mr. Peterson buy?

I A piece of metal 8.5 in. wide was cut into 13 strips of equal width. How wide was each strip?

J A man earned \$105.40 for 42.5 hr. of work. How much was he paid per hour?

K Patty spends $5\frac{1}{4}$ hr. in school each day. Each class period is $\frac{3}{4}$ hr. long. There is time for how many classes in each school day?

L 180 fl. oz. of liquid was put into bottles that held $4\frac{1}{2}$ fl. oz. each. How many bottles were used?

M A piece of land 484 ft. wide was marked off into lots. Each lot was 60.5 ft. wide. How many lots were in the piece of land?

N Mrs. Peterson has a 4-lb. roast. If she allows $\frac{1}{3}$ lb. of meat per serving, how many servings will she get?

For each exercise below, write the division in another way.

A $1.5 \div 5$

B $\frac{15}{16} \div \frac{3}{4}$

C $\dfrac{12\frac{1}{2}}{1\frac{1}{3}}$

D $20 \div 6\frac{7}{10}$

E $\dfrac{17\frac{1}{8}}{5}$

F $\frac{1}{2} \div \frac{1}{4}$

G $1.7 \div 1.6$

H $\dfrac{18}{\frac{1}{2}}$

I $\frac{19}{20} \div \frac{4}{5}$

J $\dfrac{21.6}{4.3}$

Keeping skilful

A 45 is ▭% of 15.

B 2% of 1300 is ▭.

C 59 is 100% of ▭.

D 17% of 400 is ▭.

E 23 is 50% of ▭.

F 81 is ▭% of 2700.

G 42 is ▭% of 35.

H 14% of 1000 is ▭.

I 72 is 30% of ▭.

J 28 is ▭% of 350.

K 1% of 4900 is ▭.

L 155% of 200 is ▭.

M 486 is ▭% of 648.

N 60 is 5% of ▭.

O 161 is 23% of ▭.

Learning how

Now you will learn how to divide when the numerals represent proper fractions or mixed numbers.

see

A The equation below was made for a problem.

$5\frac{1}{2} \div \frac{3}{4} = w$

The work below shows how to divide $5\frac{1}{2}$ by $\frac{3}{4}$. Use $\frac{11}{2}$ as another name for $5\frac{1}{2}$.

$\frac{\frac{11}{2}}{\frac{3}{4}}$ Express the division in this way.

$\frac{\frac{11}{2}}{\frac{3}{4} \times \frac{4}{3}}$ ← First you need to get 1 for the divisor. You multiply $\frac{3}{4}$ by $\frac{4}{3}$ to get 1.

$\frac{\frac{11}{2} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}}$ ← $\frac{3}{4}$ has been multiplied by $\frac{4}{3}$. So you must also multiply $\frac{11}{2}$ by $\frac{4}{3}$.

$\frac{\frac{11}{2} \times \frac{4}{3}}{1}$ ← You do not need to write the divisor when it is 1.

$\frac{11}{2} \times \frac{4}{3}$ ← So now you can write the computation in this way.

$\frac{11}{2} \times \frac{4}{3} = \frac{44}{6} = 7\frac{1}{3}$ You found $7\frac{1}{3}$ by multiplying $\frac{4}{3}$ by $\frac{11}{2}$.

$5\frac{1}{2} \div \frac{3}{4} = 7\frac{1}{3}$

B The equation below was also made for a problem.

$\frac{7}{8} \div 4 = d$

You can find the quotient in the way shown below.

$\frac{\frac{7}{8}}{\frac{4}{1}}$ ← Use $\frac{4}{1}$ as another name for 4.

$\frac{\frac{7}{8}}{\frac{4}{1} \times \frac{1}{4}}$ ← To get 1 for the divisor, multiply $\frac{4}{1}$ by $\frac{1}{4}$.

$\frac{\frac{7}{8} \times \frac{1}{4}}{\frac{4}{1} \times \frac{1}{4}}$ ← You multiply $\frac{7}{8}$ by $\frac{1}{4}$ because $\frac{4}{1}$ was multiplied by $\frac{1}{4}$.

$\frac{\frac{7}{8} \times \frac{1}{4}}{1}$ ← You do not need to write the divisor when it is 1.

$\frac{7}{8} \times \frac{1}{4}$ ← So now you can write the computation in this way.

$\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$ How is this numeral found?

$\frac{7}{8} \div 4 = \frac{7}{32}$

think

A $\frac{7}{10} \div 4\frac{1}{2} = a$

$\frac{7}{10}$

$\frac{9}{2}$ ← Is $\frac{9}{2}$ another name for $4\frac{1}{2}$?

$\frac{7}{10}$

$\frac{9}{2} \times \frac{2}{9}$ ← What divisor do you want to get? Why do you multiply $\frac{9}{2}$ by $\frac{2}{9}$?

$\frac{7}{10} \times \frac{2}{9}$ ← Why is $\frac{7}{10}$ also multiplied by $\frac{2}{9}$?

1 ← How is this 1 found? Can you now write the work without showing the 1?

$\frac{7}{10} \times \frac{2}{9} = \frac{14}{90}$ or $\frac{7}{45}$ ← How is $\frac{7}{45}$ found?

$\frac{7}{10} \div 4\frac{1}{2} = \frac{7}{45}$

B $\frac{5}{6} \div \frac{1}{3} = b$

$\frac{5}{6}$

$\frac{1}{3}$ ← Is it necessary to use another name for either $\frac{5}{6}$ or $\frac{1}{3}$?

$\frac{5}{6}$

$\frac{1}{3} \times \frac{3}{1}$ ← Why do you multiply $\frac{1}{3}$ by its reciprocal, $\frac{3}{1}$?

$\frac{5}{6} \times \frac{3}{1}$ ← Why is $\frac{5}{6}$ also multiplied by $\frac{3}{1}$?

1 ← You can rewrite the work without the 1.

$\frac{5}{6} \times \frac{3}{1} = \frac{15}{6} = 2\frac{1}{2}$ ← How is this numeral found?

$\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$

try

A $9 \div \frac{3}{4} = t$

$\frac{\frac{9}{1} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{\frac{9}{1} \times \frac{4}{3}}{1}$

$\frac{9}{1} \times \frac{4}{3} = \frac{36}{3}$ or 12

B $6\frac{1}{4} \div 3\frac{2}{5} = r$

$\frac{\frac{25}{4} \times \frac{5}{17}}{\frac{17}{5} \times \frac{5}{17}}$

$\frac{25}{4} \times \frac{5}{17} = \frac{125}{68}$ or $1\frac{57}{68}$

C $5\frac{7}{12} \div 3 = m$

$\frac{\frac{67}{12} \times \frac{1}{3}}{\frac{3}{1} \times \frac{1}{3}}$

$\frac{67}{12} \times \frac{1}{3} = \frac{67}{36}$ or $1\frac{31}{36}$

D $27 \div 5\frac{1}{10} = w$

$\frac{\frac{27}{1} \times \frac{10}{51}}{\frac{51}{10} \times \frac{10}{51}}$

$\frac{27}{1} \times \frac{10}{51} = \frac{270}{51}$ or $5\frac{15}{51}$ or $5\frac{5}{17}$

E $\frac{4}{5} \div \frac{5}{8} = b$

$\frac{\frac{4}{5} \times \frac{8}{5}}{\frac{5}{8} \times \frac{8}{5}}$

$\frac{4}{5} \times \frac{8}{5} = \frac{32}{25}$ or $1\frac{7}{25}$

F $\frac{24}{25} \div 8 = d$

$\frac{\frac{24}{25} \times \frac{1}{8}}{\frac{8}{1} \times \frac{1}{8}}$

$\frac{24}{25} \times \frac{1}{8} = \frac{24}{200}$ or $\frac{3}{25}$

G $40 \div \frac{5}{6} = k$

$\frac{\frac{40}{1} \times \frac{6}{5}}{\frac{5}{6} \times \frac{6}{5}}$

$\frac{40}{1} \times \frac{6}{5} = \frac{240}{5}$ or 48

H $\frac{14}{15} \div \frac{2}{5} = a$

$\frac{\frac{14}{15} \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}}$

$\frac{14}{15} \times \frac{5}{2} = \frac{70}{30}$ or $2\frac{10}{30}$ or $2\frac{1}{3}$

I $\frac{9}{16} \div 3 = z$

$\frac{\frac{9}{16} \times \frac{1}{3}}{\frac{3}{1} \times \frac{1}{3}}$

$\frac{9}{16} \times \frac{1}{3} = \frac{9}{48}$ or $\frac{3}{16}$

J $8 \div \frac{1}{10} = c$

$\frac{\frac{8}{1} \times \frac{10}{1}}{\frac{1}{10} \times \frac{10}{1}}$

$\frac{8}{1} \times \frac{10}{1} = \frac{80}{1}$ or 80

do

Show your answers in lowest terms.

A $\frac{3}{4} \div \frac{1}{8} = a$

B $6\frac{2}{3} \div \frac{3}{8} = r$

C $10 \div \frac{5}{6} = t$

D $12\frac{5}{16} \div 4\frac{1}{2} = k$

E $\frac{7}{16} \div 2 = m$

F $9\frac{1}{8} \div 4\frac{5}{6} = s$

G $\frac{7}{32} \div \frac{3}{64} = b$

H $\frac{8}{15} \div \frac{1}{3} = e$

I $7 \div 2\frac{1}{3} = z$

J $\frac{13}{20} \div 5 = f$

K $11\frac{2}{3} \div \frac{1}{3} = w$

L $4 \div \frac{7}{8} = d$

M $5\frac{1}{3} \div 4 = s$

N $9\frac{3}{4} \div \frac{4}{5} = v$

O $16\frac{1}{4} \div 13 = n$

P $\frac{21}{32} \div 1\frac{3}{4} = a$

Q $\frac{11}{12} \div \frac{3}{4} = r$

R $9 \div 4\frac{3}{8} = h$

S $2\frac{1}{4} \div 3\frac{7}{8} = c$

T $12 \div 7\frac{1}{3} = n$

A $5\frac{4}{5} \div \frac{1}{6} = c$

B $7 \div \frac{11}{12} = r$

C $3\frac{1}{3} \div 1\frac{1}{2} = w$

D $\frac{13}{16} \div 2\frac{3}{4} = a$

E $\frac{11}{24} \div \frac{3}{8} = t$

F $\frac{7}{8} \div 7 = d$

G $9\frac{5}{6} \div 3 = m$

H $8 \div \frac{1}{4} = e$

I $\frac{9}{10} \div \frac{1}{6} = k$

J $15 \div 1\frac{2}{3} = g$

K $10 \div 4\frac{1}{2} = b$

L $6\frac{9}{10} \div 13 = a$

M $\frac{9}{16} \div 3 = r$

N $\frac{7}{12} \div \frac{7}{24} = m$

O $\frac{19}{32} \div 1\frac{3}{8} = w$

P $3\frac{7}{16} \div 2\frac{4}{5} = v$

Q $14 \div \frac{7}{8} = c$

R $3\frac{1}{8} \div 20 = t$

S $1\frac{5}{16} \div 2\frac{1}{2} = n$

T $4\frac{2}{3} \div 7 = y$

Now you should be able to divide when the numerals represent fractions.

Thinking straight

A $4\frac{1}{4}+0=4\frac{1}{4}$

B $0+\frac{2}{3}=\frac{2}{3}$

C $\frac{11}{12}-0=\frac{11}{12}$

D $\frac{7}{8}\times 1=\frac{7}{8}$

E $1\times 4\frac{2}{3}=4\frac{2}{3}$

F $\frac{29}{32}\div 1=\frac{29}{32}$

G $1\frac{2}{5}\times 0=0$

H $0\times\frac{11}{16}=0$

I $0\div\frac{5}{6}=0$

J $\frac{7}{12}-\frac{7}{12}=0$

K $\frac{15}{16}\div\frac{15}{16}=1$

L $\frac{3}{8}\times\frac{8}{3}=1$

For each example at the left, find a statement below that belongs with it.

A When 0 is added to, or subtracted from, a number, the answer is the number you started with.

B When a number is subtracted from the same number, the answer is 0.

C When a number other than zero is divided by the same number, the answer is 1.

D When a number is multiplied or divided by 1, the answer is the number you started with.

E The answer is 0 when a number is multiplied by 0.

F When 0 is divided by a number, the quotient is 0.

G When a number is multiplied by its reciprocal, the product is 1.

Keeping skilful

A 24 is ▭% of 96.

B 15% of 120 is ▭.

C 60 is 2% of ▭.

D 140% of 25 is ▭.

E 114 is 120% of ▭.

F 75 is 30% of ▭.

G 48 is ▭% of 16.

H 63% of 200 is ▭.

I 1% of 300 is ▭.

J 28 is ▭% of 112.

K 150 is 20% of ▭.

L 40% of 350 is ▭.

M 19 is ▭% of 38.

A $w\times 207=11592$

B $64513\div 177=t$

C $86\times k=4250$

D $136\div m=59$

E $25850\div b=94$

F $n\times 402=25326$

G $94861\div 29=r$

H $73\times c=4258$

I $6276\times 53=m$

J $9156\div 82=v$

K $44306\div 178=t$

L $17\times e=1156$

M $28030\div z=400$

A $\frac{7}{16}\times 11\frac{2}{3}=m$

B $4\frac{4}{5}\times 5\frac{1}{2}=d$

C $3\frac{9}{10}\times 1\frac{2}{3}=w$

D $\frac{1}{16}\times\frac{2}{3}=z$

E $20\times\frac{7}{10}=a$

F $4\frac{1}{6}\times\frac{1}{5}=n$

G $\frac{15}{16}\times 40=v$

H $\frac{5}{8}\times\frac{3}{5}=k$

I $1\frac{4}{5}\times 11=b$

J $14\times 3\frac{3}{4}=g$

K $6\frac{1}{2}\times 15=f$

Using arithmetic

A Mrs. Peters used 16 yd. of cloth to make curtains. She used 2 yd. of the cloth for each curtain. How many curtains did she make?

You can write an equation of ratios for Problem A. Is the equation below correct?

$$\frac{2}{1}=\frac{16}{n}$$

When you use the ratio test, can you get the equation shown below?

$$2n=16$$

To find the numeral that replaces n, you divide 16 by ▒.
What numeral replaces n?

B Mrs. Peters used $4\frac{1}{2}$ yd. of cloth to make kitchen towels. She used $\frac{3}{4}$ yd. of cloth for each towel. How many towels did she make?

Can you write the equation of ratios below for Problem B?

$$\frac{\frac{3}{4}}{1}=\frac{4\frac{1}{2}}{w}$$

How do you get the equation below?

$$\tfrac{3}{4}w=4\tfrac{1}{2}$$

To find the numeral that replaces w, you divide $4\frac{1}{2}$ by ▒.
What numeral replaces w?

C George said, "I have read $1\frac{1}{2}$ times as many pages as you have, Jim. I have read 57 pages." How many pages had Jim read?

You know that George read $1\frac{1}{2}$ pages for each page that Jim read. Can you write this equation for Problem C?

$$\frac{1\frac{1}{2}}{1}=\frac{57}{t}$$

Does the equation below show the result when you use the ratio test?

$$1\tfrac{1}{2}t=57$$

To find the numeral that replaces t, you divide 57 by ▒.
What numeral replaces t?

D The area of a rectangular garden is 360 sq. yd. Its length is 90 yd. What is its width?

Can you write the equation below for Problem D?

$$\frac{360}{d}=\frac{90}{1}$$

Does the equation below show the result when you use the ratio test?

$$90d=360$$

To find the numeral that replaces d, you divide 360 by ▒.
What numeral replaces d?

When you solve each problem below, first write an equation of ratios.

E Jim gives his ducks about $\frac{3}{8}$ peck of feed each day. One day he noticed

that he had only about $\frac{3}{4}$ peck of feed left. This amount would make about how many feedings?

F John and Bill planned a $6\frac{2}{3}$-mile hike one Saturday. They expected to walk at a rate of about $2\frac{1}{2}$ mi. per hour. About how long should the hike have taken?

G Mr. Brown needed some poles for his tomato plants. He had some poles $3\frac{1}{2}$ yd. long. He cut each of these poles into 3 equal shorter poles. How long was each of these shorter poles?

H When Mrs. Wood made costumes for a school play, she used $2\frac{3}{8}$ yd. of cloth for each costume. She used $14\frac{1}{4}$ yd. of cloth in all. How many costumes did she make?

I Mr. Walker's pasture fence is $\frac{5}{6}$ mi. long. Mr. Baker's pasture fence is $\frac{2}{3}$ mi. long. Mr. Walker's fence is how many times as long as Mr. Baker's?

J The area of a rectangular farm is $1\frac{1}{3}$ sq. mi. The width of this farm is $\frac{3}{4}$ mi. What is its length?

K When Mrs. Castle makes an apple pie, she uses $\frac{2}{3}$ cup of lard for the crust. There are 2 cups of lard in 1 pound. She can make crust for how many apple pies with 1 pound of lard?

L Mary wanted to buy $1\frac{1}{2}$ lb. of salted peanuts. At the store, the peanuts came in $\frac{3}{8}$-pound cans. How many cans should Mary have bought?

M Mrs. Wood paid \$1.47 for $1\frac{1}{2}$ lb. of fudge. She paid how much per pound for the fudge?

N Mrs. Brown paid \$3.25 for $\frac{1}{8}$ oz. of perfume. At the same rate, how much would 1 oz. of this perfume cost?

Checking up

Test 1

A $\frac{3}{8} \div \frac{1}{4} = x$

B $\frac{2}{5} \div \frac{7}{10} = t$

C $\frac{5}{6} \div \frac{5}{6} = a$

D $\frac{11}{16} \div \frac{3}{4} = d$

E $\frac{9}{20} \div \frac{1}{2} = w$

F $\frac{7}{8} \div \frac{7}{12} = b$

G $\frac{3}{5} \div \frac{3}{10} = c$

Test 2

A $4\frac{1}{3} \div 1\frac{1}{2} = r$

B $\frac{5}{6} \div 2\frac{3}{4} = x$

C $\frac{5}{8} \div 4\frac{3}{8} = s$

D $10\frac{1}{2} \div \frac{1}{2} = b$

E $6\frac{3}{4} \div 1\frac{1}{3} = n$

F $\frac{1}{2} \div 6\frac{2}{3} = d$

G $9\frac{3}{5} \div 1\frac{1}{5} = m$

Test 3

A $\frac{7}{20} \div 2 = s$

B $11\frac{2}{3} \div 7 = a$

C $5\frac{5}{6} \div 3 = c$

D $\frac{4}{5} \div 7 = r$

E $\frac{1}{3} \div 9 = w$

F $9\frac{1}{4} \div 6 = k$

G $5\frac{3}{5} \div 4 = h$

Test 4

A $6 \div \frac{2}{3} = m$

B $5 \div 4\frac{1}{4} = t$

C $9 \div 9\frac{3}{4} = d$

D $4 \div \frac{8}{15} = f$

E $10 \div 3\frac{1}{3} = z$

F $14 \div 7\frac{3}{4} = g$

G $36 \div \frac{1}{4} = x$

Thinking straight

A $\frac{7}{12} \times \frac{8}{15} = \frac{56}{180}$ or $\frac{14}{45}$

B $\frac{7 \times 8}{12 \times 15} = \frac{7 \times \overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}} \times 15} = \frac{14}{45}$

C $\frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{2}{\cancel{8}}}{15} = \frac{14}{45}$

D $2\frac{1}{10} \times \frac{20}{49} = \frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{10}}} \times \frac{\overset{2}{\cancel{20}}}{\underset{7}{\cancel{49}}} = \frac{6}{7}$

A In Example A, how is the 56 found? How is the 180 found? In lowest terms, $\frac{56}{180}$ is $\frac{\blacksquare}{\blacksquare}$.

B In Example B, the 7 and the 8 are written with the times sign between them above the line. How are the 12 and the 15 written?

C In the work in red in Example B, both 12 and 8 are divided by 4. Why is 3 written below 12? Why is 2 written above 8? The 12 and the 8 are crossed out to show that they are replaced with 3 and $\blacksquare$.

D How is the 14 in $\frac{14}{45}$ found? How is the 45 in $\frac{14}{45}$ found?

E Example C shows that you can use the short cut without rewriting the work. Why are the 12 and the 8 crossed out? How is the product, $\frac{14}{45}$, found? Is $\frac{14}{45}$ in lowest terms?

F In Example D, the short cut is used twice. 20 and 10 are crossed out and replaced with 2 and 1 because both 20 and 10 have been divided by $\blacksquare$. Why are 21 and 49 crossed out and replaced with 3 and 7?

G Now multiply $\frac{20}{49}$ by $\frac{21}{10}$ in the way shown in Example A. The product, in lowest terms, is $\frac{\blacksquare}{7}$. Which way of finding the product is more convenient, the way shown in Example A or the way shown in Example D?

In the exercises below, you may use the short cut if you wish. Notice that sometimes you cannot use the short cut.

A $\frac{7}{10} \times \frac{5}{6} = d$

B $4\frac{1}{12} \times \frac{6}{7} = r$

C $\frac{1}{3} \times \frac{7}{10} = w$

D $3\frac{11}{16} \times 3\frac{1}{3} = n$

E $\frac{3}{4} \times \frac{3}{10} = e$

F $\frac{33}{10} \times \frac{5}{3} = a$

G $\frac{14}{15} \times \frac{1}{2} = k$

H $\frac{9}{16} \times 13\frac{1}{3} = m$

I $\frac{21}{5} \times \frac{1}{3} = t$

J $\frac{10}{21} \times \frac{15}{16} = b$

Keeping skilful

A $24\frac{3}{4} \times 86 = a$

$$\begin{array}{r} 86 \\ 24\frac{3}{4} \\ \hline 64\frac{1}{2} \\ 344 \\ 1720 \\ \hline 2128\frac{1}{2} \end{array}$$

← Multiply 86 by $\frac{3}{4}$.
← Multiply 86 by 4.
← Multiply 86 by 20.

B $38 \times 342\frac{5}{6} = w$

$$\begin{array}{r} 342\frac{5}{6} \\ 38 \\ \hline 31\frac{2}{3} \\ 2736 \\ 10260 \\ \hline 13027\frac{2}{3} \end{array}$$

← Multiply $\frac{5}{6}$ by 38.
← Multiply 342 by 8.
← Multiply 342 by 30.

C $18\frac{1}{4} \times 200 = d$
D $30\frac{1}{2} \times 76 = m$
E $50 \times 12\frac{1}{10} = t$
F $60\frac{2}{3} \times 155 = a$
G $16 \times 112\frac{1}{2} = s$
H $24\frac{3}{8} \times 160 = n$
I $28 \times 261\frac{1}{3} = g$
J $14\frac{1}{12} \times 156 = k$
K $60\frac{2}{3} \times 145 = r$
L $62\frac{7}{8} \times 360 = t$
M $100 \times 18\frac{3}{5} = b$
N $44\frac{1}{4} \times 608 = w$

A $6 \div 18 = t$

You can also write the equation this way. → $\frac{6}{18} = t$

$\frac{6}{1} \div \frac{18}{1}$ ← You can write the division in either of these two ways. → $\frac{\frac{6}{1}}{\frac{18}{1}}$

$$\frac{\frac{6}{1} \times \frac{1}{18}}{\frac{18}{1} \times \frac{1}{18}}$$

$\frac{6}{1} \times \frac{1}{18} = \frac{6}{18}$ or $\frac{1}{3}$

Look at the ways in which 6 divided by 18 is written above. Is this fraction numeral like one of them?

B $20 \div 25 = m$

$\frac{20}{25}$ ← When you divide 20 by 25, the answer is $\frac{20}{25}$. What is the answer in lowest terms?

A $8 \div 13 = v$
B $4 \div 26 = a$
C $3 \div 12\frac{1}{4} = d$
D $15 \div 25 = r$
E $2 \div 4\frac{2}{3} = x$
F $10 \div 130 = b$
G $16 \div 18 = n$
H $20 \div 24 = c$
I $5 \div 5\frac{5}{6} = f$
J $80 \div 100 = y$
K $64 \div 128 = w$
L $18 \div 18\frac{3}{4} = t$
M $16 \div 64 = s$
N $50 \div 125 = h$
O $75 \div 80\frac{5}{6} = g$
P $26 \div 39 = k$
Q $90 \div 105 = e$
R $12\frac{1}{2} \div 25 = n$

Checking up

Test 1

A Mr. Clark bought $\frac{1}{2}$ lb. of pole beans for seed. The package stated that 1 lb. would plant 110 hills of beans. How many hills should Mr. Clark plant with the seeds he bought?

B Mr. Cook paid $3 for cabbage seed priced at $.75 per ounce. How many ounces of cabbage seed did he buy?

C In the fall, Mr. Cook sold 320 heads of cabbage. The total weight of the cabbage was 1680 lb. What was the average weight per head?

D Mrs. Cook paid $5.25 for 7 packages of flower seed. Each package cost the same amount. What was the cost of each package?

E Mrs. Cook had a rectangular flower garden that was $20\frac{1}{2}$ ft. long and 16 ft. wide. What was its area in square feet?

F Mr. Clark planned to plant $\frac{1}{3}$ acre of sweet corn. He read that 12 lb. of seed would plant 1 acre. How many pounds of seed should Mr. Clark have bought?

G Mr. Clark bought lawn seed for his 3300 sq. ft. of lawn. He planned to use it at the rate of 1 lb. for every 220 sq. ft. of lawn. How much lawn seed should he have bought?

H When Mrs. Cook bought some plants, she was told that about 80% of the plants would have double flowers. About how many of her 60 plants could be expected to have double flowers?

I Mr. Cook bought 1 lb. of melon seed priced at $\frac{1}{4}$ lb. for $1.35. How much did he pay for the 1 lb. of seed?

J Mr. Clark bought 12 oz. of weed killer to use on his 3300 sq. ft. of lawn. His neighbor, Mr. Cook, said, "I have about $1\frac{1}{2}$ times as much lawn as you have. So I will need $1\frac{1}{2}$ times as much weed killer as you need." How much weed killer did Mr. Cook need?

K Mr. Cook bought 500 strawberry plants at $32.50 per 250 plants. How much did he pay for all the plants?

L One kind of flower seed that Mrs. Cook wanted was sold in $\frac{1}{4}$ ounce packages. She wanted $1\frac{1}{2}$ oz. of the seed. How many packages should she have bought?

M Mr. Clark ordered plants and seeds at a total cost of $26. He had to pay an extra 10% of this amount for packing and postage. How much money did he have to pay for packing and postage?

N Mrs. Cook's tallest sunflower plant was $8\frac{1}{4}$ ft. high. This plant was how many times as tall as the shortest plant, which was $5\frac{1}{2}$ ft. high?

O Mrs. Cook says that last spring she had 130 red tulips. These tulips were about 65% of all her tulips. About how many tulips did Mrs. Cook have?

P Mrs. Cook bought $\frac{1}{8}$ oz. of pansy seed. Mrs. Clark bought $\frac{1}{16}$ oz. of the same kind of pansy seed. Mrs. Cook bought how many times as much pansy seed as Mrs. Clark?

Q Mrs. Cook paid \$3 for $\frac{1}{8}$ oz. of pansy seed. At this rate, how much would 1 oz. of the seed cost?

Test 2

A Can the sum of two proper fractions be less than 1?

B Can the sum of two proper fractions be 1?

C Can the sum of two proper fractions be more than 1?

D Can the sum of two proper fractions be 2?

E Suppose you have six different fraction numerals, each of which has 1 for the numerator. Which fraction numeral represents the largest fraction, the one with the largest denominator or the one with the smallest denominator?

F If each of two fractions is greater than $\frac{1}{2}$, can their sum be equal to 1?

G If each of two fractions is greater than $\frac{1}{2}$, can their sum be less than 1?

H If each of two fractions is greater than $\frac{1}{2}$, is their sum greater than 1?

I When you subtract a proper fraction from a whole number, is the answer ever a whole number?

J When a mixed number is subtracted from a whole number, is the answer always a mixed number?

K If you multiply the number in each term of a fraction numeral by the same number, will the new fraction numeral be equal to, greater than, or less than the fraction numeral you began with?

L Which sentence below has more whole numbers in its solution set?

$15 > a$ $15 < a$

M Which sign or signs below can you use to make a true statement of the sentence $13 \times 3 \frown\!\!\!\!\frown 2 \times 26$?

$<$ $>$ $=$ $\neq$

N Of the ratios given below, which are equal to 4 over 9?

$\frac{8}{32}$ $\frac{1\frac{1}{3}}{3}$ $\frac{6}{11}$ $\frac{12}{27}$ $\frac{1}{2\frac{1}{4}}$ $\frac{16}{27}$

Test 3

Find the sum.

A .014, 3.686, 47.740
B 16.7, 43.2, 96.8, 12.6
C .3372, .8657, 7.4839
D 99.99, 4.65, 104.61, .85
E 1.0003, .9797, .6409
F 86.915, .742, 9.326
G 5.7777, .2223, .0006
H 48.3, 92.0, 76.4, 18.7
I 24.76, 30.05, 63.99, 89.96

Test 4

A $62.4 - x = 17.2$
B $m + .748 = 1.000$
C $39.051 - 4.626 = w$
D $d - 15.83 = 62.40$
E $.1716 + b = 2.0005$
F $g - 2.8 = 1.9$
G $11.006 - f = 4.835$
H $n + 6.78 = 111.91$
I $.8396 + .0706 = t$
J $42.1034 - k = 8.6485$

Test 5

A $m \times 87 = 3132$
B $9170 \div 189 = w$
C $593 \times d = 38605$
D $1640 \div k = 92$
E $746 \times 1862 = b$
F $a \times 49 = 6776$
G $2763 \div n = 88$
H $94 \times 385 = d$
I $r \times 63 = 50652$
J $8664 \div 49 = y$

Test 6

Find the sum.

A $6\frac{1}{2}$, $1\frac{7}{8}$
B $8\frac{1}{4}$, $7\frac{5}{12}$, $10\frac{1}{3}$
C $60\frac{1}{16}$, $120\frac{3}{8}$, 11
D $14\frac{1}{5}$, $2\frac{3}{4}$, $9\frac{4}{5}$, $8\frac{1}{4}$
E $\frac{9}{10}$, $\frac{3}{5}$, $4\frac{5}{6}$, $2\frac{2}{3}$
F $3\frac{7}{12}$, $2\frac{1}{8}$, $\frac{3}{4}$, $5\frac{1}{2}$
G $28\frac{3}{5}$, $11\frac{9}{20}$, $46\frac{7}{10}$
H $17\frac{9}{16}$, $5\frac{3}{4}$, $\frac{11}{12}$, $\frac{5}{8}$

Test 7

A $10\frac{7}{20} - 8\frac{9}{10} = x$
B $14\frac{1}{3} - 6 = t$
C $\frac{15}{32} - \frac{3}{8} = a$
D $60 - 49\frac{5}{6} = r$
E $15\frac{11}{12} - \frac{7}{8} = b$
F $21 - \frac{13}{32} = d$
G $8\frac{9}{16} - 8\frac{1}{16} = c$
H $47\frac{11}{20} - 26\frac{11}{20} = w$
I $\frac{15}{16} - \frac{19}{64} = m$

Test 8

A $\frac{5}{6} \times \frac{3}{10} = m$
B $1\frac{3}{8} \times \frac{7}{16} = r$
C $15 \times 1\frac{2}{3} = v$
D $9\frac{1}{2} \times 4\frac{8}{15} = d$
E $\frac{19}{32} \times 124 = b$
F $7\frac{3}{10} \times 6\frac{5}{8} = c$
G $45 \times 3\frac{4}{5} = k$
H $21\frac{1}{4} \times 6\frac{8}{15} = w$
I $100 \times 1\frac{7}{10} = t$

Test 9

A $\frac{21}{32} \div \frac{3}{4} = w$
B $8\frac{2}{5} \div 7 = t$
C $40 \div 1\frac{2}{3} = d$
D $9\frac{3}{8} \div 6\frac{1}{4} = m$
E $27\frac{1}{3} \div 1\frac{1}{2} = x$
F $\frac{11}{16} \div 4\frac{1}{8} = n$
G $\frac{15}{32} \div \frac{25}{48} = r$
H $18\frac{2}{3} \div 14 = b$

Test 10

A $r + 3\frac{1}{4} = 6\frac{7}{8}$
B $11 - x = 5\frac{5}{6}$
C $\frac{3}{4} \times 52 = w$
D $9\frac{15}{16} + 1\frac{5}{8} = c$
E $v - 12\frac{11}{12} = 1\frac{1}{6}$
F $10\frac{3}{10} \div \frac{7}{10} = b$
G $8\frac{1}{10} - m = 1$
H $14\frac{7}{12} - 13\frac{1}{12} = t$

Test 11

A 15 is ▭% of 20.
B 152% of 300 is ▭.
C 11% of ▭ is 22.
D 180% of ▭ is 90.
E 40% of 625 is ▭.
F 27 is ▭% of 135.
G 155% of ▭ is 465.
H 2% of 150 is ▭.
I 75 is ▭% of 15.

Thinking straight

	A	B	C
Step 1	$\frac{120}{15}=\frac{n}{1}$	$\frac{8}{1}=\frac{n}{15}$	$\frac{8}{1}=\frac{120}{n}$
Step 2	$15n=120$	$n=8\times15$	$8n=120$
Step 3	$120\div15$	8×15	$120\div8$

A Step 1 in Example A above shows an equation that was made for a problem. Notice that 1 is the second term of one ratio. Does Step 2 show what you write when you use the ratio test? What does Step 3 show?

B Look again at Steps 1 and 3 of Example A. Can you solve the equation without using the ratio test? To find the numeral that will replace n, you divide 120 by ▩.

C Study the equation in the first step of Example B. Is 1 the second term of one ratio? What does Step 2 show? What does Step 3 show?

D Can you solve the equation in Example B without using the ratio test? To find the numeral that will replace n, you multiply ▩ by ▩.

E Is 1 the second term of one ratio in the equation in Example C? What does Step 2 show? What does Step 3 show?

F Can you solve the equation in Example C without using the ratio test? To find the numeral that will replace n, you divide ▩ by ▩.

When you have an equation of ratios, and 1 is the second term of one ratio, you do not need to use the ratio test.

You can solve each equation below without using the ratio test.

A $\frac{60}{15}=\frac{b}{1}$ **D** $\frac{7}{1}=\frac{35}{r}$ **G** $\frac{95}{5}=\frac{w}{1}$

B $\frac{9}{1}=\frac{x}{50}$ **E** $\frac{14}{1}=\frac{m}{26}$ **H** $\frac{11}{1}=\frac{t}{121}$

C $\frac{25}{5}=\frac{n}{1}$ **F** $\frac{5}{1}=\frac{b}{80}$ **I** $\frac{21}{1}=\frac{105}{f}$

Exploring problems

A A farmer had $20\frac{2}{3}$ gal. of maple syrup. In one week he sold $15\frac{1}{2}$ gal. What fraction of his total supply did he sell?

When you find what fraction of the total supply the farmer sold, you are really finding what fraction of a gallon he sold for each gallon he had to start with.

First, make an equation of ratios. Each ratio will express the comparison of gallons the farmer sold with the gallons he had to start with.

$$\frac{15\frac{1}{2}}{20\frac{2}{3}} = \frac{d}{1}$$

Ratio expressing the comparison for $20\frac{2}{3}$ gal.

Ratio expressing the comparison for 1 gal.

How is the equation below found?

$20\frac{2}{3}\,d = 15\frac{1}{2}$

$15\frac{1}{2} \div 20\frac{2}{3} = \frac{31}{2} \div \frac{62}{3} = \frac{31}{2} \times \frac{3}{62} = \frac{3}{4}$

$\frac{3}{4}$ replaces d. The farmer sold $\frac{3}{4}$ gal. of syrup for each gallon he had. So we say that he sold $\frac{3}{4}$ of his supply.

When you solved Problem A, you compared the gallons that were sold with all the gallons. You compared part of a group with the whole group.

B Mr. Henry planted 34 acres in corn. Mr. Jackson planted 51 acres in corn. The amount of land Mr. Henry planted in corn was what fraction of the amount of land Mr. Jackson planted?

Problem B really asks you to find what fraction of an acre Mr. Henry planted for each acre that Mr. Jackson planted.

Again make an equation of ratios. Each ratio will compare acres planted by Mr. Henry with acres planted by Mr. Jackson.

$$\frac{34}{51} = \frac{a}{1}$$

Ratio expressing the comparison for ▒ acres

Ratio expressing the comparison for ▒ acre

How is the equation below found?

$51a = 34$

$34 \div 51 = \frac{2}{3}$

$\frac{2}{3}$ replaces a. For each acre of land that Mr. Jackson planted, Mr. Henry planted ▒ acre. Mr. Henry planted ▒ as much land as Mr. Jackson.

When you solved Problem B, you compared Mr. Henry's 34 acres with Mr. Jackson's 51 acres. You compared two groups.

C One morning a milkman started out with 25 qt. of buttermilk. By noon

he had sold $\frac{3}{5}$ of the 25 qt. How many quarts had he sold by noon?

Problem C tells you what fraction of a quart the milkman sold for each 1 qt. he had to start with. You are to find how many quarts of his total supply he had sold by noon.

Look at the equation below. Does each ratio show the comparison of quarts the milkman sold with quarts he had?

$$\frac{3}{5} = \frac{t}{25}$$

Ratio expressing the comparison for 1 qt.

Ratio expressing the comparison for 25 qt.

How do you get the equation below?

$t = \frac{3}{5} \times 25$

$\frac{3}{5} \times 25 = 15$

15 replaces t. The milkman had sold ▩ qt. of buttermilk by noon.

In Problem C, was a part of a group compared with the whole group, or were two groups compared?

D Mrs. Price bought 18 yd. of cloth. Mrs. Cook said she bought $\frac{2}{3}$ as much cloth as Mrs. Price. How many yards of cloth did Mrs. Cook buy?

Does Problem D tell you what fraction of a yard Mrs. Cook bought for each yard that Mrs. Price bought? What are you to find?

What does each ratio in the equation below show?

$$\frac{\frac{2}{3}}{1} = \frac{w}{18}$$

Ratio expressing the comparison for ▩ yd.

Ratio expressing the comparison for ▩ yd.

Is the equation below correct?

$w = \frac{2}{3} \times 18$

$\frac{2}{3} \times 18 = 12$

What numeral replaces w? Mrs. Cook bought ▩ yd. of cloth.

In Problem D, was a part of a group compared with the whole group, or were two groups compared?

E On a trip, Mr. Henry used $\frac{5}{6}$ of a full tank of gasoline. He used $12\frac{1}{2}$ gal. How many gallons does the gasoline tank hold?

You know what fraction of a gallon Mr. Henry used for each 1 gal. he had. You know how many gallons he used. So you can make an equation of ratios that compare the gallons Mr. Henry used with the gallons he had.

$$\frac{\frac{5}{6}}{1} = \frac{12\frac{1}{2}}{n}$$

Ratio expressing the comparison for 1 gal.

Ratio expressing the comparison for the total number of gallons

Now turn the page.

Is the equation below correct?

$\frac{5}{6}n = 12\frac{1}{2}$

$12\frac{1}{2} \div \frac{5}{6} = 15$

What numeral replaces n? Mr. Henry's gasoline tank holds ▒ gal.

In Problem E, was a part of a group compared with the whole group, or were two groups compared?

F Don has a collection of 200 pennies. He says that he has $\frac{2}{3}$ as many pennies as Andy has. How many pennies has Andy?

Do the ratios in the equation below express the comparison of Don's pennies with Andy's pennies?

$$\frac{\frac{2}{3}}{1} = \frac{200}{k}$$

Ratio expressing the comparison for ▒ penny

Ratio expressing the comparison for the total number of Andy's pennies

Is the equation below correct?

$\frac{2}{3}k = 200$

$200 \div \frac{2}{3} = 300$

What numeral replaces k? Andy has ▒ pennies.

In Problem F, was a part of a group compared with the whole group, or were two groups compared?

G Mrs. Price bought 15 yd. of cloth. She used $3\frac{3}{4}$ yd. of this cloth for a dress. The cloth she used was what fraction of all the cloth she bought?

THINK One ratio you use to solve Problem G is $3\frac{3}{4}$ over 15. What is the other ratio?

H Pat built 20 model ships. Jim built $\frac{1}{4}$ as many ships as Pat. How many model ships did Jim build?

THINK One ratio you use to solve Problem H is $\frac{1}{4}$ over 1. What is the other ratio?

I John Price spent \$8 for baseball equipment. This amount was $\frac{4}{5}$ of his savings. How much had he saved before he bought the baseball equipment?

THINK To solve Problem I, use $\frac{4}{5}$ over 1 as one ratio. Why can you use either 8 over r or 800 over r as the other ratio?

J Jim Cook said that in three months he had saved \$7. He also said that this amount was $\frac{1}{8}$ of the amount he had earned during the three months. How much had he earned during the three months?

K Mrs. Stone baked 36 cookies one Saturday. She said, "This is only $\frac{3}{4}$ as many cookies as I baked on Tuesday." How many cookies had she baked on Tuesday?

L On Monday, Mrs. Stone baked 60 cookies. She baked what fraction as many cookies on Saturday as she did on Monday?

M Mr. Parker had a pasture with an area of $6\frac{1}{4}$ acres. He plowed $\frac{3}{5}$ of the pasture to make a garden. How many acres did he plow for the garden?

N Mr. Parker's garden is $\frac{1}{3}$ as large as another field that is planted in potatoes. How many acres are in the field of potatoes?

O One winter High Falls had 21 in. of snow. Centreville, 20 mi. from High Falls, had only 14 in. of snow that winter. What fraction can be used to express the comparison of the snowfall in Centreville that winter with the snowfall in High Falls?

P A baker made 120 small cakes. He put chocolate icing on $\frac{5}{6}$ of them. He put chocolate icing on how many of the small cakes?

Q A road builder had $20\frac{2}{3}$ mi. of road to build. At the end of 8 months, he had finished $\frac{1}{2}$ of the road. How many miles of road had he built then?

R Nancy said her new dress cost $\frac{1}{2}$ as much as her new coat. Her coat cost \$34. How much did her dress cost?

S Nancy spent 75¢ of her allowance for magazines. Her allowance is \$2.50 per week. She spent what fraction of her allowance for magazines?

Now you should be able to solve comparison problems in which you compute with fractions.

Keeping skilful

Find the area of each parallelogram whose dimensions are given below.

A base, 2 ft.; altitude, 18 in.

B base, $1\frac{2}{3}$ mi.; altitude, $1\frac{1}{5}$ mi.

C base, 15 yd.; altitude, 20 yd.

D base, $3\frac{1}{2}$ yd.; altitude, $1\frac{1}{4}$ yd.

E base, 35 in.; altitude, 2 ft.

F base, $2\frac{3}{10}$ mi.; altitude, $1\frac{7}{10}$ mi.

G base, $4\frac{7}{12}$ in.; altitude, $2\frac{3}{4}$ in.

H base, 50 yd.; altitude, 27 ft.

I 40-ft. square

J base, 4 mi.; altitude, $3\frac{1}{2}$ mi.

K base, 60 in.; altitude, 2 yd.

L base, $1\frac{1}{3}$ in.; altitude, $1\frac{7}{8}$ in.

M $5\frac{1}{4}$-ft. square

N base, 3 mi.; altitude, $7\frac{1}{3}$ mi.

O base, 4 in.; altitude, $4\frac{1}{2}$ ft.

P $17\frac{1}{8}$-ft. square

Exploring problems

A The height of an iceberg above the ocean surface was about 270 ft. Its total height was estimated as 2430 ft. The height above the ocean was what fraction of the total height?

THINK Problem A asks you to find what fraction 270 is of 2430. You can write this equation for the problem.

$$\frac{270}{2430} = \frac{d}{1}$$

← Fraction of a foot above the water for each foot of total height

You can divide 270 by 2430 to get the answer.
270 is what fraction of 2430?

B If the iceberg described in Problem A had been floating in fresh water, about $\frac{9}{10}$ of it would have been below water. About how much of the estimated total height would have been below water?

THINK Problem B asks you to find the number that is $\frac{9}{10}$ of 2430.

$$\frac{\frac{9}{10}}{1} = \frac{k}{2430}$$

← Number of feet below water

You can multiply 2430 by $\frac{9}{10}$ to find the answer.

$\frac{9}{10}$ of 2430 is ▤.

C The height above water of a large iceberg was about 420 ft. A scientist estimated that this was only $\frac{1}{8}$ of the total height. What was the approximate height of the iceberg?

THINK 420 is $\frac{1}{8}$ of what number?

$$\frac{\frac{1}{8}}{1} = \frac{420}{t}$$

Number of feet in total height ←

You can divide 420 by $\frac{1}{8}$ to find the answer.
420 is $\frac{1}{8}$ of ▤.

D The height above water of another iceberg was estimated as 200 ft. If you estimate the height above the water as $\frac{1}{9}$ of the total height, what was the total height of this iceberg?

THINK What equation can you make to find the answer for Problem D?

Which of these two sentences is correct for Problem D?

1. 200 is $\frac{1}{9}$ of ▤.
2. $\frac{1}{9}$ of 200 is ▤.

E The weight of one large iceberg was estimated as 1,500,000 tons. At one time an estimated 20,000 tons of ice fell from this iceberg. The weight of the ice that fell was what fraction of the total weight of the iceberg?

THINK What equation can you make to find what fraction 20,000 is of 1,500,000?

Which of these two sentences is correct for Problem E?

1 1,500,000 is what fraction of 20,000?
2 20,000 is what fraction of 1,500,000?

F The iceberg described in Problem E drifted southward. Its length one Monday was estimated as 180 ft. On Wednesday, its length was estimated as $\frac{3}{5}$ of its length on Monday. What was its estimated length on Wednesday?

THINK What equation can you make to find $\frac{3}{5}$ of 180?

Which of these two sentences is correct?

1 $\frac{3}{5}$ of 180 is ▒.
2 180 is $\frac{3}{5}$ of ▒.

For each exercise below, first make an equation of ratios. Then find the answer.

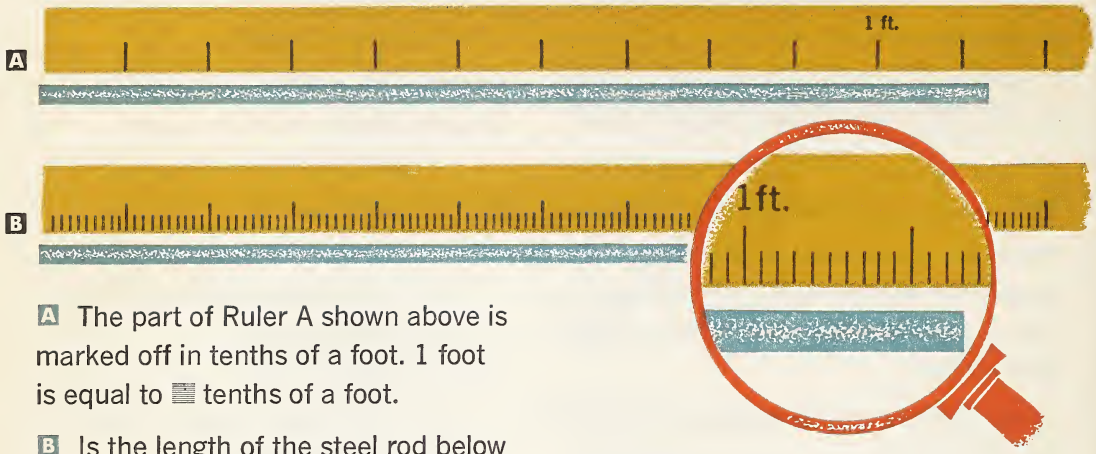
A $\frac{7}{10}$ of 600 is ▒.
B 165 is what fraction of 660?
C $12\frac{1}{2}$ is $\frac{4}{5}$ of ▒.
D $\frac{3}{4}$ of 53 is ▒.
E 48 is $\frac{3}{4}$ of ▒.
F $8\frac{3}{4}$ is what fraction of $17\frac{1}{2}$?
G 225 is what fraction of 675?
H $\frac{7}{10}$ of 15 is ▒.
I 63 is $\frac{7}{8}$ of ▒.
J 114 is what fraction of 138?
K $\frac{7}{8}$ of 1258 is ▒.
L 73 is $\frac{9}{10}$ of ▒.
M 82 is $\frac{1}{4}$ of ▒.
N 115 is what fraction of 575?

Now you know how to work with problems and sentences that use fractions to compare one number with another number.

Keeping skilful

A $v + 14862 = 21046$
B $89166 + 32844 = s$
C $a - 9215 = 846$
D $7000 - t = 1488$
E $9236 + k = 48721$
F $11000 - 999 = m$
G $r + 27535 = 50101$
H $45147 - b = 2103$
I $4890 + 45047 = c$
J $41731 - 39845 = d$
K $n - 6476 = 1548$
L $96822 + 4879 = k$
M $6340 + w = 18342$
N $7403 - t = 114$
O $b - 95865 = 796$
P $87001 + 999 = f$
Q $70507 + y = 76299$
R $g - 8373 = 812$
A $z - 42.831 = 2.695$
B $r + 695.40 = 710.01$
C $4611.5 - 3898.7 = t$
D $178.36 + d = 200.05$
E $986.005 + 14.987 = x$
F $f - 1.076 = 21.434$
G $8492.3 - b = 183.9$
H $6936.1 + q = 9023.5$
I $735.729 - 30.015 = c$

Thinking straight



A The part of Ruler A shown above is marked off in tenths of a foot. 1 foot is equal to ▒ tenths of a foot.

B Is the length of the steel rod below Ruler A nearer to 1.1 ft. or to 1.2 ft.? Is the rod about 1.1 ft. long?

C When you say that the rod is 1.1 ft. long, you are giving the length to the nearer tenth of a foot. What is the length of the rod to the nearer foot?

D Look at Ruler B. The part shown is marked off in hundredths of a foot. 1 foot is equal to ▒ hundredths of a foot. 1 tenth of a foot is equal to ▒ hundredths of a foot.

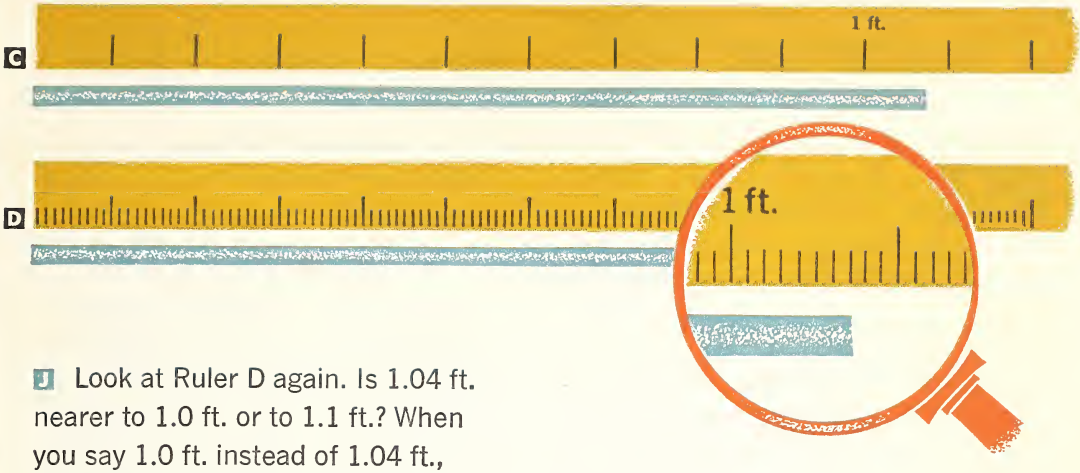
E Now look at the magnified part of Ruler B and the rod below it. Is the length of the rod nearer to 1.12 or to 1.13 ft.? Is the rod 1.13 ft. long to the nearer hundredth of a foot?

F The part of Ruler C shown on page 217 is marked off in ~~~ of a foot. What is the length of the rod below Ruler C to the nearer tenth of a foot?

G Ruler D is marked off in hundredths of a foot. Is the magnified part of this ruler also marked off in hundredths of a foot?

H Now look at the rod below the magnified part of the ruler. Is the rod nearer to 1.08 or to 1.07 ft. in length? Is it about 1.07 ft. long?

I When you describe the rod on this page and the rod on page 217 as about 1.1 ft. long, you cannot tell how much longer the first rod is than the second. When you measure the rods in hundredths of a foot, can you tell approximately how much longer the first rod is? $1.13 - 1.07 = ▒$



J Look at Ruler D again. Is 1.04 ft. nearer to 1.0 ft. or to 1.1 ft.? When you say 1.0 ft. instead of 1.04 ft., you round 1.04 ft. down to 1.0 ft. When you express a measurement as 1.0 ft., you are indicating that the measurement is made to the nearer tenth of a foot. The measurement is precise to tenths of a foot.

K Use Ruler D to decide whether .68 ft. is nearer to .6 ft. or to .7 ft. When you say .7 ft. instead of .68 ft., you round .68 ft. up to .7 ft. Is .7 ft. precise to the nearer tenth or to the nearer hundredth of a foot?

L Is .45 ft. halfway between .40 ft. and .50 ft.? Can you say that .45 ft. is about .4 ft.? Can you also say that .45 ft. is about .5 ft.?

M Some measuring instruments can be used to measure thousandths of a foot or thousandths of an inch. What cases can you think of where such precise measurements are needed?

N Suppose you are told that the diameter of a piece of wire has been measured as .02 in. to the nearer hundredth of an inch. Could the diameter of the wire be .019 in.? Could it be .024 in.? Could it be .016 in.? Could it be .020 in.? Could the diameter of the wire be anywhere between .015 and .025 in.?

O The thickness of a strip of metal is measured as .065 in. to the nearer thousandth of an inch. Could the thickness of the metal be .0654 in.? Could the thickness be .0647 in.? Could it be .0658 in.? Could the thickness of the metal be anywhere between .0645 in. and .0655 in.?

P With which can you measure more precisely, a ruler marked off in tenths of an inch or a ruler marked off in fourths of an inch?

Learning how

In this lesson you will learn how to find the product of numbers expressed with decimal fraction numerals.

see A The Wests' hallway is 7.6 ft. long and 4.25 ft. wide. What is the area of the hallway in square feet?

What equation can you make?

To solve your equation, you need to multiply 4.25 by 7.6. You can find the product by using common fraction numerals.

$$\frac{76}{10} \times \frac{425}{100} = \frac{32300}{1000}$$

$$\frac{32300}{1000} = 32\frac{300}{1000} \text{ or } 32.300$$

$$\begin{array}{r} 4.25 \\ \times\ 7.6 \end{array}$$

Instead of using common fraction numerals, you can write the numerals as shown at the left.

Now think of 4.25 as 425, and think of 7.6 as 76.

$$\begin{array}{r} 425 \\ \times\ 76 \\ \hline 2550 \\ 29750 \\ \hline 32300 \end{array}$$

You are multiplying the same numbers that you did when you found the product of the numerators of the fractions.

$$\begin{array}{r} 4.25 \\ \times\ 7.6 \\ \hline 2550 \\ 29750 \\ \hline 32.300 \end{array}$$

The product of 4.25 and 7.6 must be $\frac{32300}{1000}$ because the product of $\frac{425}{100}$ and $\frac{76}{10}$ is $\frac{32300}{1000}$. So you must use a point to show the thousandths.

The area of the hallway is 32.3 sq. ft.

B One quart of milk weighs about 2.58 lb. About how much does one gallon of milk weigh?

What equation can you make?

To solve your equation, you need to multiply 2.58 by 4. Write 2.58 and 4 as common fraction numerals and multiply.

$$\frac{4}{1} \times \frac{258}{100} = \frac{1032}{100} \text{ or } 10.32$$

You can find the product of 2.58 and 4 by writing the numerals as shown below.

$$\begin{array}{r} 2.58 \\ \times\ 4 \\ \hline 1032 \end{array}$$

When you compute, ignore the point in 2.58. You know that 1032 is too large for the product.

$$\begin{array}{r} 2.58 \\ \times\quad 4 \\ \hline 10.32 \end{array}$$

The product of 2.58 and 4 must be $\frac{1032}{100}$ because the product of $\frac{258}{100}$ and $\frac{4}{1}$ is $\frac{1032}{100}$. So you must use a point to show the hundredths.

One gallon of milk weighs about 10.32 lb.

C The Millers' sidewalk is 15 yd. long and .9 yd. wide. What is the area of the sidewalk in square yards?

What equation can you make?

To solve your equation, you multiply 15 by .9. You can write 15 and .9 as common fraction numerals and then multiply.

$\frac{9}{10} \times \frac{15}{1} = \frac{135}{10}$ or 13.5

You can find the product of 15 and .9 by writing the numerals as shown below.

$$\begin{array}{r} 15 \\ \times\quad .9 \\ \hline 135 \end{array}$$

When you compute, ignore the point in .9. You know that 135 is too large for the product.

$$\begin{array}{r} 15 \\ \times\quad .9 \\ \hline 13.5 \end{array}$$

The product of 15 and .9 must be $\frac{135}{10}$ because the product of $\frac{15}{1}$ and $\frac{9}{10}$ is $\frac{135}{10}$. So you must use a point to show the tenths.

The area of the sidewalk is 13.5 sq. yd.

think **A** Find the product of .38 and .17.

You multiply ▒ by ▒. Are .38 and .17 correctly written below as common fraction numerals?

$\frac{38}{100} \times \frac{17}{100}$

What is the denominator of the product? The product is $\frac{▒}{10000}$.

You can find the product of .38 and .17 by computing as shown below.

$$\begin{array}{r} .38 \\ \times\quad .17 \\ \hline 266 \\ 380 \\ \hline .0646 \end{array}$$

The product of 38 and 17 is 646. How do you know that 646 is too large for the product? Why must you show ten-thousandths in your answer?

You must put a zero to the left of 646 before you can put in the point to show ten-thousandths.

B Find the product of .306 and 18.

$$\begin{array}{r} .306 \\ \times\quad 18 \\ \hline 2448 \\ 3060 \\ \hline 5.508 \end{array}$$

When you multiply 306 by 18, you get 5508. How do you know that the product of .306 and 18 is $\frac{5508}{1000}$? Why do you put a point in 5508 between 5 and 508?

C Find the product of .75 and 1.5.

$$\begin{array}{r} .75 \\ 1.5 \\ \hline 375 \\ 750 \\ \hline 1.125 \end{array}$$

Is 1125 the product of 75 and 15? Why must you show thousandths in the product?

D Find the product of 2.7 and 4.3.

$$\begin{array}{r} 2.7 \\ 4.3 \\ \hline 81 \\ 1080 \\ \hline 11.61 \end{array}$$

When you multiply 27 by 43, you get ▒. Why must you show hundredths in the product?

try

A $.6 \times 7 = v$

$$\begin{array}{r} 7 \\ .6 \\ \hline 4.2 \end{array}$$

How do you know that the product is 4.2 and not 42?

B $8.2 \times 1.7 = g$

$$\begin{array}{r} 8.2 \\ 1.7 \\ \hline 574 \\ 820 \\ \hline 13.94 \end{array}$$

How do you know that the product must be expressed in hundredths?

C $1.3 \times 12.32 = s$

$$\begin{array}{r} 12.32 \\ 1.3 \\ \hline 3696 \\ 12320 \\ \hline 16.016 \end{array}$$

The product of a number expressed in hundredths and a number expressed in tenths must be expressed in ~~~.

D $1.35 \times .15 = p$

$$\begin{array}{r} 1.35 \\ .15 \\ \hline 675 \\ 1350 \\ \hline .2025 \end{array}$$

Is the point in the product in the correct position? How do you know?

E $.106 \times .3 = b$

$$\begin{array}{r} .106 \\ .3 \\ \hline .0318 \end{array}$$

How do you know that the product is not 318? Why must you put a zero to the left of 318 before you can put in the point?

F $6 \times .092 = r$

$$\begin{array}{r} .092 \\ 6 \\ \hline .552 \end{array}$$

The product of a whole number and a number expressed in thousandths must be expressed in ~~~.

do

A Bill read that the average load per freight car in Canada is 43.9 tons. At this rate, how many tons of freight will a train of 65 freight cars carry?

$$\frac{43.9}{1}=\frac{t}{65}$$

B A medium-sized household pail holds about 2.5 gallons of water. Water weighs 10 lb. per gallon. If you fill the pail with water, about how much does the water in it weigh?

$$\frac{10}{1}=\frac{n}{2.5}$$

C $12.7\times1.3=v$
D $.35\times7=n$
E $3.278\times6.1=w$
F $4.5\times3.32=a$
G $.02\times.04=k$
H $25\times8.3=y$
I $16.26\times.75=m$
J $.67\times.5=e$
K $13.1\times7.2=s$
L $3.33\times1.8=n$
M $.8\times.9=b$

N $6\times.35=p$
O $.7\times.92=z$
P $1.55\times6.3=c$
Q $.01\times.6=h$
R $.84\times.36=q$
S $27\times.195=f$
T $1.6\times7.3=r$
U $.39\times1.42=d$
V $.4\times11=n$
W $100\times.4375=t$
X $.6\times.98=g$

A $5.06\times.32=b$
B $27.1\times.9=z$
C $.35\times1.2=m$
D $48\times.5=h$
E $.6\times.1=n$
F $.08\times.07=a$
G $50\times3.8=k$
H $72\times.25=v$
I $.6\times1.8=e$
J $21.32\times.67=y$
K $33.5\times6.73=w$

Now you should be able to multiply numbers expressed with decimal fraction numerals.

Keeping skilful

Find the sum.

A $6\frac{1}{8}, 4\frac{1}{2}, 8\frac{1}{3}$
B $7\frac{2}{3}, 9\frac{1}{4}, 8\frac{1}{4}$
C $1\frac{1}{5}, 6\frac{1}{6}, 7\frac{3}{10}$
D $5\frac{3}{4}, 7\frac{5}{8}, 6\frac{1}{2}$
E $3\frac{2}{3}, 8\frac{3}{8}, 4\frac{3}{4}$
F $9\frac{3}{8}, 7\frac{7}{8}, 4\frac{7}{16}$
G $5\frac{5}{6}, 7\frac{2}{3}, 3\frac{1}{2}$
H $9\frac{1}{2}, 4\frac{1}{4}, 7\frac{1}{8}, 3\frac{1}{3}$

A $4\frac{1}{2}\times\frac{2}{3}=m$
B $3\frac{1}{8}\times5\frac{3}{5}=y$
C $19\times\frac{3}{4}=d$
D $\frac{5}{6}\times\frac{1}{3}=r$
E $2\frac{7}{10}\times10\frac{5}{8}=z$
F $6\frac{1}{4}\times7\frac{1}{2}=b$
G $42\times\frac{1}{6}=s$
H $\frac{4}{5}\times\frac{1}{10}=g$
I $20\frac{2}{3}\times9\frac{3}{5}=k$

A $1\frac{1}{2}\div\frac{3}{4}=p$
B $6\div2\frac{1}{3}=k$
C $\frac{3}{5}\div\frac{1}{5}=v$
D $\frac{5}{8}\div5=n$
E $7\frac{2}{3}\div1\frac{1}{4}=r$
F $13\frac{1}{2}\div2=q$
G $7\frac{3}{4}\div8\frac{1}{3}=b$
H $25\div\frac{5}{12}=x$
I $19\div1\frac{1}{3}=a$

A $6.754+3.089=x$
B $12.3-.8=q$
C $17.35-a=12.63$
D $.3219+2.8476=n$
E $t+.6=7.1$
F $z-3.48=2.97$
G $14.238-10.255=b$
H $181.5+e=193.5$
I $7.39+11.04=n$
J $w+47.792=58.173$

Using arithmetic

A The Stone School swimming pool is 60 ft. long. Jim swam 14 lengths of the pool. How many feet did he swim?

You can make an equation of ratios for Problem A.

$$\frac{60}{1}=\frac{t}{14}$$

How do you find the numeral that replaces t?

B It takes Jim 14.8 seconds to swim 1 length of the pool. At this rate, how many seconds does it take him to swim 4 lengths?

Can you make the equation below for Problem B?

$$\frac{14.8}{1}=\frac{k}{4}$$

What numeral replaces k?

C In a swimming test, Bob stayed under water 3 times as long as John. John stayed under water 26.8 seconds. How long did Bob stay under water?

You know that Bob stayed under water 3 seconds for every second that John stayed under water. Does the equation below show how to solve Problem C?

$$\frac{3}{1}=\frac{r}{26.8}$$

What numeral replaces r?

D The coach clocked Bob's swimming speed and found that it was .74 yard per second. At this rate, how many yards can Bob expect to swim in 32.4 seconds?

First write an equation of ratios for Problem D. Use m to hold a place for the numeral you are to find. How do you find the numeral that replaces m?

When you know the numerals in both terms of one ratio, and one of these numerals is 1, do you need to write an equation of ratios?

E Pete swims faster than Bob. His average rate is .975 yards per second. How far can Pete swim in .8 second?

F The coach found that when Bob used the backstroke, he swam at a speed of .38 yd. per second. At that rate, how many yards could Bob swim in 37 sec.?

G When Bob used the side stroke, he swam 7.6 feet per stroke. How far could Bob swim in 13 strokes?

H The water in the pool is replaced at the rate of 12,148.9 gallons per hour. The coach said that the water is replaced completely every 6 hours. How many gallons of water does the pool contain?

I In some swimming meets, distances are measured in metres. A metre is 1.094 yards. How many yards does a swimmer go in a 100-metre race?

J A world record for fast swimming was set by a man who swam 880 yards at an average rate of .655 seconds per yard. How many seconds did it take him to swim the 880 yards?

K A record for swimming the English Channel was set by a woman who swam the distance of 20 mi. at an average rate of .7 hr. per mile. How long did it take her to swim the Channel?

L When Marilyn Bell swam across Lake Ontario from Youngstown, New York, to Toronto, her average speed was about 2 miles per hour. She hoped to reach Toronto in 16 hours. How many miles did she expect to swim?

M She thought that in long-distance swimming she could average 3600 strokes per hour. She swam for 22.1 hr. About how many strokes had she taken during the time she was swimming? How many miles more than she expected did the currents and winds force her to swim?

Checking up

Test 1

A $12 \times 6.75 = p$
B $38 \times .4 = n$
C $42.03 \times 96 = w$
D $132 \times .005 = a$
E $.29 \times 748 = k$
F $.6 \times 463 = s$
G $79 \times 6.112 = y$
H $4.1 \times 17 = e$
I $52 \times .3847 = m$
J $35 \times 6.95 = r$
K $12.66 \times 48 = z$
L $419 \times .363 = n$
M $76 \times 29.4 = c$
N $.186 \times 300 = q$
O $.278 \times 69 = k$

Test 2

A $.75 \times .75 = v$
B $6.83 \times 7.1 = h$
C $11.3 \times .652 = t$
D $8.71 \times 7.43 = k$
E $5.412 \times 3.6 = b$
F $10.9 \times 2.8 = n$
G $6.21 \times .25 = d$
H $9.3 \times 1.73 = s$
I $.065 \times 28.3 = w$
J $4.27 \times 3.13 = p$
K $119.5 \times .2 = a$
L $7.3 \times .96 = m$
M $.688 \times 1.4 = z$
N $2.3 \times 8.7 = e$
O $.048 \times .51 = h$

Test 3

A $.31 \times .2 = q$
B $1.6 \times .06 = y$
C $.21 \times .14 = n$
D $.3 \times .261 = h$
E $.76 \times .13 = w$
F $.27 \times .27 = r$
G $3.45 \times .02 = c$
H $.19 \times .4 = z$
I $.32 \times .28 = n$
J $.098 \times .7 = s$
K $.21 \times .43 = a$
L $.06 \times .15 = p$
M $.19 \times .19 = k$
N $.03 \times .02 = t$
O $.06 \times .05 = a$

Thinking straight

A $43 \times 10 = 430$
B $4.3 \times 10 = 43.0$ or 43
C $.43 \times 10 = 4.30$ or 4.3
D $.043 \times 10 = .430$ or .43
E $.0043 \times 10 = .0430$ or .043

F $43 \times 100 = 4300$
G $4.3 \times 100 = 430.0$ or 430
H $.43 \times 100 = 43.00$ or 43
I $.043 \times 100 = 4.300$ or 4.3
J $.0043 \times 100 = .4300$ or .43

K $43 \times 1000 = 43000$
L $4.3 \times 1000 = 4300.0$ or 4300
M $.43 \times 1000 = 430.00$ or 430
N $.043 \times 1000 = 43.000$ or 43
O $.0043 \times 1000 = 4.3000$ or 4.3

P $14.3 \times 10 = 143.0$ or 143
Q $1.43 \times 100 = 143.00$ or 143
R $.143 \times 1000 = 143.000$ or 143
S $.0143 \times 10000 = 143.0000$ or 143

Use the examples above to answer the following questions.

A How many zeros do you put at the right of a whole number when you multiply the whole number by 10? By 100? By 1000?

B When a decimal fraction expressed in tenths is multiplied by 10, is the product a whole number?

C When a decimal fraction expressed in hundredths is multiplied by 100, is the product a whole number?

D When a decimal fraction expressed in thousandths is multiplied by 1000, is the product a whole number?

E When a decimal fraction expressed in ten-thousandths is multiplied by 10,000, is the product a whole number?

F When a decimal fraction expressed in hundredths is multiplied by 10, what decimal fraction is in the product?

G When a decimal fraction expressed in thousandths is multiplied by 10, what decimal fraction is in the product?

H When a decimal fraction expressed in thousandths is multiplied by 100, what decimal fraction is in the product?

Multiply each number below by 10.
A 63.5 B 27.98 C .3 D 44.325

Multiply each number below by 100.
E .76 F 7.1 G 1.0058 H 3.45

Multiply each number below by 1000.
I 1.39 J .469 K .001 L 3.227

Thinking straight

A $168 \div 12 = n$

B $\frac{168}{12} = \frac{14}{1} = 14$

C $\frac{168 \times 2}{12 \times 2} = \frac{336}{24} = \frac{14}{1} = 14$

D $1.68 \div .12 = n$

E $\frac{1.68 \times 100}{.12 \times 100} = \frac{168}{12} = \frac{14}{1} = 14$

A The first part of Example B shows another way to write the division in Example A. How is 14 over 1 found?

B In Example C, both the 12 and the 168 are multiplied by ▒. Do $336 \div 24$ and $168 \div 12$ have the same quotient?

C If you multiply 168 and 12 by .01, do you get the numbers in Example D?

D Look at Example E. 1.68 and .12 have been multiplied by ▒. Is the quotient of $1.68 \div .12$ the same as the quotient of $168 \div 12$?

E If you multiply 168 and 12 by .001, what numbers do you get? What is the quotient of .168 and .012?

F Why should $.0168 \div .0012$ equal 14?

G The quotient of $1728 \div 144$ is 12. If you multiply both 1728 and 144 by .01 and then divide, will the quotient be 12? If you multiply both 1728 and 144 by .001 and then divide, what will the quotient be?

Look at Exercises A to D below.

A	B	C	D
$\frac{64}{5}$	$\frac{1.45}{1.9}$	$\frac{6 \times 7}{14}$	$\frac{3.27}{.054}$

For each exercise above, find others below that will have the same quotient. You should not do any computing.

E $\frac{1.45 \times 4}{1.9 \times 4}$

F $\frac{64 + 10}{5 + 10}$

G $\frac{6 \times 7 \times 10}{14 \times 10}$

H $\frac{3.27 \times 1000}{.054 \times 1000}$

I $\frac{64 \div 2}{5 \div 2}$

J $\frac{1.45 \div 3}{1.9 \div 3}$

K $\frac{3.27 \times 6}{.054 \times 6}$

L $\frac{6 \times 7 \times .01}{14 \times .01}$

M $\frac{6 \times 7 \div 2}{14 \div 2}$

N $\frac{64 \times .1}{5 \times .01}$

O $\frac{32.7}{54}$

P $\frac{6.4}{.5}$

Q $\frac{145}{190}$

R $\frac{14.5}{19}$

S $\frac{327}{5.4}$

T $\frac{.6 \times 7}{1.4}$

Learning how

In this lesson you will learn how to divide numbers expressed by decimal fraction numerals.

see **A** The equation below was made for a problem.

$24.48 \div 1.7 = z$

Division will be easier if you change the divisor to a whole number. To do this, multiply 1.7 by 10. You must also multiply 24.48 by 10. $244.8 \div 17 = z$

You can divide by using common fraction numerals. $\frac{2448}{10} \div \frac{17}{1}$

$$\frac{\overset{144}{\cancel{2448}}}{10} \times \frac{1}{\underset{1}{\cancel{17}}} = \frac{144}{10} = 14.4$$

You can also divide by using decimal fraction numerals. You divide just as you do with whole numbers.

$$\begin{array}{r|l} 17\overline{)244.8} & 100 \\ 1700 & \\ \hline 748 & 40 \\ 680 & \\ \hline 68 & 4 \\ 68 & \\ \hline & 14.4 \end{array}$$

You divided as though there were no tenths. The work with common fraction numerals shows that your answer should be expressed in tenths. ← tenths.

$24.48 \div 1.7 = 14.4$

B The equation below was made for a problem.

$60.25 \div 5 = d$

The divisor is a whole number. You can divide by using common fraction numerals. $\frac{6025}{100} \div \frac{5}{1}$

$$\frac{\overset{1205}{\cancel{6025}}}{100} \times \frac{1}{\underset{1}{\cancel{5}}} = \frac{1205}{100} = 12.05$$

You can also divide by using decimal fraction numerals. Divide just as you do with whole numbers.

$$\begin{array}{r|l} 5\overline{)60.25} & 1000 \\ 5000 & \\ \hline 1025 & 200 \\ 1000 & \\ \hline 25 & 5 \\ 25 & \\ \hline & 12.05 \end{array}$$

You divided as though there were no hundredths. The work with common fraction numerals shows that your answer should be expressed in hundredths. ← hundredths.

$60.25 \div 5 = 12.05$

In Example B, the number you divided was expressed in hundredths. So the quotient was expressed in hundredths.

C This equation was also made for a problem.

$2 \div 80 = p$

To solve the equation, you divide 2 by 80. Before you can divide, you need another name for 2. Use 2.000 as another name for 2.

You can now change the numerals to common fraction numerals and divide. $\frac{2000}{1000} \div \frac{80}{1}$

$$\frac{\overset{25}{\cancel{2000}}}{1000} \times \frac{1}{\underset{1}{\cancel{80}}} = \frac{25}{1000} = .025$$

You can also divide just as you do with whole numbers.

```
80)2.000 | 20
   1600  |
   ----  |
    400  |  5
    400  |
    ---- |
         | .025
```

You divided as though there were no thousandths. The work with common fraction numerals shows that your answer should be expressed in thousandths.

$2 \div 80 = .025$

Both the number you divided and the answer were expressed in thousandths.

In Example A on page 226, the number you divided was finally expressed in tenths. The answer was also expressed in ~~~.

In Example B, both the number you divided and the answer were expressed in ~~~.

---

think **A** $201.6 \div 2.88 = n$

Can the equation below be used in place of the equation above?

$20160 \div 288 = n$

What do you do to get 20160 and 288?

```
288)20160 | 60
   17280  |
   -----  |
    2880  | 10
    2880  |
    ----  |
          | 70 ← Why is no decimal fraction expressed in the answer?
```

$201.6 \div 2.88 =$ ▒

B $23.4204 \div 3.48 = w$

Before you divide, multiply both 3.48 and 23.4204 by ▒.

$2342.04 \div 348 = w$

Are 2342.04 and 348 correctly written below as fraction numerals? $\frac{234204}{100} \div \frac{348}{1}$

$$\frac{\overset{673}{\cancel{234204}}}{100} \times \frac{1}{\underset{1}{\cancel{348}}} = \frac{673}{100} = 6.73$$

Why was 234204 replaced by 673? Why was 348 replaced by 1?

Now turn the page.

Now divide 2342.04 by 348 just as you do with whole numbers.

```
348)2342.04|600
    208800
    -------
     25404 | 70
     24360
     -----
      1044 |  3
      1044
      ----
           |----
           |6.73
```

The number that is divided is expressed in ~~~. The answer is also expressed in ~~~.

23.4204 ÷ 3.48 = ▩

C .702 ÷ 35.1 = s

Can the equation below be used in place of the equation above?

7.02 ÷ 351 = s

Now divide just as you do with whole numbers.

```
351)7.02 |  2
   702   |----
         | .02
```

The number that is divided is expressed in ~~~. So the answer is expressed in ~~~.

.702 ÷ 35.1 = ▩

---

try

A 8021 ÷ 24.68 = p

```
2468)802100|300
     740400
     ------
      61700|  20
      49360
      -----
      12340|   5
      12340
      -----
           |----
           |325
```

B 3.00 ÷ 12 = z

Is 3.00 another name for 3?

```
12)3.00|20
   240
   ---
    60|  5
    60
    --
      |--
      |.25
```

C 3.588 ÷ .13 = n

```
13)358.8|200
   2600
   ----
    988| 70
    910
    ---
     78|  6
     78
     --
       |----
       |27.6
```

Why is no decimal fraction expressed in the answer?

D $2.660 \div 35 = b$

$$
\begin{array}{r|l}
35\overline{)2.660} & 70 \\
\underline{2\,450} & \\
210 & 6 \\
\underline{210} & \\
0 & .076
\end{array}
$$

E $67.50 \div 1.8 = h$

$$
\begin{array}{r|l}
18\overline{)675.0} & 300 \\
\underline{5\,400} & \\
1\,350 & 70 \\
\underline{1\,260} & \\
90 & 5 \\
\underline{90} & \\
0 & 37.5
\end{array}
$$

F $25.76 \div .28 = y$

$$
\begin{array}{r|l}
28\overline{)2\,576} & 90 \\
\underline{2\,520} & \\
56 & 2 \\
\underline{56} & \\
0 & 92
\end{array}
$$

do

A $13.5 \div .18 = z$
B $28.50 \div 475 = w$
C $93.98 \div 3.7 = k$
D $44.676 \div 7.3 = a$
E $11.1936 \div 8.48 = n$
F $75.0 \div 10 = e$
G $652.8 \div 38.4 = v$
H $92.88 \div .129 = r$

I $3.6608 \div 4.4 = b$
J $5652 \div 6.28 = q$
K $3.1356 \div 78.39 = n$
L $.0144 \div .12 = s$
M $18.564 \div .39 = d$
N $254.7 \div .3 = z$
O $.2407 \div .29 = t$
P $8368.1 \div 10.66 = m$

A $275.2 \div 16 = c$
B $55.5546 \div 17.47 = n$
C $105.4 \div .136 = y$
D $17.76 \div .3 = h$
E $39.6 \div 11 = r$
F $.9248 \div .272 = f$
G $22.5 \div 1.5 = w$
H $1396.83 \div 30.3 = a$

Now you should be able to divide numbers expressed by decimal fraction numerals.

Keeping skilful

Find the sum.

A 3.682, 7.425, 8.399
B .0037, 25.4292
C 17.93, .02, .64, 1.77
D 37.1, 19.8, 2.6, 107.3
E .1894, .3625, .1818
F 37.56, 7.39, 9.28
G .745, .631, .892
H 7.2, .6, 4.5, .9
I 19.52, 17.48, 3.12

Find the difference.

A 7.39, .78
B .055, .493
C 21.3, 6.7
D 3.7748, 3.2798
E 51.16, 14.02
F 8.5363, 4.5607
G 3.49, 18.76
H 1.784, .939
I 124.8, 125.3

A $36.46 - z = 18.95$
B $6.2 + 7.5 = n$
C $w + .3596 = 6.0085$
D $a - 3.345 = 9.728$
E $6.38 + m = 17.92$
F $1.349 - .895 = y$
G $b - 6.72 = 3.49$
H $72.04 + 16.93 = r$
I $.6832 - n = .0055$
J $3.227 - 1.895 = x$

Thinking straight

I know that there will be hundredths in the answer when there are hundredths in the number I am dividing. So I know the answer is 3.83 and not 383.
67/256.61 300
20100
5561 80
5360
201 3
201
3.83
A
That means that if I want hundredths in the answer, I must have hundredths in the number I am dividing. But here I have tenths. How can I get hundredths in the answer?
26/126.3
B
Well, Sam, you know that 126.30 is another name for 126.3. So you can just put a zero at the right of the 126.3.
I see! Now when I divide, I will have hundredths in my answer.
26/126.30
C
I have hundredths in my answer. But I also have a remainder. What shall I do with it, Miss Allen?
26/126.30 400
10400
2230 80
2080
150 5
130
20 4.85
D
You can use the remainder to round off the answer. If the remainder is equal to, or greater than, one half of the divisor, you round up the answer. If the remainder is less than one half of the divisor, you ignore it. What will you do in this example?
26/126.30 400
10400
2230 80
2080
150 5
130
20 4.85
E
Well, the remainder, 20, is greater than one half of the divisor, 26. So I round up the answer to 4.86. But if the remainder had been less than 13, I would ignore it.
26/126.30 400
10400
2230 80
2080
150 5
130
20 4.85 4.86
F

A Now divide 126.3 by 26 so that you will have thousandths in the answer. You will need thousandths in the number you are dividing. So use 126.300 as another name for 126.3.

```
26)126.300 | 4000
   104000
    22300 |  800
    20800
     1500 |   50
     1300
      200 |    7
      182
       18 | 4.857
          | 4.858
```

Round the answer up to 4.858 because 18 is greater than one half of the divisor, 26. Would the answer to the nearer tenth be 4.9 or 4.8? Why?

B Divide 69.62 by 7.25 so that you will have tenths in the answer. First multiply both 7.25 and 69.62 by 100. Why? Since you will need tenths in the number you are dividing, use 6962.0 as another name for 6962.

```
725)6962.0 | 90
   65250
    4370 |  6
    4350
      20 | 9.6
```

You can ignore the remainder because 20 is less than one half of 725, the divisor.

C Divide 37.204 by 19.8 so that you will have hundredths in the answer. Why do you first multiply 19.8 and 37.204 by 10? Now you must find the quotient of 372.04 ÷ 198. Is the number to be divided expressed in hundredths?

```
198)372.04 | 100
    19800
    17404 |  80
    15840
     1564 |   7
     1386
      178 | 1.87
          | 1.88
```

Why do you round up the answer to 1.88? What would the answer be to the nearer tenth?

D Divide 623 by 78. Find the answer first to tenths, then to hundredths, and then to thousandths.

THINK What three names do you use for 623?

Find the answer for each exercise below. The word in parentheses tells what decimal fraction you are to have in the answer.

- **A** $8.6 \div 75 = b$ (hundredths)
- **B** $492.6 \div 8.9 = z$ (tenths)
- **C** $897.6 \div 84 = n$ (thousandths)
- **D** $23 \div 120 = s$ (tenths)
- **E** $35.756 \div 1.8 = g$ (hundredths)
- **F** $95.38 \div 6 = y$ (thousandths)

Using arithmetic

For the problems below, express each answer with a decimal fraction numeral if necessary. Each problem tells you how to express the answer.

A John had 5 cu. in. of lead that he found weighed 2 lb. What is the weight of 1 cu. in. of lead? Find the answer to tenths.

You can write an equation of ratios for Problem A. What does each term in the equation below stand for?

$$\frac{2}{5}=\frac{n}{1}$$

What numeral replaces n?

B John read in a science book that 1 cu. in. of water weighs approximately .04 lb. One cu. in. of lead is about how many times as heavy as 1 cu. in. of water?

$$\frac{.4}{.04}=\frac{s}{1}$$

Without dividing, how do you know that your answer will be a whole number? What numeral replaces s?

C John also read that 1 gal. of water weighs 10 lb. How much does 1 pt. of water weigh? Find the answer to hundredths.

THINK There are how many pints in 1 gallon?

In some of the following problems you will need to understand what 1 cubic foot means. Think of 1 cubic foot as meaning a cube 1 ft. long, 1 ft. wide, and 1 ft. high.

D John's science book also stated that 1 cubic foot of water weighs about 62.3 lb. and that mercury, when it is a liquid, weighs 847.4 lb. per cubic foot. Mercury is how many times as heavy as water? Find your answer to tenths.

E John's teacher said that there are about 6.23 gal. in 1 cubic foot. How much will 1 gal. of mercury weigh? Find the answer to hundredths.

THINK How do you know from Problem D that 6.23 gal. of mercury weigh 847.4 lb.?

F Aluminum weighs 168.5 lb. per cubic foot. This metal is how many times as heavy as water? Find the answer to tenths.

THINK Which problem tells you the weight of 1 cubic foot of water?

G 1 cubic foot of walnut wood weighs 40.25 lb. The weight of 1 cubic foot of walnut wood is what fraction of the weight of 1 cubic foot of water? Find the answer to hundredths.

H Uranium, which is a heavy metal, weighs 1167.4 lb. per cubic foot. How many cubic feet of water are equal in weight to 1 cubic foot of uranium? Find the answer to tenths.

I Cork is a very light wood. It weighs only 15 lb. per cubic foot. 1 cubic foot of water is how many times as heavy as 1 cubic foot of cork? Find the answer to tenths.

J Ice, which is water in a solid state, is lighter than water as a liquid. Ice weighs 56.06 lb. per cubic foot. What fraction of a cubic foot of water will make 1 cubic foot of ice? Find the answer to hundredths.

K Lithium is a very light metal. 12.5 cubic feet of this metal weigh 416.25 lb. What is the weight of 1 cubic foot of lithium? Find the answer to hundredths.

L Balsa, a very light wood, weighs only 7.3 lb. per cubic foot. How many cubic feet of balsa weigh 100 lb.? Find the answer to tenths.

M 6.5 cubic feet of ebony weigh 539.5 lb. How much does 1 cubic foot of ebony weigh?

N White pine weighs about 22 lb. per cubic foot. What fraction of a cubic foot of water is equal in weight to 1 cubic foot of white pine? Find the answer to tenths.

O Slate weighs about 162.8 lb. per cubic foot. What fraction of a cubic foot of slate will weigh 100 lb.? Find the answer to hundredths.

Checking up

The word in parentheses after each exercise tells what decimal fraction you are to have in the quotient.

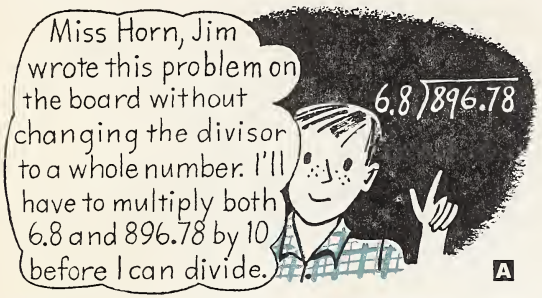
Test 1

A $3 \div 4 = m$ (hundredths)
B $38.64 \div 9 = w$ (thousandths)
C $16 \div .9 = e$ (thousandths)
D $8.43 \div 3.622 = n$ (tenths)
E $5.8 \div 123 = p$ (hundredths)
F $9 \div 27 = f$ (thousandths)
G $11 \div .016 = d$ (tenths)
H $.27 \div 12.3 = z$ (ten-thousandths)

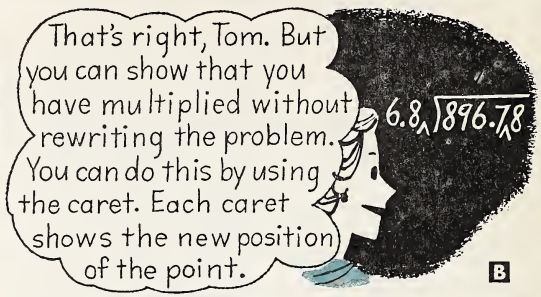
Test 2

A $14.4 \div 6 = n$ (tenths)
B $3 \div .27 = y$ (hundredths)
C $14.9 \div 6.2 = t$ (thousandths)
D $21 \div 24 = a$ (tenths)
E $.3 \div .04 = s$ (hundredths)
F $172.6 \div 45 = q$ (hundredths)
G $28 \div 375 = b$ (ten-thousandths)
H $8 \div .26 = x$ (thousandths)

Side trip


Miss Horn, Jim wrote this problem on the board without changing the divisor to a whole number. I'll have to multiply both 6.8 and 896.78 by 10 before I can divide.
6.8)896.78

A


That's right, Tom. But you can show that you have multiplied without rewriting the problem. You can do this by using the caret. Each caret shows the new position of the point.
6.8^)896.78^

B

You can use the caret to show how you would rewrite numerals for division.

A In the example below, why do you write a zero at the right of 75.4 before putting in the caret? Will there be any decimal places in the answer?

.8 3)7 5.4 .8 3^)7 5.4 0^

B Now look at the example below. What has been done to the example shown in Exercise A? Where should you put the point in your answer?

.8 3^)7 5.4 0^0 0

For each exercise below, rewrite the numerals for division. Use the caret. Then find the answer. The word in parentheses tells how many decimal places you are to have in the answer.

A .3 8)7.6 8 4 (hundredths)

B 2.9)1 1 3 7 (tenths)

C 3.4 6).5 9 3 7 (thousandths)

D .4 7 2)6.7 (tenths)

E 1.8 5)1 3.9 2 (hundredths)

F 3 6.2)2 7 8.4 4 (hundredths)

G .0 2)4 7 (tenths)

Keeping skilful

A $3\frac{3}{4} \times \frac{1}{8} = v$

B $2 \times 2\frac{2}{3} = m$

C $1\frac{1}{2} \times 6\frac{7}{8} = e$

D $\frac{9}{10} \times \frac{4}{5} = p$

E $8\frac{3}{5} \times 7\frac{1}{3} = s$

F $6\frac{5}{6} \times \frac{1}{2} = a$

G $14\frac{7}{12} \times 6 = z$

H $\frac{3}{8} \times 7 = q$

I $11\frac{1}{3} \times 5\frac{4}{9} = d$

J $3\frac{1}{2} \times 7\frac{1}{2} = n$

A $4\frac{1}{4} \div 1\frac{1}{8} = t$

B $18 \div \frac{9}{10} = c$

C $7\frac{4}{5} \div 3\frac{2}{3} = w$

D $\frac{7}{12} \div 3 = n$

E $6\frac{1}{2} \div 4\frac{1}{6} = z$

F $\frac{3}{5} \div \frac{1}{5} = b$

G $5\frac{5}{8} \div 2 = k$

H $8 \div \frac{3}{4} = r$

I $2\frac{9}{10} \div 1\frac{1}{10} = x$

J $17\frac{1}{9} \div 4 = f$

Side trip

On pages 162 and 163, you learned another way to do division of whole numbers. You can also use this method for division of decimal fractions.

A Divide 63.97 by 75.4 so that you will have hundredths in the answer.

You can use the caret to show where the points are in the numerals you use in your computation.

$$75.4\overset{\wedge}{)}63.97\overset{\wedge}{0}$$

You want hundredths in the answer. So you need hundredths in the number you divide.

$$\begin{array}{r} .85 \\ \cancel{.84} \\ 75.4\overset{\wedge}{)}63.97\overset{\wedge}{0} \\ 6032 \\ \hline 3650 \\ 3016 \\ \hline \cancel{634} \end{array}$$

The remainder, 634, is greater than one half of the divisor, 754. So you write the answer as .85, and not .84.

B Divide 1.151 by 11.98 so that you will have thousandths in the answer.

Is the caret used correctly below?

$$11.98\overset{\wedge}{)}1.15\overset{\wedge}{1}00$$

Why are 2 zeros written here? ↑

Is the answer correct? → .096

$$\begin{array}{r} 11.98\overset{\wedge}{)}1.15\overset{\wedge}{1}00 \\ 10782 \\ \hline 7280 \\ 7188 \\ \hline \cancel{92} \end{array}$$

Why can you cross out the remainder without changing the answer? → 92

For each exercise below, write your computation in the way shown in this lesson. Find each answer to hundredths.

A $8732.5 \div 1.9 = s$
B $92 \div 453 = y$
C $7003.6 \div 11.63 = n$
D $433.69 \div .752 = h$
E $2.885 \div 3.1 = r$
F $42.96 \div .8 = c$
G $697.43 \div 24.2 = w$
H $8.473 \div 9.36 = p$
I $27.008 \div 15.4 = b$

Exploring problems

A The Jackson baseball team has played 40 games so far this year. They have won .600 of these games. How many games has the team won?

When you say that the team has won .600 of its games, you may think of this as meaning that the team has won .600 of a game for each game played. You are to find how many of the 40 games the team has won.

First make an equation of ratios. Each ratio will express the comparison of games won with games played.

$$\frac{.600}{1} = \frac{t}{40}$$

Ratio expressing the comparison for 1 game

Ratio expressing the comparison for 40 games

$.600 \times 40 = 24.000$ or 24

24 replaces t. The Jackson baseball team has won 24 games.

B One player on the Jackson team has made 49 hits in 147 times at bat. This player's hits are what fraction of his number of times at bat? Find the answer to thousandths.

You are to find what fraction the player's hits are of the number of times at bat. You may think of this as finding what fraction of a hit he made for each time at bat.

Again make an equation of ratios. Each ratio will express the comparison of hits with times at bat.

$$\frac{49}{147} = \frac{z}{1}$$

Ratio expressing the comparison for ▤ times at bat

Ratio expressing the comparison for ▤ time at bat

$49 \div 147 = .333$

.333 replaces z. The player's hits are .333 of his number of times at bat.

C One player has made 16 single-base hits. These hits are .8 of all his hits. How many hits in all has this player made?

You can say that, for each hit, the player has made .8 of a single-base hit. You know he made 16 single-base hits. So you can make an equation of ratios that compare his single-base hits with all his hits.

$$\frac{.8}{1} = \frac{16}{k}$$

Ratio expressing the comparison for ▤ hit

Ratio expressing the comparison for the total number of hits

$16 \div .8 = 20$

In the equation for Problem C, what numeral replaces k? The player has made ▒ hits.

D The team that is first in the league has won 32 of the 42 games it has played. This team has won what fraction of the games it has played? Find the answer to thousandths.

THINK You want to find what fraction of its games this team has won for each game it has played.

$$\frac{32}{42} = \frac{w}{1}$$

$\frac{32}{42}$: Ratio expressing the comparison for ▒ games

$\frac{w}{1}$: Ratio expressing the comparison for ▒ game

What numeral replaces w? This team has won what fraction of its games?

E Each year, the Jackson team plays .65 of its games at night. The team plays 78 games at night. How many games in all does the Jackson team play each year?

THINK You know what fraction of a game the Jackson team plays at night for each game that it plays. You also know how many games the team plays at night. In the equation below, what comparison does each ratio express?

$$\frac{.65}{1} = \frac{78}{b}$$

What numeral replaces b in the equation for Problem E? The Jackson team plays ▒ games each year.

F One player on the Jackson team has made 50 hits so far this year. Of the 50 hits, .32 have been extra-base hits. How many extra-base hits has this player made?

THINK You know that for each hit the player has made, he has made .32 of an extra base hit. You know how many hits he has made in all. In the equation below, what comparison does each ratio express?

$$\frac{.32}{1} = \frac{d}{50}$$

What numeral replaces d? The player has made ▒ extra-base hits.

G So far, the Jackson team has played what fraction of its total number of games for the year? Find the answer to hundredths.

H One of the pitchers for the Jackson team has a record of 6 wins and 3 losses. This pitcher's wins are what fraction of his total number of wins and losses? Find the answer to thousandths.

THINK How can you find the total number of wins and losses? What equation of two ratios can you make for this problem?

I Another Jackson pitcher has a record of 4 wins and 5 losses. This pitcher's wins are what fraction of his total wins and losses? Find the answer to thousandths.

J The Jackson second baseman has fielded correctly .980 of the balls that have been hit to him. He has fielded 147 balls correctly. How many balls have been hit to him in all?

K The player who is first in the league in hitting has made a hit .380 of the times he has been at bat. He has made 57 hits. How many times has this player been at bat?

L The team that is last in the league has played 40 games so far. It has won .375 of these games. How many games has this team won?

M The Jackson team has made 435 hits in 1562 times at bat. The hits are what fraction of the number of times at bat? Find the answer to thousandths.

N Attendance at the Jackson games this year has been about .95 of what it was for the same number of games last year. By this time last year, attendance had been about 80,000. What has the approximate attendance been so far this year?

Now you should be able to solve comparison problems in which you use decimal fraction numerals to represent fractions.

Keeping skilful

In each sentence below, replace t with a numeral that will make a true statement.

A $24 \times t < 75$

B $3\frac{4}{5} + t \neq 6$

C $75 \div t = 7\frac{1}{2}$

D $48 - 13 > t$

E $t \times 5\frac{1}{2} \neq 10$

F $19.8 \times 75.6 = t$

G $13\frac{9}{10} + t > 17\frac{7}{8}$

H $t + 2 = 2$

I $7\frac{1}{4} \div t \neq 3\frac{1}{8}$

J $29.8 + t > 45.7$

K $3\frac{3}{5} - 2\frac{3}{8} < t$

L $95 \times t = 114$

M $37.23 - 19.64 > t$

N $t \times 6 = 6$

O $5\frac{3}{4} \times t < 7\frac{1}{3}$

P $t - 137.02 = .95$

Q $29 \times t = 0$

R $28 \times 7\frac{1}{2} < t$

S $t + 35 < 119\frac{9}{10}$

T $10.80 \div 7.2 > t$

U $43.1 + t \neq 50.1$

V $t - 113 = 0$

W $19\frac{2}{3} \div 7\frac{7}{12} \neq t$

X $15\frac{1}{2} \div t < 3$

Exploring problems

A The sixth graders were weighed in health class. Jim weighs 76 lb. Carol weighs 68 lb. Carol's weight is what fraction of Jim's weight? Express the answer with a decimal fraction numeral in hundredths.

THINK To solve Problem A, you must find what fraction 68 is of 76. You can write the equation below for Problem A.

$$\frac{68}{76} = \frac{n}{1}$$ ← Fraction of a pound Carol weighs for each pound Jim weighs

To get the answer, you can divide 68 by 76.

68 is what fraction of 76?

B Jim's weight one year ago was .875 of his weight now. What was Jim's weight one year ago?

THINK In Problem B, you are to find the number that is .875 of 76. You can write the equation below.

$$\frac{.875}{1} = \frac{k}{76}$$ ← Number of pounds Jim weighed one year ago

To get the answer, you can multiply 76 by .875.

▒ is .875 of 76.

C One year ago Cal was 57 in. tall. His height then was .95 of his height now. What is his height now?

THINK 57 is .95 of what number? You can write the equation below.

$$\frac{.95}{1} = \frac{57}{d}$$ ← Cal's height now

To get the answer, you can divide 57 by .95.

57 is .95 of ▒.

D Ann is 53.5 in. tall. Sue is 57 in. tall. Ann's height is what fraction of Sue's height? Find the answer to hundredths.

THINK What equation can you make to find what fraction 53.5 is of 57?

Which of these sentences is correct for Problem D?

1 57 is what fraction of 53.5?

2 53.5 is what fraction of 57?

E The sixth-grade class found that the average height of the boys is 58 in. They also found that the average height of the girls is .95 of the average height of the boys. What is the average height of the girls?

Now turn the page.

THINK What equation can you make to find .95 of 58?

Which of these sentences is correct for Problem E?

1 .95 of 58 is ▭.
2 58 is .95 of ▭.

F Alice's weight is .89 of George's weight. Alice weighs 66 lb. What does George weigh? Find the answer to tenths.

THINK 66 is .89 of what number? What equation can you write for Problem F?

Which of these sentences is correct for Problem F?

1 .89 of 66 is ▭.
2 66 is .89 of ▭.

For each exercise below, first make an equation of ratios. Then find the answer.

A ▭ is .48 of 72.
B 95 is what fraction of 127.3? Find the answer to hundredths.
C 39 is .125 of ▭.
D 12.8 is what fraction of 49? Find the answer to tenths.
E .6 of 127.2 is ▭.
F .322 of ▭ is 9. Find the answer to hundredths.
G ▭ is .33 of 98.7.
H 11.6 is what fraction of 1160? Find the answer to hundredths.
I .75 of ▭ is .892. Find the answer to thousandths.
J ▭ is .24 of 38.79.

Now you know how to work with problems and sentences that use decimal fraction numerals to compare one number with another number.

Keeping skilful

If there is a decimal fraction numeral in the answer, express it in hundredths.

A 21 is ▭% of 75.
B 45% of 113 is ▭.
C 128% of ▭ is 16.4.
D ▭ is 67% of 92.
E ▭% of 250 is 325.
F 37 is 5% of ▭.

A $75.4 \times .83 = r$
B $.93 \times .6 = y$
C $10.3 \times .672 = n$
D $46 \times 3.4 = z$
E $.126 \times .7 = c$
F $.75 \times .95 = s$
G $.03 \times .04 = k$
H $1.9 \times .37 = w$
I $.086 \times 4.17 = b$

Find each answer to hundredths.

A $14 \div 386 = v$
B $77.2 \div 3.58 = q$
C $1.934 \div .11 = t$
D $532 \div 6.2 = b$
E $88.78 \div 3 = n$
F $42.6 \div .25 = y$
G $9 \div 63 = p$

Thinking straight

A $\frac{3 \div 8}{8 \div 8} = \frac{3 \div 8}{1}$

$$
\begin{array}{r|l}
8\overline{)3.000} & 300 \\
\underline{2400} & \\
600 & 70 \\
\underline{560} & \\
40 & 5 \\
\underline{40} & \\
0 & .375
\end{array}
$$

$\frac{3}{8} = \frac{.375}{1}$ or .375

A Suppose you want to find a decimal fraction numeral to use instead of the common fraction numeral $\frac{3}{8}$. To do this, first divide the 8 in the denominator by 8 to get 1 as a new denominator. Why must you also divide the 3 in the numerator by 8? Look at the work labelled A. Does this show the result of dividing both the number in the denominator and the number in the numerator by 8?

B Now you must divide 3 by 8. You know that the answer can be expressed as a decimal fraction numeral. Usually, you will want to divide to thousandths. Is 3.000 another name for 3? Look at the work shown in red. Why is the answer expressed as .375 and not as 375?

C You can now write $\frac{.375}{1}$ in place of $\frac{3 \div 8}{1}$. Is .375 another name for $\frac{.375}{1}$? .375 is another name for $\frac{3}{8}$.

B $\frac{2 \div 5}{5 \div 5} = \frac{2 \div 5}{1}$

$$
\begin{array}{r|l}
5\overline{)2.000} & 400 \\
\underline{2000} & \\
0 & .400
\end{array}
$$

$\frac{2}{5} = \frac{.400}{1}$ or .400 or .4

D The work labelled B shows how to find a decimal fraction numeral to use instead of the common fraction numeral $\frac{2}{5}$. Why are both the 5 in the denominator and the 2 in the numerator divided by 5?

E In the computation shown in red, 2.000 is used as another name for ▒. How do you know that the quotient is .400, and not 400?

F Is $\frac{.400}{1}$ another name for $\frac{2}{5}$? Is .400 another name for $\frac{2}{5}$? Is .4 another name for .400? Is .4 another name for $\frac{2}{5}$?

G Suppose you had used 2.0 instead of 2 when you divided 2 by 5. What would your answer have been?

H The work below shows how to find a decimal fraction numeral to use in place of $\frac{1}{3}$.

$\frac{1 \div 3}{3 \div 3} = \frac{1 \div 3}{1}$

$$
\begin{array}{r|l}
3\overline{)1.000} & 300 \\
\underline{900} & \\
100 & 30 \\
\underline{90} & \\
10 & 3 \\
\underline{9} & \\
1 & .333
\end{array}
$$

$\frac{1}{3} = \frac{.333}{1}$ or .333

I Can .333 be used instead of $\frac{1}{3}$? Can you use .33 instead of $\frac{1}{3}$? Can you also use .3?

J Suppose you want to find a decimal fraction numeral expressed in ten-thousandths to use instead of $\frac{1}{3}$. How do you know, without computing, that the decimal fraction numeral will be .3333?

K Use the way shown in this lesson to show that .1667 can be used for $\frac{1}{6}$. Can you use .167? Can you also use .17?

L When you find a decimal fraction numeral to use instead of $\frac{7}{8}$, why do you divide 7 by 8? $\frac{7}{8}$ is equal to what decimal fraction numeral expressed in thousandths?

For each common fraction numeral below, find a decimal fraction numeral expressed in thousandths that can be used in its place.

A $\frac{1}{2}$	**D** $\frac{1}{9}$	**G** $\frac{9}{10}$	**J** $\frac{3}{7}$
B $\frac{1}{4}$	**E** $\frac{5}{6}$	**H** $\frac{4}{15}$	**K** $\frac{5}{12}$
C $\frac{3}{8}$	**F** $\frac{2}{3}$	**I** $\frac{1}{20}$	**L** $\frac{1}{11}$

A Find the common fraction numeral that is equal to .75. Express the fraction numeral in lowest terms.

You know that .75 can be written as $\frac{75}{100}$.

$\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

Is $\frac{3}{4}$ another name for .75?

B Find the common fraction numeral that is equal to .125. Express the fraction numeral in lowest terms.

.125 can be written as $\frac{\blacksquare}{\blacksquare}$.

$\frac{125}{1000} = \frac{125 \div 125}{1000 \div 125} = \frac{1}{8}$

Is $\frac{1}{8}$ another name for .125?

For each common fraction numeral, find a decimal fraction numeral expressed in thousandths that can be used instead. For each decimal fraction numeral, find a common fraction numeral that is equal. Express it in lowest terms.

A .36	**D** .85	**G** $\frac{5}{8}$	**J** .27
B $\frac{9}{16}$	**E** $\frac{7}{12}$	**H** .44	**K** $\frac{6}{25}$
C .56	**F** .8	**I** $\frac{11}{24}$	**L** .95

Thinking straight

A $25\% = \frac{25}{100} = \frac{.25}{1}$

B $80\% = \frac{80}{100} = \frac{.80}{1} = \frac{.8}{1}$

C $\frac{.25}{1} = \frac{\frac{1}{4}}{1}$

D $\frac{.8}{1} = \frac{\frac{8}{10}}{1} = \frac{\frac{4}{5}}{1}$

A Look at Example A above. You know that 25% is a ratio that can be written as 25 over 100. Use the ratio test to show that the ratio .25 over 1 is equal to the ratio 25 over 100. Both 25 and 100 can be divided by ▬ to get the ratio .25 over 1. Can 25% also be expressed by the ratio .25 over 1?

B Look at Example B. Use the ratio test to show that the ratio .80 over 1 is equal to the ratio 80 over 100. Can .80 be written as .8? Can 80% also be expressed by the ratio .8 over 1?

Write the following per cents as ratios with a decimal fraction numeral in the first term and 1 in the second term.

C 42%	**G** 139%	**K** 6%	**O** 18%
D 63%	**H** 372%	**L** 85%	**P** 3%
E 1%	**I** 248%	**M** 50%	**Q** 55%
F 20%	**J** 19%	**N** 99%	**R** 70%

A Look at Example C. Is $\frac{1}{4}$ another name for .25? Can the ratio .25 over 1 be expressed as $\frac{1}{4}$ over 1?

B In Example A you saw that 25% can be expressed as .25 over 1. Can 25% also be expressed as $\frac{1}{4}$ over 1?

C Now look at Example D. Is $\frac{8}{10}$ another name for .8? Is $\frac{4}{5}$ another name for $\frac{8}{10}$? Can the ratio .8 over 1 be expressed as $\frac{4}{5}$ over 1? Can 80% be expressed as $\frac{4}{5}$ over 1?

For each exercise below, write a ratio with a decimal fraction numeral in the first term and 1 in the second term. Then write a ratio with a common fraction numeral in lowest terms in the first term and 1 in the second term.

D 15%	**G** 128%	**J** 76%	**M** 45%
E 30%	**H** 150%	**K** 35%	**N** 8%
F 93%	**I** 236%	**L** 2%	**O** 90%

Exploring problems

A The average yearly precipitation at Saint John is about 48 in. About 65% of this precipitation comes during the 6 months from November to April. About how much precipitation does Saint John have during these 6 months?

THINK 65% can be expressed in the following ways:

$$65\% = \frac{65}{100} = \frac{.65}{1} = \frac{\frac{13}{20}}{1}$$

So you can write these three sentences for Problem A.

1 65% of 48 is ▒.
2 .65 of 48 is ▒.
3 $\frac{13}{20}$ of 48 is ▒.

You can compute in the three ways shown below to solve Problem A.

1 $\frac{65}{100} = \frac{s}{48}$ $\quad 100s = 65 \times 48$

$\frac{65 \times 48}{100} = 31.20$ or 31.2

65% of 48 is 31.2.

2 $\frac{.65}{1} = \frac{s}{48}$ $\quad s = .65 \times 48$

$.65 \times 48 = 31.20$ or 31.2

.65 of 48 is 31.2.

3 $\frac{\frac{13}{20}}{1} = \frac{s}{48}$ $\quad s = \frac{13}{20} \times 48$

$\frac{13}{20} \times 48 = 31\frac{2}{10}$ or 31.2

$\frac{13}{20}$ of 48 is 31.2.

B In the summer of 1954 there were 175 oil wells in Manitoba. This was 70% of the number of oil wells in Manitoba by the fall of 1954. How many oil wells were there in Manitoba that fall?

THINK 70% can be expressed in the following ways.

$$70\% = \frac{70}{100} = \frac{.70}{1} = \frac{.7}{1} = \frac{\frac{7}{10}}{1}$$

So you can write these three sentences for Problem B.

1 175 is 70% of ▒.
2 175 is .7 of ▒.
3 175 is $\frac{7}{10}$ of ▒.

You can compute in the three ways shown below to solve Problem B.

1 $\frac{70}{100} = \frac{175}{n}$ $\quad 70n = 100 \times 175$

$\frac{100 \times 175}{70} = 250$

175 is 70% of 250.

2 $\frac{.7}{1} = \frac{175}{n}$ $\quad .7n = 175$

$\frac{175}{.7} = 250$

175 is .7 of 250.

3 $\frac{\frac{7}{10}}{1} = \frac{175}{n}$ $\quad \frac{7}{10}n = 175$

$\frac{175}{\frac{7}{10}} = 250$

175 is $\frac{7}{10}$ of 250.

C The Millers have a monthly income of $475. They spend an average of $133 per month for food. What part of their monthly income do the Millers spend for food?

THINK You are asked to find what part of their income the Millers spend for food. The answer to Problem C can be expressed as a per cent, as a decimal fraction numeral, and as a common fraction numeral.

So you can write the three sentences below for Problem C.

1 133 is ▒% of 475.

2 133 is what fraction of 475? Express the answer as a decimal fraction numeral in hundredths.

3 133 is what fraction of 475? Express the answer as a common fraction numeral in lowest terms.

You can compute in the three ways shown below to solve Problem C.

1 $\frac{133}{475}=\frac{d}{100}$ $\quad 475d = 133 \times 100$

$\frac{133 \times 100}{475} = 28$

133 is 28% of 475.

2 $\frac{133}{475}=\frac{d}{1}$ $\quad 475d = 133$

$\frac{133}{475} = .28$

133 is .28 of 475.

3 $\frac{133}{475}=\frac{d}{1}$ $\quad 475d = 133$

$\frac{133}{475} = \frac{7}{25}$

133 is $\frac{7}{25}$ of 475.

D The population per square mile in Ontario is now about 16 persons. The population per square mile 15 years ago was about 75% of what it is now. What was the population per square mile 15 years ago?

THINK 75% can be expressed in the following ways.

$$75\% = \frac{75}{100} = \frac{.75}{1} = \frac{\frac{3}{4}}{1}$$

Are the three sentences below correct for Problem D?

1 75% of 16 is ▒.

2 .75 of 16 is ▒.

3 $\frac{3}{4}$ of 16 is ▒.

Write an equation of ratios for each sentence above. Then compute to solve each equation.

E Honolulu, Hawaii, has an average yearly rainfall of about 24 in. An average of 8 in. of rainfall comes during the months of January and December. About what part of the average yearly rainfall in Honolulu comes during these two months?

Now turn the page.

THINK You are asked to find what part of the rainfall comes during January and December. You know that the answer can be expressed as a per cent, or as a decimal fraction numeral, or as a common fraction numeral.

Are the three sentences below correct for Problem E?

1. 8 is ▒% of 24. Find the answer to the nearer per cent.
2. 8 is what fraction of 24? Express the answer as a decimal fraction numeral in hundredths.
3. 8 is what fraction of 24? Express the answer as a common fraction numeral in lowest terms.

Write an equation of ratios for each sentence above. Then compute to solve each equation.

F The population of 56 persons per square mile in the United States is approximately 10% of the population per square mile in Great Britain. What is the approximate population per square mile in Great Britain?

THINK 10% can be expressed in the following ways.

$$10\% = \frac{10}{100} = \frac{.1}{1} = \frac{\frac{1}{10}}{1}$$

Are the following sentences correct for Problem F?

1. 56 is 10% of ▒.
2. 56 is .1 of ▒.
3. 56 is $\frac{1}{10}$ of ▒.

Write an equation of ratios for each sentence above. Then compute to solve each equation.

Make three sentences for each problem below. Make an equation for each sentence and solve each equation.

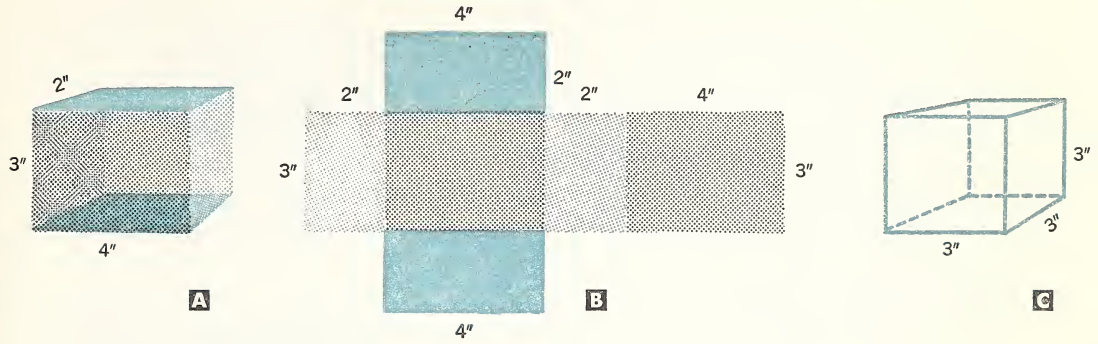
G The distance across Lake Erie is about 35% of the distance across Lake Superior. The distance across Lake Superior is 160 miles. What is the distance across Lake Erie?

H The population per square mile in Canada is 5 persons. The population per square mile in Brazil is 20 persons. What per cent, common fraction numeral, and decimal fraction numeral can you use to compare Canada's population per square mile with Brazil's?

I The area of Yoho and Kootenay National Parks together is 25% of the area of Jasper National Park. Yoho and Kootenay cover 1050 sq. mi. What is the area of Jasper?

Now you should be able to write equations using per cents, decimal fraction numerals, and common fraction numerals when you solve per cent problems.

Thinking straight



A Picture A shows a rectangular prism. You can describe this prism as being 4 inches long. What is the width of Prism A? What is the height of Prism A?

B Prism A has how many faces? Two of these faces are usually called bases. Each face has the shape of a ~~~.

C Diagram B shows one way that Prism A would look if it were flattened out. The rectangles shown in solid blue represent the bases of Prism A. What do the four other rectangles represent?

D Can you use the equation below to find the area of one base of Prism A?

$$\frac{4}{1}=\frac{t}{2}$$

E You can also use the equation $4 \times 2 = t$ to find the area of one base. The area is ▒ sq. in. What is the total area of both bases?

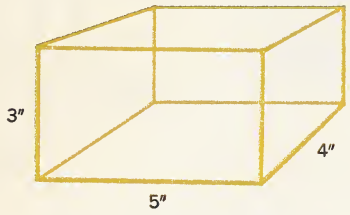
F Now suppose you want to find the total area of the other faces. These faces form a rectangle $12'' \times$ ▒". Can you use the equation $12 \times 3 = n$ to find the area of this rectangle?

G You can find the total area of the faces of Prism A by adding 16 sq. in. and ▒ sq. in. The total surface area is 52 sq. in.

H Picture C shows a cube. What is the area of one face? Why is the total surface area of this cube 6 times the area of one face?

Exploring problems

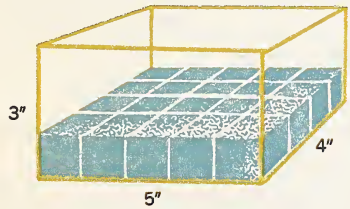
A A box is in the shape of a rectangular prism with a base 5″ x 4″ and a height of 3″. What is its volume in cubic inches?



Imagine that the box is being filled with 1-inch cubes. There are 20 cubes in the first layer.

Is the number of cubes in 1 layer the same as the number of square inches in the area of the base of the prism? Can you represent the number of cubes in 1 layer as 5×4?

B This is a rate of (5×4) cubic inches per 1 layer. Express this rate with a ratio.

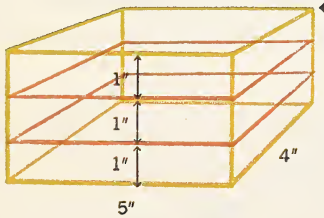


$$\frac{5 \times 4}{1}$$

5×4 ← Number of cubic inches in each layer

1 ← Number of layers

C Now you must find how many layers of cubic inches there will be. The height of the prism is 3″. Each layer will use 1″ of the height. There will be 3 layers.



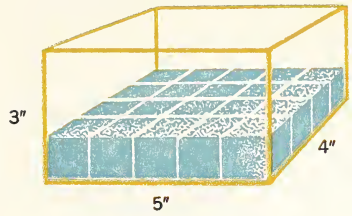
You do not know how many cubic inches will be used in all 3 layers, but you can express the rate as w cubic inches per 3 layers.

$$\frac{w}{3}$$

w ← Total number of cubic inches

3 ← Total number of layers

D



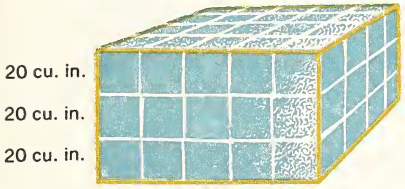
The rate at which cubic inches will be used will be the same for the 3 layers as for 1 layer. So you can make an equation of the ratios.

$$\frac{5 \times 4}{1} = \frac{w}{3}$$

Do these terms stand for number of cubic inches or number of layers? You must find the numeral that replaces w.

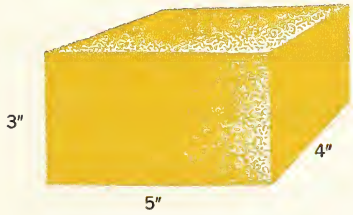
What do these terms stand for?

E



20 cubic inches are used for 1 layer. You can see that 60 cubic inches are used for all 3 layers. You can also compute to find how many cubic inches there are in the prism.

F



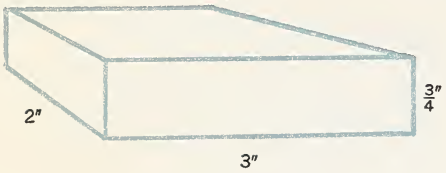
When you use the ratio test, you get this equation.

$W = (5 \times 4) \times 3$ or $W = 5 \times 4 \times 3$

60 replaces w.

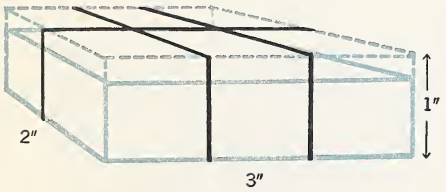
The volume of the box is 60 cu. in.

G A piece of cheese is in the shape of a rectangular prism 3 in. long, 2 in. wide, and $\frac{3}{4}$ in. high. This piece contains how many cubic inches of cheese?



The blue dotted lines in Picture H show how the prism would look if it were 1 in. high. Imagine that you are cutting the prism into 1-inch cubes. The number of cubes in a 1-inch layer will be the same as the number of square inches in the area of the base of the prism. Can you represent the number of cubes as 3×2?

H



The rate is (3×2) cubic inches per 1-inch layer. Express this rate with a ratio.

$\frac{3 \times 2}{1}$ ← Number of cubes in 1 layer

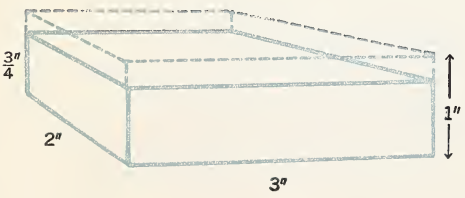
← Number of 1-inch layers

The prism is $\frac{3}{4}$ in. high. There will be $\frac{3}{4}$ cu. in. for every cubic inch in the 1-inch layer. Why can the rate also be expressed as d cubic inches per $\frac{3}{4}$ layer?

I

d ← What does this term stand for?

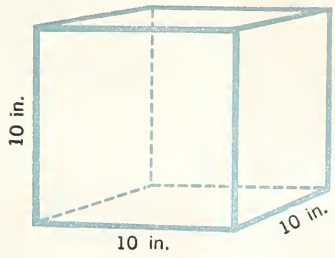
$\frac{3}{4}$ ← What does this term stand for?



Cubic inches will be used at the same rate for $\frac{3}{4}$ layer as for 1 layer.
How do you get the equation below?

$d = (3 \times 2) \times \frac{3}{4}$ or $d = 3 \times 2 \times \frac{3}{4}$

There are ▒ cubic inches in the piece of cheese.



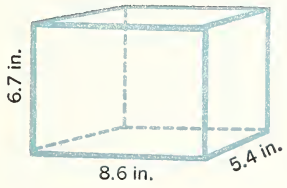
J

What are the dimensions of Cube J? What is the volume of this cube in cubic inches?

Why can you use the equation below to find the volume?

$n = (10 \times 10) \times 10$

The volume of Cube J is ▒ cu. in.



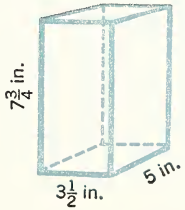
K

Prism K is 8.6″ long, 5.4″ wide, and 6.7″ high. Find the volume in cubic inches.

Why can you use the equation below to find the volume?

$m = (8.6 \times 5.4) \times 6.7$

Is the volume of Prism K about 311 cu. in.?



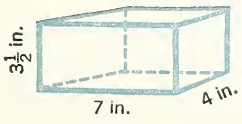
L

Prism L has a base $3\frac{1}{2}$ in. by 5 in. It has a height of $7\frac{3}{4}$ in. What is the volume of this prism in cubic inches?

How do you get the equation shown below?

$h = (3\frac{1}{2} \times 5) \times 7\frac{3}{4}$

The volume of Prism L is ▒ cu. in.



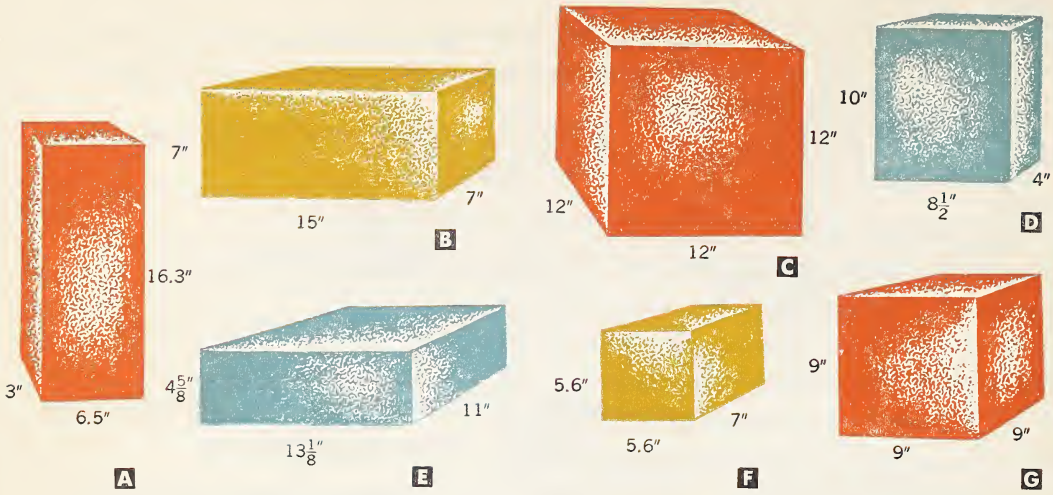
M

Prism M is 7 in. long, 4 in. wide, and $3\frac{1}{2}$ in. high. Find the volume in cubic inches.

How do you get the equation shown below?

$e = (7 \times 4) \times 3\frac{1}{2}$

The volume of Prism M is ▒ cu. in.



Each diagram above represents a rectangular prism. Find the volume of each prism in cubic inches.

In all the problems on this page, remember that you are working with rectangular prisms.

A A solid glass paperweight is a cube with an edge of 4 in. Its volume is how many cubic inches?

B Helen has a fish tank 14″ long, 8″ wide, and 10″ high on the inside. She keeps the tank filled with water to a depth of 9″. How many cubic inches of water are in the tank?

THINK The water is a prism ▤″ high.

C There are about 277 cubic inches in 1 gallon. To the nearer gallon, about how many gallons of water does Helen keep in her fish tank?

$$\frac{277}{1} = \frac{1008}{k}$$

D Martha has a fish tank 20″ long, 10″ wide, and 12″ high. She keeps the fish tank filled to a depth of 10″. How many cubic inches of water are in this tank? To the nearer gallon, how many gallons of water are in the tank?

E One box measures $7\frac{1}{2}$″ x 15″ x 11″. Another box measures 8″ x $9\frac{1}{2}$″ x $13\frac{1}{2}$″. Which box has the greater volume? Its volume is how many cubic inches greater?

F What is the volume of a metal cube with an edge of 6.2 inches?

Now you should be able to find the volumes of rectangular prisms.

Moving forward

In this lesson you will learn more about the measurement of volume.

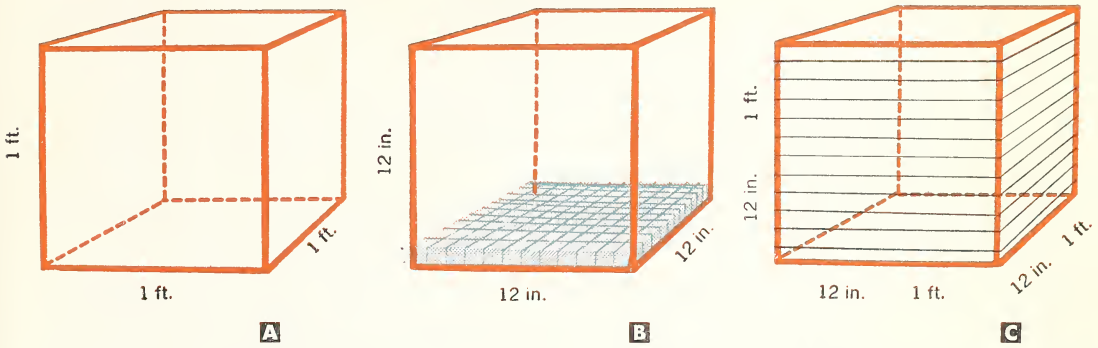


Diagram A represents a box that is a 1-foot cube. The volume of the cube is 1 cubic foot. The work below shows how to find the volume in cubic inches.

A Why can you think of each dimension of the box as 12 in.? Look at Diagram B. If you fill the box with 1-inch cubes, there will be (12×12) cubes in 1 layer. This is a rate of ▒ cu. in. per 1 layer.

$\frac{144}{1}$ ← What does 144 represent?

← What does 1 represent?

B Look at Diagram C. How do you know that there will be 12 layers of cubes? The rate can also be expressed as z cubic inches per 12 layers.

What will the numeral that replaces z represent? ← $\frac{z}{12}$

What does this 12 represent? ←

C Cubic inches will be used at the same rate for 12 layers as for 1 layer.

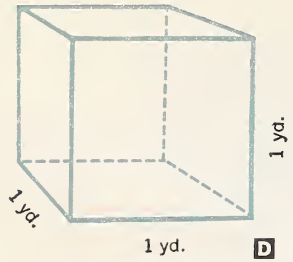
How do you find the numeral that replaces z? When you find this numeral, you will know how many cubic inches there are in 12 layers.

$$\frac{144}{1} = \frac{z}{12}$$

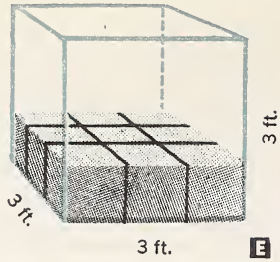
Why can you also use the equation $z = 144 \times 12$? Can you use the equation $z = 12 \times 12 \times 12$?

The volume of the box is ▒ cu. in.

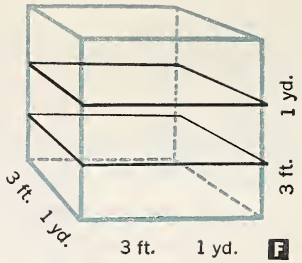
1 cubic foot = 1728 cubic inches



D



E



F

Diagram D represents another box in the shape of a cube. What are the dimensions of this box? The volume of the box is 1 cubic yard. The work below shows how to find the volume in cubic feet.

D Why can you think of each dimension of the box as 3 ft.? Look at Diagram E. In one layer of 1-foot cubes, there are (▒ × ▒) cubes. The rate is ▒ cu. ft. per 1 layer.

E Look at Diagram F. How many layers of cubes will there be? The rate can now be expressed as k cubic feet per ▒ layers.

F Cubic feet will be used at the same rate for 3 layers as for 1 layer.

What does 9 represent?

How do you find the numeral that replaces k? When you find this numeral, you will know how many cubic feet there are in 3 layers.

$$\frac{9}{1} = \frac{k}{3}$$

Why can you also use the equation $k = 9 \times 3$? Can you use $k = 3 \times 3 \times 3$?

The volume of the box is ▒ cu. ft.
1 cubic yard = 27 cubic feet

G Diagram G on page 255 represents a packing case that has the shape of a rectangular prism. What are its dimensions? You can express its volume as 12 cu. yd. How do you get the 12?

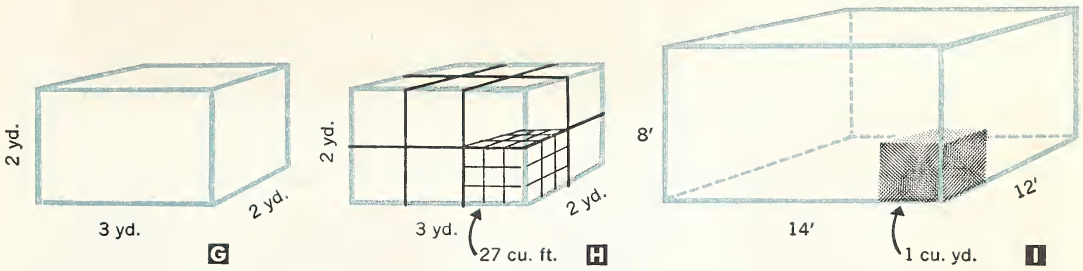
H You can also express the volume of the case in cubic feet. Find how many cubic feet there are in 12 cu. yd.

The rate of cubic feet per cubic yards is always the same. The rate of cubic feet for 1 cu. yd. is expressed as 27 per 1. Look at Diagram H. Can the rate of cubic feet for 12 cu. yd. be expressed as e per 12?

You can use this equation to find the number of cubic feet in 12 cu. yd.

$$\frac{27}{1} = \frac{e}{12}$$

Can you multiply 12 by 27 to find the numeral that replaces e?
12 cu. yd. = 324 cu. ft.



I A piece of wood has a volume of 1.5 cu. ft. What is the volume of this piece of wood in cubic inches?

In 1 cu. ft. there are ▩ cu. in.

$$\frac{1728}{1}=\frac{a}{1.5}$$

Can you find the number of cubic inches by finding the product of 1728 and 1.5?

J Diagram I represents a room. What are the dimensions of this room? You can say that the room contains 1344 cu. ft. of space. How do you get the 1344?

K You can also express the volume of the room in cubic yards. Find how many cubic yards there are in 1344 cubic feet.

Does 27 per 1 express the rate of cubic feet per cubic yards? There are 1344 cu. ft. in the room. Does 1344 per b also express the rate of cubic feet per cubic yards?

You can use the equation below to find how many cubic yards there are in 1344 cu. ft.

$$\frac{27}{1}=\frac{1344}{b}$$

$27b = 1344$. To find the numeral that replaces b, divide ▩ by ▩.
1344 cu. ft. $= 49\frac{7}{9}$ cu. yd.

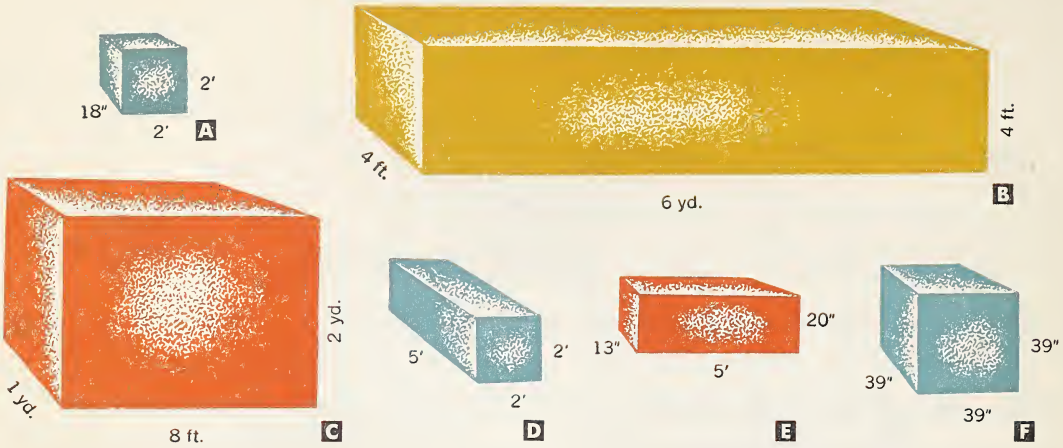
L A chest of drawers is 32″ wide, 18″ deep, and 44″ high. How many cubic feet of space does the chest of drawers occupy?

First find the volume of the chest in cubic inches. The volume is 25,344 cu. in. How do you get the 25,344?

Next express 25,344 cu. in. as cubic feet. ▩ cu. in. will be used for each cubic foot.

$$\frac{1728}{1}=\frac{25344}{W}$$

$1728w = 25344$. To find the numeral that replaces w, divide ▩ by ▩.



M Each diagram above represents a rectangular prism. Look at Diagram A. Two of its dimensions are given in feet, but the third dimension is given in inches. To find the volume in cubic inches, you can use the dimensions 24″, 24″, and 18″. Why? What is the volume in cubic inches?

N You can also find the volume of Prism A in cubic feet. To do this, you can express 10,368 cu. in. as cubic feet. Or you can use the dimensions 2′, 2′, and $1\frac{1}{2}$′. What is the volume in cubic feet?

O Find the volume of each of the other prisms above. You may have to change one dimension before you make your equation.

P A box containing a kitchen stove is 3′ wide, 2′ deep, and 42″ high. How many cubic feet of space does the box occupy?

THINK Why can you use $3\frac{1}{2}$ feet as one dimension of the box?

Q Mr. Block has a rectangular pool about 4 yd. long, 2 yd. wide, and 2 ft. deep in his yard. He wants to fill it in to make a rock garden there. About how many cubic yards of fill will he need?

THINK Why can you use $\frac{2}{3}$ yd. as one dimension of the pool?

A 21 cu. yd. = ▭ cu. ft.
B 3 cu. ft. = ▭ cu. in.
C 1180 cu. ft. = ▭ cu. yd.
D 8640 cu. in. = ▭ cu. ft.
E 405 cu. ft. = ▭ cu. yd.

Now you should be able to find the volumes of rectangular prisms in cubic inches, cubic feet, and cubic yards.

Using arithmetic

A A straight driveway is 9 ft. wide and 60 ft. long. It is to be paved with concrete 6 in. thick. How many cubic feet of concrete will there be in the driveway?

THINK Why can you think of the depth of the concrete as $\frac{1}{2}$ ft.?

B Sometimes length is measured in rods. A rod is equal to $16\frac{1}{2}$ ft. How many rods of fencing will be needed for a square garden, each side of which is 132 ft.?

C Mr. Williamson has a rectangular market garden with an area of 1 acre. Its area can also be expressed as 160 square rods. The garden is 20 rods long. How wide is it?

D Another rectangular market garden is 25 rods wide and $68\frac{1}{2}$ rods long. Is the area of this garden more than or less than 10 acres? The area is how much more than or less than 10 acres?

E A steel bar is in the shape of a rectangular prism. It is $\frac{3}{4}''$ wide, $\frac{3}{4}''$ high, and 8′ long. How many cubic inches of steel are there in the bar?

F A garden 50 ft. long and 20 ft. wide is to be covered with black dirt to a depth of 8 in. About how many cubic feet of dirt are needed? About how many cubic yards of dirt are needed?

THINK When you find the number of cubic feet in Problem F, why can you use $\frac{2}{3}$ ft. as one dimension?

G Bob made a chest for his little sister's toys. He wanted to know the capacity of the chest, or how much it would hold. So he found the inside dimensions of the chest. These dimensions were 36″, $16\frac{1}{2}''$, and 16″. What was the capacity of the chest in cubic inches?

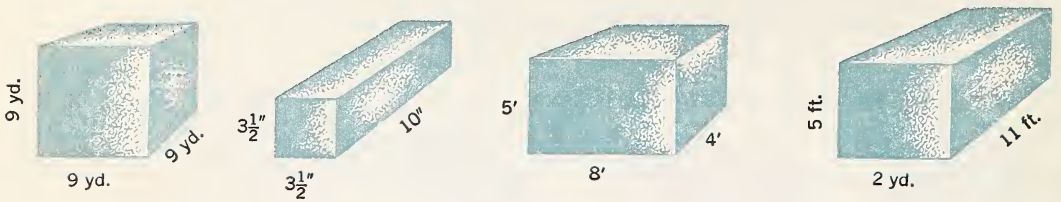
H A water tank in the shape of a rectangular prism is 28″ long, $19\frac{1}{2}''$ wide, and 26″ high. What is the capacity of this tank in cubic inches?

I 277 cu. in. of water equal about 1 gal. When the tank described in Problem H is filled, about how many gallons of water will it hold?

J Another water tank in the shape of a rectangular prism is 4 ft. long, 3.5 ft. wide, and 3.5 ft. high. When this tank is filled with water, how many cubic feet of water does it hold?

K A cubic foot of water weighs 62.3 lb. What is the weight of the water in the tank described in Problem J?

Checking up



A B C D

Test 1

Find the volume of each rectangular prism shown above.

Test 2

Find the volume of each rectangular prism whose dimensions are given below.

	Dimensions of base	Height
A	4 ft. by 9 ft.	14 ft.
B	$1\frac{3}{4}$ in. by $2\frac{1}{2}$ in.	6 in.
C	1.2 yd. by 3.5 yd.	5 yd.
D	42 in. square	$6\frac{1}{2}$ ft.
E	$9\frac{1}{4}$ ft. by 4 ft.	3 yd.
F	25 ft. square	10 yd.

Test 3

A 3 cu. ft. = ▭ cu. in.
B 45 cu. ft. = ▭ cu. yd.
C 864 cu. in. = ▭ cu. ft.
D $2\frac{1}{3}$ cu. yd. = ▭ cu. ft.
E 78 cu. ft. = ▭ cu. yd.
F 1512 cu. in. = ▭ cu. ft.
G 80 cu. yd. = ▭ cu. ft.
H $14\frac{1}{2}$ cu. ft. = ▭ cu. in.
I 144 cu. ft. = ▭ cu. yd.
J 756 cu. in. = ▭ cu. ft.
K 75 cu. ft. = ▭ cu. in.
L $15\frac{1}{4}$ cu. ft. = ▭ cu. in.
M 774 cu. ft. = ▭ cu. yd.

Keeping skilful

Find each quotient to hundredths.

A $1565.25 \div 42.5 = r$
B $3.5 \div .146 = d$
C $.624 \div k = 4.2$
D $8.65 \div 14 = m$
E $24 \div n = .36$
F $.522 \div 8.7 = t$

Find the sum.

A .338, .160, .276
B .0438, 1.6429
C .29, .84, 7.62
D 84.66, 1.37, 96.45
E .19, 1.63, .55, 8.26
F 4.0064, 91.8936

Find the difference.

A 8.0491, .6597
B .7726, .1489
C 20.119, 19.879
D 1.3201, .8569
E 14.073, 8.295
F 16.3333, 15.8497

Exploring problems

A Dick has a collection of 246 rocks. He also has 131 shells. He has how many more rocks than shells?

In Problem A, do you know the size of each group? You are to find the difference of the numbers 246 and 131.

Number in larger group

$$246 - 131 = t$$

Number in smaller group

Number that tells the difference

Dick has ▒ more rocks than shells.

B Dick has a collection of 246 rocks. He has 115 more rocks than shells. Dick has how many shells?

In Problem B, do you know the size of the larger group or of the smaller group? Do you know the difference in the size of the groups? You are to find the size of which group?

Number in larger group

$$246 - n = 115$$

Number in smaller group

Number that tells the difference

Dick has ▒ shells.

C Dick has a collection of rocks. He also has a collection of 131 shells. He has 115 fewer shells than rocks. Dick has how many rocks?

In Problem C, do you know the size of the larger group or of the smaller group? Do you know the difference in the size of the groups? You are to find the size of which group?

Number in larger group

$$k - 131 = 115$$

Number in smaller group

Number that tells the difference

Dick has ▒ rocks.

D May collects Canadian and United States stamps. She has 260 United States stamps. She has 127 more Canadian stamps than United States stamps. She has how many Canadian stamps?

THINK Which do you have to find for Problem D, the difference, the size of the larger group, or the size of the smaller group?

$r - 260 = 127$

In the equation above, what does r represent? What numeral replaces r?

E Tony has a collection of United States and Canadian pennies. He has 156 Canadian pennies and 139 United States pennies. He has how many fewer United States pennies than Canadian pennies?

THINK For Problem E, which do you have to find, the size of the smaller group, the size of the larger group, or the difference?

$156 - 139 = d$

What does d represent in the equation above? What numeral replaces d?

F Bob has a collection of beetles and butterflies. He has 82 beetles. He has 15 fewer butterflies than beetles. Bob has how many butterflies?

THINK Which do you have to find for Problem F, the size of the larger group, the size of the smaller group, or the difference?

$82 - w = 15$

In the equation above, what does w represent? What numeral replaces w?

For each problem that follows, first decide which you are to find, the size of the larger group, the difference, or the size of the smaller group. Then make an equation and find the answer for the problem.

G Paul collects stamped and cancelled envelopes called covers. He has 212 Canadian covers and 86 covers from other places. He has how many more Canadian covers than covers from other places?

H Linda has many pictures of large cities. She has 63 pictures of Canadian cities. She has 18 more pictures of American cities than of Canadian cities. Linda has how many pictures of American cities?

I Chuck collects autographs of baseball and football players. He has 49 football autographs. He has 18 fewer baseball autographs than football autographs. He has how many baseball autographs?

J Chuck also collects baseball and football cards. He has 119 football cards. He has 33 more football cards than baseball cards. Chuck has how many baseball cards?

K Ann has a scrapbook with pictures of flowers and leaves in it. She has 44 flower pictures. She has 16 more leaf pictures than flower pictures. Ann has how many leaf pictures in her scrapbook?

L Jack and Don collect pictures of ships. Jack has 128 ship pictures. He has 36 more ship pictures than Don. Don has how many ship pictures?

M Chuck has 77 pictures of old autos. He has 28 fewer pictures of old autos than Paul has. Paul has how many pictures of old autos?

N Chuck also has 38 pictures of old sailing ships. He has how many more pictures of old autos than of old sailing ships?

O Sam has 208 match-book covers. He has 38 fewer match-book covers than Bill has. Bill has how many match-book covers?

P Carol has 95 glass buttons. She has 27 more glass buttons than pearl buttons. Carol has how many pearl buttons?

Now you should know how to solve problems in which two groups are compared by using the difference.

Keeping skilful

A 75% of 48 is ▭.
B 305.28 is ▭% of 318.
C 9.7 is 16% of ▭.
D 2.16 is ▭% of 72.
E 147% of 147 is ▭.
F 49 is 50% of ▭.
G 1.21 is ▭% of 11.
H 38 is 275% of ▭.
I 1% of 300 is ▭.
J 32.3 is 6% of ▭.
K 138.32 is ▭% of 76.
L 25% of 69 is ▭.
M 14% of 244 is ▭.
N 75 is 175% of ▭.
O 8.41 is ▭% of 29.
P 88 is 88% of ▭.
Q 7.2 is ▭% of 16.
R 63% of 42 is ▭.
S 77 is 16% of ▭.

A 6 gal. = ▭ pt.
B 140 pk. = ▭ bu.
C 16 T. = ▭ lb.
D 16 fl. oz. = ▭ tbs.
E 8 hr. = ▭ min.
F 15 yd. = ▭ in.
G 8800 yd. = ▭ mi.
H 64 qt. = ▭ bu.
I 3 mi. = ▭ ft.
J 832 wk. = ▭ yr.
K 75 lb. = ▭ oz.
L 120 pt. = ▭ qt.
M 6 pk. = ▭ qt.
N 12 doz. = ▭ things
O 147 da. = ▭ wk.
P 45 c. = ▭ qt.
Q 10 rd. = ▭ ft.
R 1800 yd. = ▭ ft.
S 15 mi. = ▭ yd.

A 16 sq. ft. = ▭ sq. in.
B 162 sq. ft. = ▭ sq. yd.
C 9 cu. ft. = ▭ cu. in.
D 1584 sq. in. = ▭ sq. ft.
E 216 cu. ft. = ▭ cu. yd.
F 5184 cu. in. = ▭ cu. ft.
G 38 sq. yd. = ▭ sq. ft.
H 45 sq. ft. = ▭ sq. in.
I 6 cu. yd. = ▭ cu. ft.
J 108 cu. ft. = ▭ cu. yd.
K 2880 sq. in. = ▭ sq. ft.
L 172,800 cu. in. = ▭ cu. ft.
M 7 sq. yd. = ▭ sq. ft.
N 12 sq. ft. = ▭ sq. in.
O 19 cu. ft. = ▭ cu. in.
P 27 cu. yd. = ▭ cu. ft.
Q 621 sq. ft. = ▭ sq. yd.
R 12 sq. yd. = ▭ sq. ft.
S 6 sq. ft. = ▭ sq. in.

Side trip

A

B

C

A How do you find the perimeter of Polygon A?

B You can write the length of one of the sides as $4\frac{4}{12}$ ft. Write the other three dimensions in this way.

C Examples A, B, and C below show three ways to do the work to find the perimeter. When you find the sum in each example, why is the 17 replaced by 5? Why is the 15 replaced by 16?

D Do the answers agree in all three ways of doing the work?

A

$4\frac{5}{12}$ ft.
$4\frac{4}{12}$ ft.
$5\frac{1}{12}$ ft.
$2\frac{7}{12}$ ft.
$15\frac{17}{12}$ ft.
$16\frac{5}{12}$ ft.

B

4 ft. 5 in.
4 ft. 4 in.
5 ft. 1 in.
2 ft. 7 in.
15 ft. 17 in.
16 ft. 5 in.

C

ft.	in.
4	5
4	4
5	1
2	7
15	17
16	5

E What is the length of the longest side in Polygon A? What is the length of the shortest side? How can you find the difference in the lengths?

F Examples D, E, and F below show three ways to do the work to find the difference. In Example D, is $4\frac{13}{12}$ another name for $5\frac{1}{12}$? Study the subtraction. Is it correct?

G In Example E, why are 4 and 13 written above 5 ft. 1 in.?

H Look at Example F. Is 4 ft. 13 in. another name for 5 ft. 1 in.? Does the answer in this example agree with the answers for Examples D and E?

D

$4\frac{13}{12}$
$5\frac{1}{12}$ ft.
$2\frac{7}{12}$ ft.
$2\frac{6}{12}$ ft.

E

4 13
5 ft. 1 in.
2 ft. 7 in.
2 ft. 6 in.

F

ft.	in.
4	13
5	1
2	7
2	6

G

$5\frac{2}{3}$ yd.
4
$20\frac{8}{3}$ yd.
$22\frac{2}{3}$ yd.

H

5 yd. 2 ft.
4
20 yd. 8 ft.
22 yd. 2 ft.

I

5)11 ft. 3 in. | 2 ft.
10 ft.
1 ft. 3 in.
15 in. | 3 in.
15 in.
2 ft. 3 in.

I Polygon B on page 262 is a square. How can you use multiplication to find the perimeter of a square?

J Can 2 ft. be written as $\frac{2}{3}$ yd.? Can 5 yd. 2 ft. be written as $5\frac{2}{3}$ yd.?

K In Example G above, $5\frac{2}{3}$ has been multiplied by 4. Is the product $20\frac{8}{3}$ correct? Is $22\frac{2}{3}$ another name for $20\frac{8}{3}$? What is the perimeter of Polygon B?

L Look at Example H. Is 20 yd. 8 ft. the product of 5 yd. 2 ft. and 4? Why is it correct to replace 20 yd. 8 ft. by 22 yd. 2 ft.?

M Polygon C on page 262 has 5 equal sides. Its perimeter is 11 ft. 3 in. How can you find the length of a side?

N Look at Example I above. How is the 10 ft. found? How is 1 ft. 3 in. found? Why is 1 ft. 3 in. replaced by 15 in.? Why is 3 in. then written in the quotient? Each of the equal sides in Polygon C is 2 ft. 3 in. long.

O Mr. Page is planning a trip. If he goes by air, it will take 2 hr. 25 min. If he goes by train, it will take him 7 hr. 15 min. How much longer will the trip take if he goes by train?

THINK What numbers can you subtract to find this difference?

Can you subtract 25 min. from 15 min.? Is 6 hr. 75 min. another name for 7 hr. 15 min.? Can you subtract now? The trip by train will take ▒ hr. ▒ min. longer than the trip by air.

P Mrs. Cook has a recipe for a fruit drink that calls for 1 pt. 1 c. of grape juice. This recipe makes 1 gal. of fruit drink. How many cups of grape juice will Mrs. Cook need to make 2 gal. of the fruit drink?

Q Mrs. Cook bought 4 chickens. They weighed 2 lb. 12 oz., 3 lb. 7 oz., 2 lb. 9 oz., and 3 lb. What was the average weight of the chickens?

R John has a board 5 ft. 4 in. long. How much should he cut from the board to get a board 3 ft. 6 in. long?

S A racing car was driven 5 laps around a track in 6 min. 30 sec. What was the average time per lap?

Find the sum.

A
3 wk. 2 da.
1 wk. 4 da.
2 wk. 1 da.

B
1 T. 1300 lb.
1400 lb.
2 T. 500 lb.

C
1 min. 28 sec.
2 min. 49 sec.
1 min. 52 sec.

Find the difference.

D
2 yd. 32 in.
1 yd. 25 in.

E
2 T. 380 lb.
1500 lb.

F
$14'\ 7\frac{1}{2}''$
$8'\ 9\frac{1}{4}''$

Find the product.

G
2 qt. 1 pt. × 5

H
7 yr. 2 mo. × 5

I
5 gal. 3 qt. × 7

Find the quotient.

J $5\overline{)16 \text{ yd. } 2 \text{ ft.}}$

K $3\overline{)5 \text{ yr. } 3 \text{ mo.}}$

L $10\overline{)30 \text{ hr. } 50 \text{ min.}}$

Keeping skilful

A $\frac{15}{45}=\frac{x}{3}$

B $\frac{b}{25}=\frac{100}{50}$

C $\frac{32.3}{1}=\frac{f}{43}$

D $\frac{2.3}{1.2}=\frac{v}{2.4}$

E $\frac{6.24}{z}=\frac{14}{28}$

F $\frac{14}{100}=\frac{a}{65}$

G $\frac{43.1}{16}=\frac{431}{n}$

H $\frac{14}{56}=\frac{c}{40}$

I $\frac{42.35}{1}=\frac{d}{4.2}$

J $\frac{100}{73}=\frac{5}{w}$

Find each quotient to thousandths.

A $6.2p=85$

B $42.3-r=6.5$

C $45\div k=4$

D $38.2\times .4=h$

E $5.4\div 3.2=t$

F $4z=.5$

G $8.2+c=17.6$

H $38.7\div w=5$

I $38.7+s=56.0$

J $3.26\div n=6.5$

K $t-.68=13.35$

L $m+.344=.782$

M $16\times p=45.2$

N $75a=477$

O $3.88-1.09=q$

P $.075+3.425=n$

Q $6.4\div 7.35=v$

R $17f=38.6$

S $25\times 6.8=y$

T $g-7.85=18.00$

Thinking straight

Look at the number line above. At the right of zero this number line is like the others you have studied. At the left of zero this number line has been extended in the opposite direction. When the number line is extended in this way, other names are needed for the points on the line.

A The name for Point p can be written as shown at the right. You can read it "negative 1." How do you read the numerals below?

-1

-2 -3 -9 -4 -7 -5

B What name can you use for Point t?

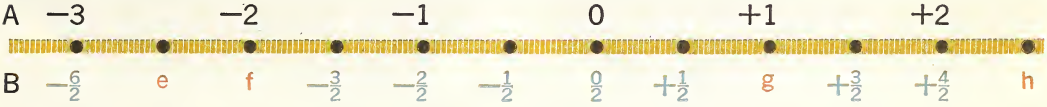
C When the number line is extended in the positive direction, the name for Point r can be written as shown at the right. You can read it "positive 4." How do you read the numerals below?

$+4$

$+2$ $+3$ $+8$ $+6$ $+1$ $+7$

D What name can you use for Point m? For Point v?

E Could you extend the line to show points for $+10$, -20, $+500$, and -1500?

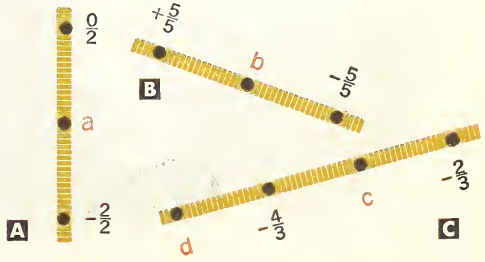


F More points are now shown on the number line. What kind of numerals have been put in the set in Row B to represent the points?

G What numerals can be used in Row B at Points e, f, g, and h?

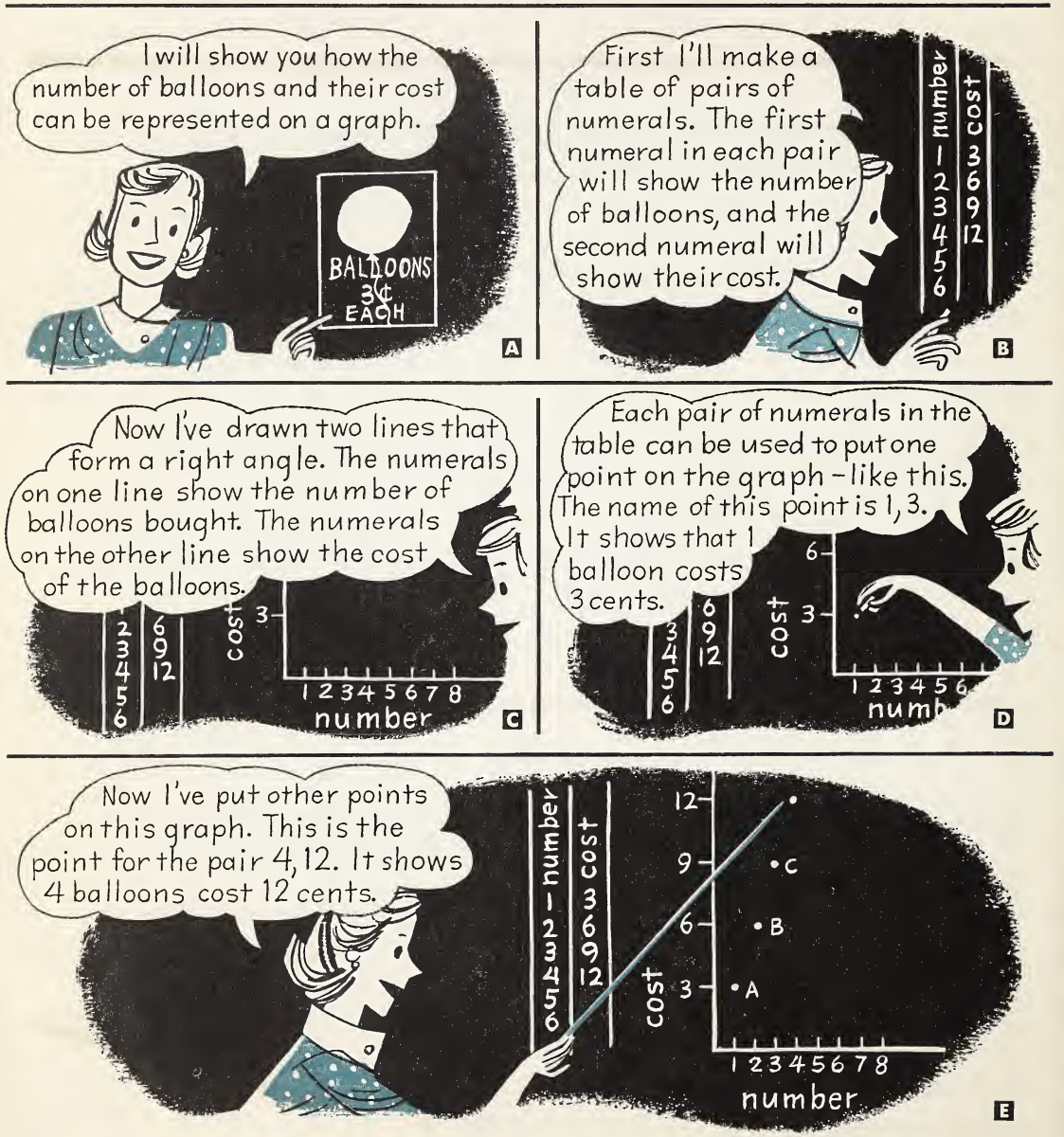
H Could you put points on this line for $-\frac{1}{3}$, $-\frac{5}{6}$, $+\frac{13}{32}$, and $-\frac{13}{32}$?

Replace each letter with a pair of numerals that belongs in the set.



Moving forward

Now you will learn how pairs of numbers can be expressed in a graph.


I will show you how the number of balloons and their cost can be represented on a graph.
BALLOONS 3¢ EACH
A
First I'll make a table of pairs of numerals. The first numeral in each pair will show the number of balloons, and the second numeral will show their cost.
number
1 2 3 4 5 6
cost
3 6 9 12
B
Now I've drawn two lines that form a right angle. The numerals on one line show the number of balloons bought. The numerals on the other line show the cost of the balloons.
2 3 4 5 6
6 9 12
cost
3
1 2 3 4 5 6 7 8
number
C
Each pair of numerals in the table can be used to put one point on the graph – like this. The name of this point is 1, 3. It shows that 1 balloon costs 3 cents.
3 4 5 6
6 9 12
cost
6
3
1 2 3 4 5 6
D
Now I've put other points on this graph. This is the point for the pair 4, 12. It shows 4 balloons cost 12 cents.
number
1 2 3 4 5 6
cost
3 6 9 12
12
9
6
3
C
B
A
1 2 3 4 5 6 7 8
number
E

A Look at Picture A on page 266. The poster shows that 1 balloon costs 3¢. Does the equation of ratios below show how to find the cost of 4 balloons?

$$\frac{1}{3}=\frac{4}{n}$$

B Now look at Picture B. Which pair of numerals in the table shows that 4 balloons cost 12 cents?

C For each expression below give the pair of numerals that belongs in the table.

5 balloons and their cost
6 balloons and their cost
7 balloons and their cost
8 balloons and their cost
10 balloons and their cost
13 balloons and their cost

D Find at least two more pairs of numerals that belong in the table.

E In Picture C, notice the two lines that form a right angle. On which line are the numerals that show number of balloons? Why are there no numerals on this line for $\frac{1}{2}$, $1\frac{1}{2}$, or $2\frac{1}{2}$ balloons?

F On which line are the numerals that show the cost of the balloons? Why are there no numerals on this line for 1 cent, 2 cents, or 4 cents?

G Look at Picture E. What point is the teacher talking about? You can write the name of this point as shown at the right. The first numeral of the pair stands for the number of balloons. The second numeral stands for the number of cents these balloons cost. This pair shows that 4 balloons cost ▒ cents.

(4, 12)

H Find the point called 1, 3. How do you write its name? This point shows that ▒ balloon costs ▒ cents.

I Find Point B and Point C. How do you write the name of each of these points? These pairs of numerals can be thought of as part of a set of equal ratios. Give five other pairs that belong to this set of equal ratios. Could these five pairs of numerals be shown in this graph?

J Suppose you picked the pair (10, 30), which shows that 10 balloons cost 30¢. What would you have to do to the graph to show this point?

K There are many ratios equal to those in the table, but it would not be sensible to show some of these pairs of numerals in this graph. For example, the ratio $2\frac{1}{2}$ per $7\frac{1}{2}$ is equal to the ratio 1 per 3. Use the ratio test to show that these ratios are equal. Why would you not show Point $(2\frac{1}{2}, 7\frac{1}{2})$ in this graph? Why would you not show Point (7, 20)?

Examine the table of number pairs below. In each pair, the first numeral stands for the number of minutes travelled. The second numeral stands for the number of miles travelled. The table shows a relation of time travelled to distance travelled. This relation is expressed by a set of equal ratios or pairs of numerals.

MINUTES	10	21	35	b	c	17
MILES	5	$10\frac{1}{2}$	a	$22\frac{1}{2}$	30	d

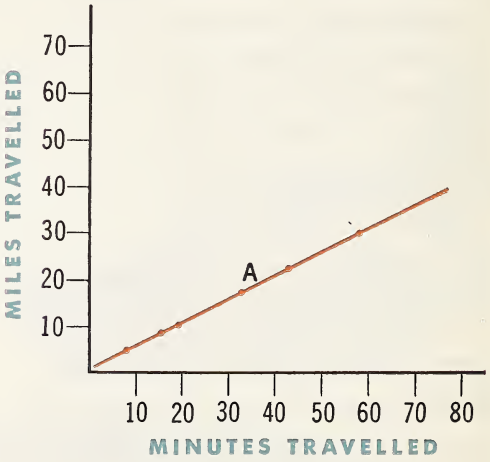
A What distance is travelled in 10 minutes? Write a pair of numerals to show that this distance is travelled in 10 minutes. What is used to show miles travelled in 35 minutes?

B Each pair of numerals in the table can be thought of as a ratio. Use the ratio test to show that the first two pairs are equal ratios.

C If the pairs of numerals in the table above express equal ratios, what numeral replaces a? Replaces b? Replaces c? Replaces d? Be prepared to explain how you found these numerals.

D Find four other pairs of numerals that belong in the table. Is there any end to the pairs of numerals that could be put in this table?

E Look at the graph below. Notice the two lines that form a right angle. Find the line that shows time travelled. There are numerals on this line for 10 minutes, 20 minutes, 30 minutes, and so on. There could be numerals on the line for 5 minutes, 6 minutes, 22 minutes, and 38 minutes. Where would you put each of these numerals? Could you travel for $2\frac{1}{2}$ minutes? For $5\frac{1}{4}$ minutes? Could the numerals $2\frac{1}{2}$ and $5\frac{1}{4}$ be put on this line?



F Could you travel 25 minutes and travel $12\frac{1}{2}$ miles? Could you put a point in this graph to represent this pair of numerals?

G It is not sensible to represent pairs like $(2\frac{1}{2}, 7\frac{1}{2})$ in the graph on page 266. Why is it sensible to represent pairs like $(25, 12\frac{1}{2})$ in this graph?

H Look at the table again. Is each pair of numerals in the table represented in this graph?

I Is there any end to the number of points that can be shown in this graph at the left of Point A? Could the names of all these points be put in the table? Why?

J If it were possible to put all the points in the graph, would you have a line like the one shown?

K Give the names for five of these other points. Use the ratio test with any two of these pairs of numerals to show that they are equal ratios.

L At the top of the next column there are five rates. Write at least six pairs of numerals to express each one of these five rates. For example, you might write (2, 10) as one pair of numerals for Exercise 1.

1. Balloons at 5 cents each
2. Pencils at 3 for 10 cents
3. A speed of 5 miles in 5 minutes
4. Postage stamps at 4 cents each
5. Meat at 50 cents per pound

M In which of your sets of pairs of numerals would it be sensible to have fraction numerals?

Each of the four sets of pairs of numerals below expresses a rate. For each set, make a statement like the ones given in Exercise L.

N	O	P	Q
(1, 6)	(3, 25)	(2, 1)	($\frac{1}{2}$, 30)
(2, 12)	(6, 50)	(6, 3)	($\frac{3}{4}$, 45)
(3, 18)	(12, 100)	(10, 5)	($1\frac{1}{2}$, 90)
(4, 24)	(18, 150)		(4, 240)

Now you should see how pairs of numbers can be expressed in a table and in a graph.

Keeping skilful

A 150 is ▭% of 75.
B 122 is 200% of ▭.
C 62% of 26.6 is ▭.
D 16 is ▭% of 1.
E 2.3 is 10% of ▭.
F 2.75 is ▭% of 6.25.
G 80% of ▭ is 100.
H 20% of 50.4 is ▭.

A $.63 \times 2.5 = n$
B $42.5 \times 6 = c$
C $.02 \times .04 = t$
D $62.53 \times 2.83 = w$
E $6 \times 4.2 = y$
F $100 \times .63 = d$
G $.26 \times 3 = b$
H $.002 \times 100 = k$

Find each answer to hundredths.

A $6.2 \times m = 16.48$
B $.35 \times e = .25$
C $y \times .483 = 6$
D $2 \times z = 2.6$
E $42.2 \times h = 163$
F $a \times 40.3 = 685.02$

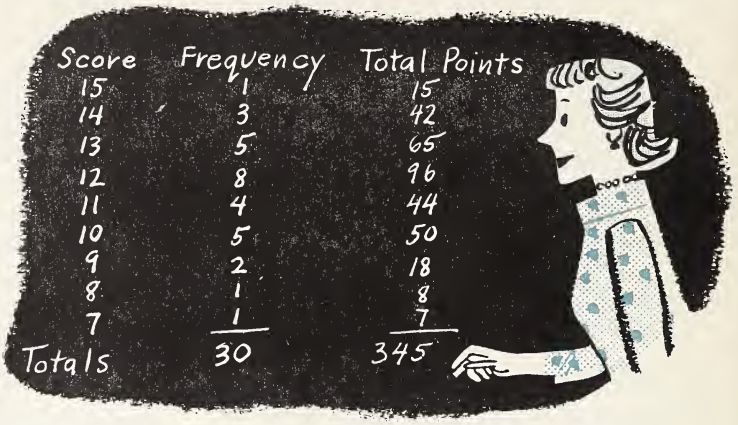
Moving forward

Now you will learn more about tables and graphs.

Arithmetic Test Scores

Section A Monday

John	12
Mary	14
Jim	11
Joe	14
Ann	11
Dee	12
Nancy	10
Alice	12
Martha	10
Linda	10
David H.	10
George	12
Henry	9
Clarence	15
Bill K.	10
Jack	13
Jane	13
Phyllis	13
Kathy	7
Mike	13
Betty	13
Janice	12
Dick	11
Pete	12
Ross	11
Peggy	14
Paul	8
Don	12
Bill W.	12
David Y.	9



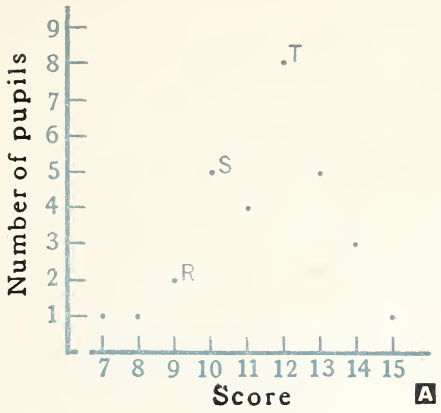
A The test scores at the left were made by Miss Clark's arithmetic class. What was the highest score in the test? What was the lowest score? How many pupils had a score of 12? How many had a score of 13?

B How many pupils had a score of 8? Of 15? The number of pupils having a certain score is called the frequency for that score. What is the frequency for the score of 8? For the score of 15? Which score in the test had the highest frequency?

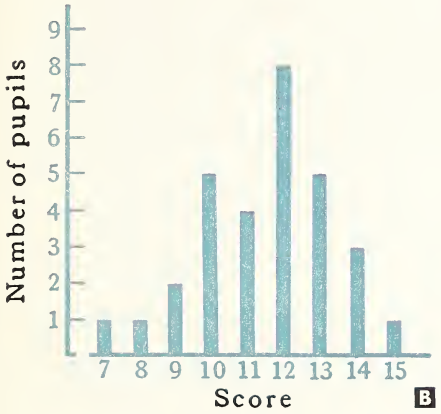
C The table above shows the frequency for each score in the test. Are the frequencies correct?

D Which scores have the same frequency? Why is no frequency shown for a score of 6 or lower?

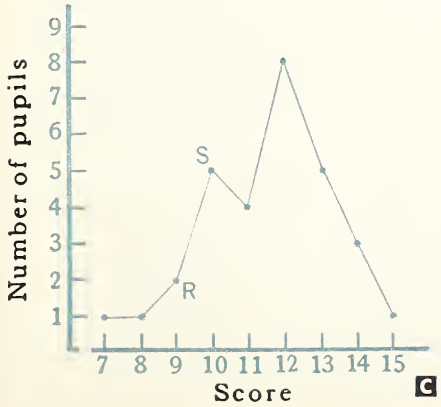
E Look at the numerals in the column labelled "Total Points." The total points for the score of 11, for example, are 4×11, or 44, because 4 pupils had a score of 11 points. Find the total points for the score of 10. For the score of 12. For the score of 15.



A



B



C

F Look again at the table on page 270. The total number of points scored by all the pupils is shown in the table. What is this total? Can you find the total number of pupils from this table? Use these totals to find the class average. Did any pupil have this average score?

G Graph A shows the frequencies for the test scores on page 270. Point R on the graph shows that 2 pupils had a score of 9. We call this point (9, 2). What does Point S show? What is it called? What point represents the score with the highest frequency?

H Graph B is called a bar graph. It represents the same pairs of numbers as those shown on Graph A. How is Point R shown on this graph? How is Point S shown? Which bars represent scores with the same frequency?

I Could other bar graphs be made that would represent these same pairs of numbers? Would the bars have to be the same length as those in Graph B? Would they have to be the same width?

J Graphs are used to make comparisons. Which graph makes it easier to compare the frequencies of the scores, Graph A or Graph B?

K Graph C also represents the same pairs of numbers as those in Graphs A and B. If you put a point on the line between R and S, will it represent a frequency for a score in this test?

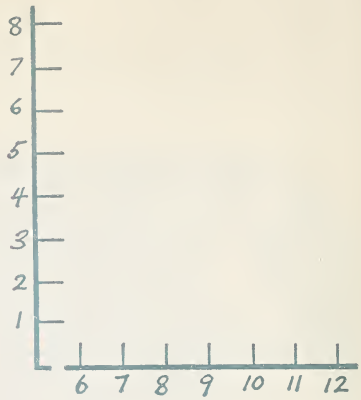
L Look at Graphs A and C again. Why do you think the points are joined by lines in Graph C?

Arithmetic Test Scores
Section A Friday

John	10
Mary	11
Jim	9
Joe	12
Ann	9
Dee	9
Nancy	9
Alice	10
Martha	8
Linda	7
David H.	8
George	8
Henry	7
Clarence	12
Bill K.	8
Jack	11
Jane	11
Phyllis	10
Kathy	6
Mike	10
Betty	9
Janice	8
Dick	9
Pete	10
Ross	9
Peggy	11
Paul	6
Don	8
Bill W.	9
David Y.	7

Score	Frequency	Total Points
12		
11		
10		
9		
8		
7		
6		
Totals		

A



B

M The scores shown at the left were made by the arithmetic class in another test. Copy Table A above. Then complete the table, using the scores at the left. How is your completed table different from the table on page 270?

N Next find the class average for the test given on Friday. Which two numbers from your table do you use?

O Now make three copies of the number lines shown in Picture B. Use the information in Table A to make three graphs. Make a graph like Graph A on page 271. Make one like Graph B. Make one like Graph C.

Now you have learned how to show information in a table and in a graph.

Keeping skilful

A $\frac{2}{3} \times n = 15$	**F** $\frac{2}{3} \times m = \frac{5}{8}$	**K** $\frac{6}{5} \times m = 72$	**P** $\frac{3}{16} \times \frac{2}{15} = z$
B $y \times \frac{3}{4} = 32$	**G** $p \times \frac{3}{7} = \frac{2}{3}$	**L** $t \times \frac{3}{12} = 6$	**Q** $n \times \frac{8}{9} = 1$
C $\frac{5}{8} \times w = 40$	**H** $\frac{1}{2} \times b = 1$	**M** $\frac{2}{3} \times \frac{7}{14} = s$	**R** $\frac{11}{12} \times \frac{2}{7} = h$
D $\frac{2}{3} \times \frac{5}{8} = m$	**I** $c \times 6 = \frac{3}{5}$	**N** $\frac{6}{3} \times y = \frac{5}{8}$	**S** $7 \times d = 1$
E $\frac{1}{16} \times p = 48$	**J** $s \times \frac{2}{3} = 9$	**O** $\frac{2}{11} \times a = \frac{2}{11}$	**T** $\frac{10}{16} \times e = \frac{5}{16}$

Side trip

A There are other systems of measures besides the one you use. Scientists in our country and all people in some countries use the metric system of measures. In this system, weights are measured in grams and kilograms instead of in ounces and pounds. 1 kilogram is about 2.2 lb. You can use the equation below to find the number of pounds in 100 kilograms.

$$\frac{1}{2.2}=\frac{100}{t}$$

B Use the equation below to find the number of kilograms in 100 lb. Find the answer to hundredths.

$$\frac{1}{2.2}=\frac{n}{100}$$

C Use the equation below to find the number of kilograms in 1 pound. Find the answer to thousandths.

$$\frac{1}{2.2}=\frac{n}{1}$$

D 1 kilogram is equal to 1000 grams. 1 lb. is about how many grams?

E There are about how many grams in 1 oz.? Find the answer to tenths.

F 1 gram is about how many ounces? Find the answer to thousandths.

THINK You can use 28.4 over 1 for one ratio. What will the other ratio be?

G Distances are measured in kilometres, metres, and centimetres. 1 metre is approximately 39.37 in. About how many yards are there in one metre? Find the answer to hundredths.

THINK You can use 36 over 1 for one ratio. What will the other ratio be?

H A man who is 5 ft. 11 in. tall is about how many metres tall? Find the answer to tenths.

THINK 5 ft. 11 in. = ▒ in.

I 1 centimetre is equal to .01 of a metre. 1 centimetre is about how many tenths of an inch?

J There are how many centimetres in 1 inch? Find the answer to tenths.

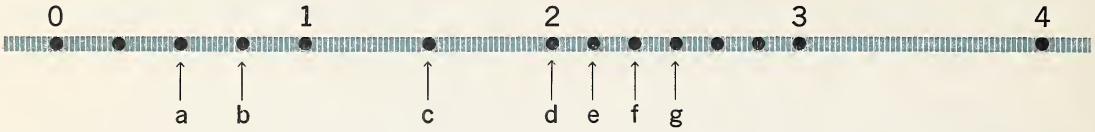
K 1 kilometre is equal to 1000 metres. This is about 3281 feet. A kilometre is about what fraction of a mile? Find the answer to hundredths.

L There are about how many kilometres in 1 mile? Find the answer to tenths.

M An automobile travelling at a speed of 45 miles per hour is travelling about how many kilometres per hour?

N Liquids are measured in litres. 1 litre is about .88 qt. Find the approximate number of litres in 1 qt. Find the answer to hundredths.

Looking back



A Look at the number line above. Give three fraction numerals that are names for each point that is indicated by a letter.

B Which of the following fraction numerals are names for Point b?

$\frac{30}{40}$ $\frac{18}{24}$ $\frac{28}{36}$ $\frac{3}{4}$ $\frac{8}{12}$ $\frac{45}{60}$ $\frac{12}{16}$

C Give the names for the points halfway between the following points.

1. a and b
2. a and c
3. c and d
4. d and e
5. c and e
6. b and g

D Find the names for two other points that are between the following points on this line.

1. a and b
2. c and d
3. f and g
4. a and g
5. b and c
6. d and e

E If a point is to the right of Point c on the number line, does it represent a number that is less than or greater than the number for Point c?

F Give a name for a point to the left of zero on this line.

G Give the names for any three points that are to the left of each of the following points.

1. 4
2. 15
3. 6.1
4. .03
5. 1.2
6. .001
7. 0
8. .6
9. .01

H What number is halfway between the following numbers?

1. 5 and 6
2. 3 and 10
3. $\frac{1}{2}$ and $\frac{5}{8}$
4. .05 and .06
5. 1000 and 10,000
6. $2\frac{1}{2}$ and 3

I How many tens are there in each number below?

1. 60
2. 520
3. 1000
4. 868
5. 100
6. 740
7. 25
8. 3
9. .1

J How many thousands are there in each number below?

1. 50,000
2. 1,000,000
3. 6500
4. 400

K The second number is how many times as large as the first number?

1. 3.6 and 36
2. 40 and 4000
3. $\frac{1}{4}$ and $\frac{1}{2}$
4. $\frac{2}{3}$ and $4\frac{1}{3}$
5. $\frac{5}{8}$ and $\frac{15}{8}$
6. $\frac{5}{8}$ and $\frac{5}{4}$

L Look at the number line on page 274. Is $2\frac{1}{3}$ a name for Point f? Which of the following decimal fraction numerals is best to use as a name for Point f?

.33 2.34 .34 2.33

M Which of the following can be used as names for Point e?

2.16 2.17 2.167 2.1667

N Is it possible to write a decimal fraction numeral that is exactly equal to $\frac{1}{6}$? Why?

O For each point labelled with a letter on the number line on page 274, write three decimal fraction numerals to name the point.

P Write with words.

1. 6250
2. 402
3. 40,650
4. 4.65
5. .365
6. .0362
7. 465.06
8. 4,000,000
9. 1.0002

Q Write with numerals.

1. 6 thousand 45
2. 400 and 6 tenths
3. 32 and 85 thousandths
4. 160 million
5. 300 ten-thousandths
6. 6 and 6 thousandths
7. 9 and 253 ten-thousandths
8. 760 and 2 hundredths

R Make a true statement for each of the following exercises, using the sign $<$, or $>$, or $=$.

1. $28 \sim 27$
2. $.0625 \sim .625$
3. $\frac{1}{1000} \sim \frac{1}{100}$
4. $2.25 \sim 2.205$
5. $\frac{3}{8} \sim \frac{3}{4}$
6. $\frac{25}{8} \sim 2\frac{9}{8}$
7. $.2 \times 0 \sim 2 \times 0$
8. $.333 \sim .33$
9. $.02 \sim .002$
10. $\frac{1}{4} \sim .25$
11. $3 \times \frac{3}{2} \sim 3 + \frac{3}{2}$
12. $.333 \sim \frac{1}{3}$

S Find one solution for each sentence below.

1. $r + 5 > 10$
2. $12 > a + 3$
3. $\frac{1}{2} + d = 2$
4. $.03 + k = 1$
5. $\frac{9}{17} \times p = 1$
6. $\frac{5}{8} \times n = 5$
7. $.23 + r = .65$
8. $.02 \times m > 6$
9. $1.3 \div n = .5$
10. $\frac{1}{2} \times t + 3 < 5$

T If two equal fraction numerals have equal numerators, what must be true of their denominators?

U If two fraction numerals are equal, and the numerator of one is twice the numerator of the other, what must be true of their denominators?

V If two fraction numerals are equal, and the numerator of one is zero, what must be true of the numerator of the other?

W If two fraction numerals are equal, and one is an improper fraction numeral, what is true of the other?

X What must be true of two equal fractions if their sum is equal to 1?

Looking back

A Look at Polygons A, B, C, D, and E. Which polygons have right angles? Which polygons have parallel sides?

B Which polygons are parallelograms? Which are rectangles? Are there any squares? Any triangles?

C Polygon C has sides that are equal in length. Polygon E also has equal sides. Find the perimeter of Polygons A, B, C, D, and E.

D Find the area of Polygon A, of Polygon B, and of Polygon D.

Find the area of each rectangle whose dimensions are given below.

E 10′ x 23′
F 23″ x 46″
G 2 yd. by 7 ft.
H 2 ft. by 5 in.

I Pictures F, G, and H represent rectangular prisms. Which of these prisms have bases that are squares? Which prism is a cube?

J The area of the base of Prism F is how many times as large as the area of the base of Prism G? The height of Prism F is how many times as great as the height of Prism G? The volume of Prism F is how many times as large as the volume of Prism G?

K Prism F could contain how many cubes like Prism H? Prism G could contain how many cubes like Prism H?

Find the volume of each rectangular prism whose dimensions are given below.

L 6′ x 9′ x 5′
M 3 yd. by 2 ft. by 1 ft.
N 14 in. by 5 in. by 7 in.
O 7″ x 7″ x 7″

Looking back

Three equations are shown for each statement below. Copy each equation that fits the statement.

A 50% of 60 is ▩.

$\frac{50}{100}=\frac{n}{60}$ $\frac{50}{100}=\frac{60}{n}$ $\frac{.5}{1}=\frac{n}{60}$

B .23 of ▩ is 30.

$\frac{.23}{30}=\frac{n}{1}$ $\frac{\frac{23}{100}}{1}=\frac{30}{n}$ $\frac{23}{1}=\frac{n}{30}$

C $\frac{3}{4}$ of 12 is ▩.

$\frac{\frac{3}{4}}{1}=\frac{12}{n}$ $\frac{.75}{1}=\frac{n}{12}$ $\frac{\frac{3}{4}}{1}=\frac{n}{12}$

D 80 is ▩% of 40.

$\frac{40}{80}=\frac{n}{100}$ $\frac{80}{40}=\frac{n}{100}$ $\frac{80}{40}=\frac{n}{1}$

E 25% of ▩ is 352.

$\frac{\frac{1}{4}}{1}=\frac{352}{n}$ $\frac{.25}{1}=\frac{352}{n}$ $\frac{352}{n}=\frac{25}{100}$

F 60 is what fraction of 100?

$\frac{60}{100}=\frac{n}{1}$ $\frac{100}{60}=\frac{n}{1}$ $\frac{n}{60}=\frac{100}{1}$

G .92 of 65 is ▩.

$\frac{.92}{1}=\frac{65}{n}$ $\frac{.92}{100}=\frac{n}{65}$ $\frac{.92}{1}=\frac{n}{65}$

H $\frac{1}{2}$ of ▩ is 60.

$\frac{\frac{1}{2}}{1}=\frac{n}{60}$ $\frac{.5}{1}=\frac{60}{n}$ $\frac{60}{n}=\frac{\frac{1}{2}}{1}$

I Which of the following ratios are equal to the ratio 4 over 7?

$\frac{2}{3\frac{1}{2}}$ $\frac{20}{35}$ $\frac{45}{75}$ $\frac{1\frac{1}{3}}{2\frac{1}{3}}$ $\frac{.04}{.07}$ $\frac{.4}{.07}$

J Use the correct sign (=, ≠) for each of the following exercises.

1. $\frac{.50}{1}$ ~~ $\frac{400}{200}$
2. $\frac{.006}{.3}$ ~~ $\frac{.02}{1}$
3. $\frac{2\frac{1}{3}}{9}$ ~~ $\frac{7}{27}$
4. $\frac{\frac{2}{3}}{\frac{1}{2}}$ ~~ $\frac{\frac{2}{6}}{\frac{1}{3}}$
5. $\frac{.42}{1}$ ~~ $\frac{84}{100}$
6. $\frac{\frac{15}{17}}{15}$ ~~ $\frac{\frac{1}{17}}{1}$

K Per cent is expressed by a ratio in which the second term is always ▩.

L Write the words you would say in reading each of the following ratios.

$75\%=\frac{75}{100}=\frac{3}{4}=\frac{\frac{3}{4}}{1}=\frac{.75}{1}$

M For each of these ratios, write five other ratios that are equal.

1. 40%
2. 20%
3. 50%
4. 65%
5. 120%
6. 6%
7. 14%
8. 200%
9. 25%

N Write each ratio below as a per cent.

1. $\frac{30}{100}$
2. $\frac{.25}{1}$
3. $\frac{3}{5}$
4. $\frac{3}{4}$
5. $\frac{72}{100}$
6. $\frac{.50}{1}$
7. $\frac{\frac{4}{5}}{1}$
8. $\frac{2}{1}$

Looking back

A When you multiply two numbers, the answer is called the ~~~.

B When you subtract a number from another number, the answer is called the ~~~.

C When you add two or more numbers, the answer is called the ~~~.

D In the fraction numeral $\frac{2}{3}$, the 2 is called the ~~~ and the 3 is called the ~~~.

E The pair of numerals that expresses a rate is called a ~~~.

F Two fraction numerals that are names for the same point are ~~~.

G If a comparison is expressed by a ratio that has 100 in its second term, the ratio may be called a ~~~.

H Two ratios can be shown to be equal by using the ~~~.

I If the product of two fractions is equal to 1, one fraction is said to be the ~~~ of the other.

J If two fraction numerals have the same numerator, the fraction numeral with the larger denominator expresses the ~~~ fraction.

K When you divide 6 by 4, the remainder is ~~~.

L The set of numbers that makes true statements for the sentence $x + 2 > 6$ is called the ~~~ set.

M The sum of the length of the sides of a rectangle is the ~~~ of the rectangle.

N A square is a rectangle whose sides are ~~~ in length.

O A cube is a rectangular prism whose faces are ~~~.

In some of the exercises below, you will have fraction numerals in your answers.

A 64 qt. = ▒ pt.
B 7 min. = ▒ sec.
C 81 cu. ft. = ▒ cu. yd.
D 5 rods = ▒ ft.
E 400 yd. = ▒ in.
F $2\frac{1}{2}$ hr. = ▒ min.
G $\frac{3}{4}$ T. = ▒ lb.
H 5 mi. = ▒ ft.
I 54 oz. = ▒ lb.
J 3 sq. yd. = ▒ sq. ft.
K 26 qt. = ▒ gal.
L 3456 cu. in. = ▒ cu. ft.
M 6 yr. = ▒ wk.
N 560 sec. = ▒ min.
O 50 ft. = ▒ yd.
P 720 sq. in. = ▒ sq. ft.
Q 32 c. = ▒ fl. oz.

Checking up

Test 1

A Mr. Kerr can drive an average of 16.5 miles on one gallon of gasoline. At this rate, how far can he drive on 15.8 gal. of gasoline?

B 35% of the people who work in Mr. Miller's office drive their cars to work. 120 persons work in Mr. Miller's office. How many of these people drive their cars to work?

C Mrs. Will made a skirt and a blouse for Peggy. She used $1\frac{1}{2}$ yd. of cloth for the skirt. She used $\frac{1}{3}$ as much cloth for the blouse as she used for the skirt. How much cloth did she use for the blouse?

D During one school week Sue spent $1\frac{1}{2}$ hr. on her homework on Monday, $1\frac{3}{4}$ hr. on Tuesday, $\frac{3}{4}$ hr. on Wednesday, and $2\frac{1}{4}$ hr. on Thursday. She spent an average of how many hours per day on her homework during the 5 days of the school week?

E Tom is 12 years old. His father is $3\frac{1}{2}$ times as old as Tom. How old is Tom's father?

F One evening Carol read 45 pages of a book in 2 hours. The book has 215 pages. At this rate, how long will it take her to read the whole book?

G Mrs. Jackson paid $2.25 for a 3-lb. roast of beef at the Beach Store. Mrs. Phillips paid $2.04 for a 3-lb. roast of beef at the Sanderson Store. The price of roast beef per pound was how much more at the Beach Store than at the Sanderson Store?

H 28 of the 32 sixth graders went on a trip to the zoo. What fraction of the sixth graders went to the zoo? Express the answer as a common fraction numeral in lowest terms.

I Jim's aquarium has the shape of a rectangular prism. The aquarium measures 20 in. x 10 in. x 12 in. What is the volume of the aquarium in cubic inches?

J Ted spent .3 of his savings for a new fishing rod. He spent $9 for the fishing rod. What was the amount of Ted's savings before he bought the fishing rod?

K Mr. Downs bought 3 rolls of black and white film at a total cost of $3.15. He also bought 3 rolls of color film at a cost of $2.75 per roll. He paid how much more for the 3 rolls of color film than he paid for the 3 rolls of black and white film?

Test 2

A The air mattress that Cal wants to buy costs $14.95. His father has promised to pay $5 of the cost, and Cal has saved $6.35. How much more money does Cal need before he can buy the air mattress?

B In a basketball game, George made 6 baskets out of 20 tries. George's baskets were what per cent of his total tries?

C Mr. May bought 75 ft. of garden hose that was priced at $6.30 for 50 ft. At this rate, how much did he have to pay for the 75 ft. of hose?

D Mrs. Town needs eight $6\frac{1}{2}$-foot lengths of cloth for curtains. She needs to buy how many yards of cloth?

E Mrs. Anderson paid $45 for a radio that was on sale. The price she paid was 60% of the regular price. What was the regular price?

F Mr. Andrews can drive to work in about 35 min. less than the time it takes when he rides on the bus. He can drive to work in about 45 min. How long does it take Mr. Andrews to get to work when he rides on the bus?

G Mrs. Town made a salad dressing for which she used $\frac{1}{2}$ as much lemon juice as salad oil. She used $\frac{3}{4}$ cup of salad oil. How much lemon juice did she use in the dressing?

H On a map that Mr. Marks was using, 1 in. represented 30 mi. The distance on the map from Saskatoon to Regina was $5\frac{5}{8}$ in. What is the distance in miles from Saskatoon to Regina?

I Susan earns $.60 an hour for baby sitting. She worked for $3\frac{2}{3}$ hr. on Friday night and $4\frac{2}{3}$ hr. on Saturday night. How much did Susan earn on these two nights?

J George delivers newspapers to 35 customers each day. He also has 15 customers to whom he delivers only week-end papers. His customers who take only week-end papers are what per cent of all his customers?

K Alice missed 9 days of school during the entire school year. She missed 5% of the days in the school year. There are how many days in the school year?

L Ann's allowance is $.85 per week. Her allowance is $.35 less per week than her brother Bill's allowance. What is Bill's allowance per week?

M On Highway 28, the distance from Hometown to High Falls is $12\frac{3}{4}$ mi. farther than it is on Highway 11. The distance on Highway 11 is $36\frac{1}{2}$ mi. What is the distance on Highway 28?

Test 3

Find the sum.

A 44, 18, 206, 628, 30
B 1163, 4809, 215, 99
C 27638, 49544, 766
D 863, 2079, 633, 74
E 3675, 11483, 9084
F 89, 17, 206, 53, 973
G 94112, 78683, 468
H 316, 528, 49872, 39

Test 4

A $87046 - 29548 = t$
B $10063 - 9985 = d$
C $7582 - 1864 = a$
D $49331 - 38475 = r$
E $11000 - 9999 = w$
F $53006 - 48881 = a$
G $29438 - 19549 = h$
H $11111 - 3429 = z$
I $12006 - 7948 = b$

Test 5

A $25 \times 406 = n$
B $721 \times 86 = w$
C $468 \times 1732 = m$
D $893 \times 694 = d$
E $29 \times 8470 = y$
F $905 \times 436 = a$
G $386 \times 1174 = z$
H $48 \times 967 = b$
I $896 \times 473 = g$

Test 6

A $16074 \div a = 19$
B $250610 \div 724 = g$
C $50742 \div 683 = m$
D $5656 \div w = 119$
E $83927 \div 89 = b$
F $294850 \div h = 725$
G $17591 \div 49 = t$
H $42293 \div d = 83$
I $71906 \div 187 = f$
J $74746 \div r = 983$

Test 7

A $n + 9382 = 15464$
B $763 \times k = 64092$
C $58662 - y = 17946$
D $36859 + 7986 = a$
E $82477 \div m = 89$
F $w - 6385 = 15420$
G $489 \times 1563 = d$
H $50461 - t = 32574$
I $65003 - 9834 = a$
J $56816 \div 67 = r$

Test 8

A $40 + \frac{11}{16} = b$
B $\frac{19}{20} + \frac{3}{5} + \frac{1}{4} = r$
C $9\frac{5}{12} + 2\frac{7}{8} = k$
D $\frac{11}{15} + 5\frac{9}{10} = w$
E $21\frac{5}{6} + 13\frac{1}{6} = m$
F $18\frac{5}{9} + 6 = d$
G $\frac{14}{15} + \frac{2}{3} + 1\frac{1}{6} = n$
H $63\frac{3}{16} + 11\frac{5}{8} = a$
I $4\frac{1}{4} + \frac{7}{12} + \frac{5}{8} = x$

Test 9

A $\frac{15}{16} - \frac{1}{4} = x$
B $10\frac{7}{12} - 8 = r$
C $8 - \frac{9}{20} = a$
D $20\frac{1}{12} - 19\frac{1}{3} = w$
E $15 - 7\frac{5}{6} = t$
F $13\frac{7}{8} - 4\frac{7}{8} = m$
G $\frac{19}{20} - \frac{1}{5} = d$
H $47\frac{2}{9} - 11\frac{2}{3} = k$
I $60 - 24\frac{7}{10} = v$

Test 10

A $\frac{5}{12} \times \frac{13}{15} = m$
B $\frac{1}{8} \times 10\frac{2}{5} = t$
C $4 \times 3\frac{1}{8} = r$
D $1\frac{5}{6} \times 4\frac{4}{5} = d$
E $16 \times 3\frac{1}{8} = n$
F $\frac{9}{20} \times 60 = w$
G $14\frac{2}{5} \times 4\frac{3}{8} = s$
H $12\frac{1}{2} \times 2\frac{18}{25} = a$
I $14\frac{1}{4} \times 64 = k$

Test 11

A $\frac{11}{12} \div 1\frac{1}{3} = b$
B $14 \div 3\frac{1}{2} = r$
C $\frac{9}{20} \div 3 = m$
D $8\frac{1}{3} \div 1\frac{1}{5} = w$
E $10\frac{5}{8} \div \frac{14}{15} = c$
F $\frac{9}{16} \div \frac{3}{8} = t$
G $21 \div \frac{11}{12} = a$
H $6\frac{3}{10} \div 7 = n$
I $34\frac{4}{5} \div 7\frac{1}{4} = d$

Test 12

A $n - 1\frac{5}{8} = 11\frac{1}{2}$

B $20\frac{1}{4} \div t = 6\frac{1}{2}$

C $8\frac{11}{16} + 7\frac{1}{4} = d$

D $\frac{31}{32} - r = \frac{7}{8}$

E $w + 4\frac{7}{12} = 8\frac{1}{3}$

F $30 - 29\frac{3}{10} = a$

G $5\frac{3}{5} \times 1\frac{1}{4} = v$

H $18\frac{1}{3} + m = 27\frac{9}{10}$

I $60\frac{3}{4} \div 1\frac{11}{16} = f$

Test 13

Find the sum.

A .06, 6.87, .36, .80

B 7.3, 11.9, 13.2, 4.6

C .0463, .9215, .3301

D 9.124, .708, 4.635

E .8635, .0101, .6287

F 14.8, 34.6, .5, 92.7

G 8.44, 11.07, 9.32

H 21.6, 8.4, 7.1, .9

I .7063, .5230, 4.6264

Test 14

A $9.3343 - 8.6475 = d$

B $.6537 - .1837 = k$

C $200.82 - 15.74 = x$

D $4.3106 - 3.4296 = r$

E $.7412 - .6596 = t$

F $8.3055 - 2.2969 = m$

G $6000.4 - 418.6 = w$

H $1.725 - .938 = a$

I $24.0061 - 4.3285 = s$

J $15.928 - 14.979 = g$

Test 15

A $.013 \times 60 = w$

B $.6 \times 18.5 = t$

C $17 \times .005 = r$

D $42.18 \times .95 = a$

E $.379 \times .8 = d$

F $56 \times 1.678 = m$

G $8.725 \times 2.6 = k$

H $1.007 \times 11 = n$

I $.0009 \times 32 = b$

J $1.4 \times .009 = y$

Test 16

Find each quotient below to hundredths.

A $1240 \div 6.3 = x$

B $.213 \div .05 = r$

C $4.43 \div 7 = d$

D $2 \div 40 = t$

E $32.6 \div 5.1 = m$

F $.0986 \div 1.43 = a$

G $9.13 \div .208 = k$

H $4 \div .0009 = n$

Test 17

A $w + 4.3842 = 7.1160$

B $.075 \times 4.8 = t$

C $.0086 - .0079 = n$

D $s - 1.95 = 8.07$

E $8 \times .75 = r$

F $4.361 - 2.794 = d$

G $.9320 - x = .0659$

H $11.06 + r = 14.91$

I $g + .0369 = 1.0000$

J $c - 24.6 = 51.3$

Test 18

A 3 is $\frac{2}{3}$ of ▭.

B 48 is $\frac{4}{5}$ of ▭.

C $\frac{1}{3}$ of 124 is ▭.

D $20\frac{1}{2}$ is what fraction of 82?

E $\frac{3}{8}$ of 136 is ▭.

F $1\frac{1}{3}$ is what fraction of 3?

G $\frac{4}{5}$ of 120 is ▭.

H $20\frac{2}{3}$ is $\frac{1}{3}$ of ▭.

I $18\frac{2}{3}$ is what fraction of 21?

Test 19

A 45% of 80 is ▭.

B 35 is 14% of ▭.

C 164 is ▭% of 82.

D 325 is ▭% of 312.5.

E 80% of 420 is ▭.

F 180 is 25% of ▭.

G 45 is ▭% of 300.

H 24.6 is 123% of ▭.

I 130% of 60 is ▭.

J 14 is ▭% of 280.

Tables of measures

imperial liquid measures*

2 tablespoonfuls =
1 fluid ounce
8 fluid ounces = 1 cup
2½ cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
8 pints = 1 gallon

*Canada uses imperial liquid measures.
Canada: 1 pt. = 20 fl. oz.
United States: 1 pt. = 16 fl. oz.

dry measures

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel
32 quarts = 1 bushel

clock measures

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

calendar measures

7 days = 1 week
52 weeks = 1 year
12 months = 1 year
365 days = 1 year
366 days = 1 leap year

length

12 inches = 1 foot
36 inches = 1 yard
3 feet = 1 yard
16½ feet = 1 rod
5280 feet = 1 mile
1760 yards = 1 mile
320 rods = 1 mile

weight

16 ounces = 1 pound
100 pounds =
1 hundredweight
2000 pounds = 1 ton

square measures

144 square inches =
1 square foot
9 square feet =
1 square yard
160 square rods = 1 acre
43,560 square feet = 1 acre

cubic measures

1728 cubic inches =
1 cubic foot
27 cubic feet =
1 cubic yard

metric measures

1 gram = .035 ounce
1 metre = 39.37 inches
1 kilogram = 2.2 pounds
1 litre = .88 quarts
1 kilometre = ⅝ mile
1000 metres = 1 kilometre

Abbreviations

acre or acres—A.
bushel or bushels—bu.
cubic—cu.
cup or cups—c.
day or days—da.
dozen or dozens—doz.
fluid ounce or
fluid ounces—fl. oz.
foot or feet—ft.
gallon or gallons—gal.
gram or grams—g.
hour or hours—hr.
hundredweight or
hundredweights—cwt.
inch or inches—in.
kilogram or kilograms—kg.
kilometre or kilometres—km.
litre or litres—l.
metre or metres—m.
mile or miles—mi.
minute or minutes—min.
month or months—mo.
ounce or ounces—oz.
peck or pecks—pk.
pint or pints—pt.
pound or pounds—lb.
quart or quarts—qt.
rod or rods—rd.
second or seconds—sec.
square—sq.
tablespoonful or
tablespoonfuls—tbs.
ton or tons—T.
week or weeks—wk.
yard or yards—yd.
year or years—yr.

Table of contents

This chart shows the organization of the arithmetic content of this book and indicates the kinds of social interests included in the pictures and problems. The book has been organized to keep the subject matter of arithmetic the centre of the child's interest. Social material of interest to the child is contained in the pictures and verbal problems. The child's own social interests and experiences are brought to the book through these pictures and problems.

pages	skills and concepts	social interest
3-5	Awareness of geometric shapes	Home environment
6-9	Identification of prism, cylinder, pyramid, cone, and sphere	Mathematical interest
10-12	Dimensions and measurement of geometric shapes	Mathematical interest
13-16	Concept of volume	Mathematical interest
17	Vocabulary of dimensions	Mathematical interest
19	Rounding numbers	Mathematical interest
20-23	Division: 2-figure and 3-figure divisors	Mathematical interest
26-30	Problem solving: multiple-step problems; averages	Home activities
31	Associative law	Mathematical interest
32-37	Rate: meaning of; expressing	Home activities
38-42	Comparison: meaning of; expressing comparisons	Hobbies
43-45	Fractions: meaning of; expressing fractions	Mathematical interest
46	Distinguishing among rates, comparisons, and fractions	Mathematical interest
49-51	Problem solving: using a variety of symbols in multiple-step problems	Buying activities
52-55	Reduction of ratios and fraction numerals	Mathematical interest
56-58	Finding the unknown numeral in one term of a ratio or fraction numeral when an equal ratio or fraction numeral is known	Mathematical interest
59-62	Using the remainder to make a fraction numeral in the quotient	Measuring activities
63-65	Developing judgment about the use of the remainder in division	Home activities
66-69	Fractions: common denominator	Mathematical interest
70-71	Fractions: reduction of improper fraction numerals	Mathematical interest
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80-85	Fractions: addition involving mixed numbers	Home activities
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93-97	Fractions: subtraction involving mixed numbers	Home activities
104-106	Introduction to signs of inequality	Mathematical interest
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110	Mental computation: multiplication	Mathematical interest
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112-116	Problem solving: using ratios in equations to solve rate problems	Gardening activities
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125	Distinguishing between comparison of two groups and part of a group with the whole group	Mathematical interest
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162-164	Division: traditional method	Mathematical interest
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195	Fractions: idea of reciprocal	Mathematical interest
196-197	Fractions: methods of indicating division	Home activities
198-200	Fractions: division	Mathematical interest
201	Fractions: generalizations about 0 and 1	Mathematical interest
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265	Number system: positive and negative numbers	Mathematical interest
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Headings

Learning how
Lessons using the four-step teaching method to develop or review computational skills

Moving forward
Lessons developing other new material, including number system, fractions, ratios, and measurement

Exploring problems
Lessons developing specific problem-solving techniques

Using arithmetic
Maintenance and practice in solving verbal problems

Keeping skilful
Maintenance and practice in computation, measurement, number system, and solutions of equations

Checking up
Tests of various types: inventory, achievement, diagnostic (End-of-block), and problem-solving

Thinking straight
Lessons involving special aspects of quantitative thinking

Looking back
Lessons involving reteaching or review of previously taught skills

Side trip
Lessons on various topics related to the arithmetic program: traditional method of division, miscellaneous information about numerals and signs, use of the caret in division involving decimal fraction numerals, operations with compound denominate measures, and metric measures

Summary of mathematical content

Numerals and the number system
- Rounding off numbers (reteaching)
- Common fraction numerals (reteaching)
- Decimal fraction numerals (reteaching and extension to ten-thousandths)
- Positive and negative numbers
- Regrouping as required in carrying and borrowing (involving decimal fraction numerals)
- Number line—naming points; sets of names

Division of whole numbers
- Division by two-figure and three-figure divisors (reteaching)
- Estimating partial quotients
- Traditional method of division
- Mental computation (one-figure divisors)

Fractions (involving common fraction numerals)
- Concept of fractions (reteaching and extension)
- Reduction
- Comparison of fractions
- Addition of proper fractions and mixed numbers
- Subtraction of proper fractions and mixed numbers
- Multiplication of proper fractions and mixed numbers
- Division of proper fractions and mixed numbers
- Using the remainder in division to make a fraction numeral in the quotient

Fractions (involving decimal fraction numerals)
- Extension of the number system to include decimal fraction numerals to ten-thousandths
- Addition
- Subtraction
- Multiplication
- Division
- Expressing the remainder in division
- Reduction of common fraction numerals to decimal fraction numerals

Per cent
- (See Ratios)

Ratios
- (Also see Problem solving)
- Ratios as expressions of rate (reteaching)
- Ratios as expressions of comparison (reteaching)
- Per cents as ratios expressing comparison
- Reduction of ratios (reteaching)

Measurement
- Measures of area—square inch, square foot, square yard, square rod, acre (reteaching)
- Measures of volume—cubic inch, cubic foot, cubic yard
- Reduction of measures (reteaching and extension)
- Precision in measurement
- Metric measures—gram, metre, litre, kilogram, kilometre centimetre

Graphs
- Graphs representing rates
- Statistical graphs

Geometry
- Parallel lines and right angles (reteaching and extension)
- Recognition of polygons and circle (reteaching)
- Recognition of cylinder, various prisms, cone, pyramid, sphere, half sphere
- Concept of area (reteaching and extension)
- Concept of volume
- Measurement of area—rectangles and other parallelograms (reteaching and extension)
- Measurement of volume—rectangular prisms
- Perimeters
- Surface area of rectangular prisms

Problem solving
- Review and reteaching of previously taught multiple-step problems
- Using a variety of symbols as place holders in equations for multiple-step problems
- Using ratios to solve problems involving rate
- Using ratios to solve problems involving comparison
- Using ratios to solve problems involving comparison expressed in terms of "times as many as"
- Using ratios to solve comparison problems involving per cent
- Using the ratio test to solve equations for rate and comparison problems
- Using fraction numerals (common and decimal) as terms of ratios in equations for rate and comparison problems
- Making abstract statements for comparison problems involving common fraction numerals, decimal fraction numerals, and per cents
- Comparison problems solved by addition and subtraction

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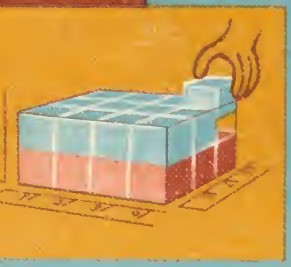
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